The Behavioral Firm and Its Internal Game: Evolutionary Dynamics of Decision Making

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The Behavioral Firm and its Internal Game: Evolutionary Dynamics of Decision Making

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Abstract

In this paper the firm is analyzed and modeled as a set of different subcoalitions (agents) each with their own objectives. It examines how the goals can be conflicting and in turn how this influences the payoff structure of the subcoalitions given that they follow 'simple' decision rules, i.e. rules of thumb. This implies that the subcoalitions act in a boundedly rational way. To see how these decision making procedures evolve we make use of an (evolutionary) dynamic game theoretical framework. Consequently, the main aim is to address the issue of modeling the dynamic and adaptive nature of the subcoalitions.

Key words: Dynamic game, theory of the firm, decision making.

Contents

List of Symbols

t	time
$\dot{x}$	time derivative of x
$f'(x)$	first derivative of $f(x)$
$f''(x)$	second derivative of $f(x)$
π	profit
$\tilde{\pi}$	aspiration level of profits
r	revenues
$\bar{c}$	total production cost
c	unit production cost
$\tilde{c}$	aspiration level of unit production cost
q	number of production
ϵ	price elasticity of demand
w	level of sales
$\tilde{w}$	aspiration level of sales
z_q	index production slack
z_w	index sales slack
x_q	percentage contribution of production slack in total cost
x_w	level of sales promotion in terms of sales slack
θ	fixed cost
η	variable cost
u	control variable
μ	$x-$switching curve
ν	$p-$switching curve
s	aspiration level
$\mathcal{P}$	payoff function
b	parameter
ξ	parameter
γ	parameter
ϕ	parameter
λ	parameter
α	parameter
β	parameter
δ	parameter

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1 Introduction

The neoclassical theory of the firm analyses the decisions of the firm under a varying market structure with a focus on output, price and profits. In the elementary neoclassical models the firm is supposed to maximize a profit function subject to a number of constraints. From the first order conditions, output, price and profits can be calculated.

The model is a simplified representation of the real world. In the neoclassical theory of the firm the major simplifications are the following:

- The firm is supposed to have one center that fully coordinates all decisions. This implies a holistic conception of the firm.
- The coordination center has perfect information on demand and cost conditions.
- The firm is at any moment fully X-efficient; there is no waste and no slack.
- The firm is a profit maximizer. It has the information and the capacity to identify and attain the maximum position due to the three properties mentioned above.
- Because the firm is a maximizer and has perfect information it realizes its equilibrium position immediately. In other words, the time dimension can be neglected. The static analysis provides the equilibrium position.

The neoclassical theory of monopoly that will be presented in Section 2 clearly shows all the five features. The theory has been criticized for going too far in its simplifications. Very articulate in their criticism have been [Cyert, March 1963]. More important, next to being critical they have been constructive by developing a *behavioral* theory of the firm. The assumptions about the firm diverge on all points from the neoclassical ones:

- The modern large firm consists of a number of groups, e.g. departments like production, sales and a central management, which have to cooperate in some way. However, each group has its own objectives. They are not fully coordinated by the central management. This implies a pluralistic conception of the firm.
- Information is imperfect and scattered among the participants in the firm.

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- There exists slack. That is, the participants are each not fully X-efficient at any moment of time. How large the amount of slack is depends on the pressure on the group to improve performance.
- Due to above three properties of the firm, there cannot be maximization of one single objective function of the management. Instead, each group within the firm has its own aspiration level. Instead of maximizing behavior the firm shows satisficing behavior. That is to say, efforts are made to satisfy the aspiration levels. In case of conflicts between the various objectives and aspiration, reduction of slack may help to solve the dilemma.
- The various groups interact with each other and so the decision of one group can depend on the decision of another group. Decision making goes in a stepwise iterative way. The groups more or less search to find their 'local optimum'. The analysis is dynamic.

Cyert and March [1963] have developed a framework for analyzing decision making within the firm by using computer simulation. The main problem with using such a technique is that is very difficult to make generalizations. In this paper an effort will be made to develop an analytical framework in order to determine the long run behavior of a system of departments within the firm based on the assumptions of the behavioral theory. That is, decision making is boundedly rational and in a stepwise manner. A same line of approach can be found in e.g. [Kryazhimskii, Tarasyev 1998] and [Kryazhimskii, Nentjes, Shibayev, Tarasyev 1998]. The first research question is to determine whether equilibria exist and given the existence of an equilibrium to analyze the properties of that equilibrium compared to the equilibrium predicted by the neoclassical theory of the firm. The second research question is how relevant variables like costs, price, sales and profits evolve over time if the the firm is not in equilibrium. In this way it can also be assessed to what extend the equilibria (if there are any) are stable or unstable. As a first step we try to answer this question for a monopoly.

The paper is organized as follows. In section 2 the basic economic model of monopoly will be described and will be functioning as a reference point for the analysis. Section 3 discusses the behavioral model of the firm. Then in section 4 the model will be transformed into a continuous time setting. Subsequently, a mathematical analysis is applied in section 5. The paper concludes with conclusions and discussion in section 6.

2 The Monopoly Case

In this section the neoclassical theory of a monopoly firm will be presented. This will function as a reference point for the model we will develop in section 2 and 3 in order to determine whether there are actual differences between the final outcome of our model and the monopoly equilibrium of neoclassical economic theory.

In a monopoly the firm basically takes decisions regarding the product price p and the quantity of production q. Assume that the firm faces an inverse demand function $p(q)$ where $p'(q) < 0$. The revenues $r(q)$ of the firm are $p(q)q$. The total cost function $c(q)$ is convex. It should be noted that in neoclassical theory $c(q)$ is the efficient cost curve. For every unit of output costs cannot be lower than defined by the cost function. The inverse demand function $p(q)$ defined the maximum price that can be fetched for any given quantity. The firm wants to maximize profit π which equals:

$$\pi = r(q) - c(q) = p(q)q - c(q). \tag{2.1}$$

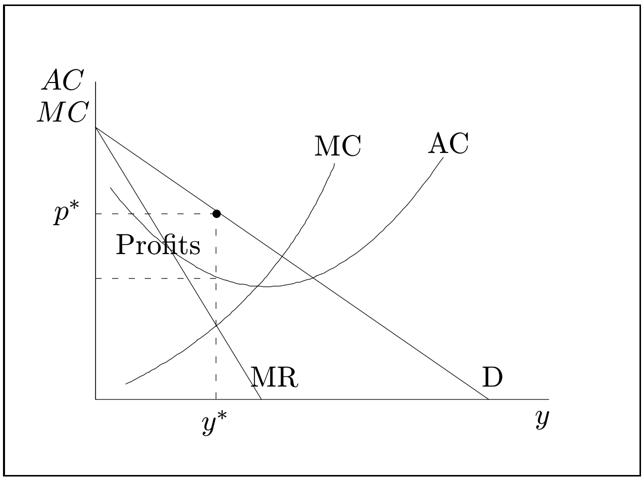

Figure 1: Profit maximizing monopoly firm.

The first order condition for this maximization problem is that marginal revenue dr/dq equals marginal cost dc/dq :

$$\frac{dp}{dq}q + p(q) = \frac{dc}{dq}. \tag{2.2}$$

One can also find the solution by solving:

$$p(q)\left(1 + \frac{1}{\epsilon(q)}\right) = \frac{dc}{dq}, \tag{2.3}$$

where the left-hand-side of (2.3) represents marginal revenue and

$$\epsilon(q) = \frac{p}{q}\frac{dq}{dp} < 0. \tag{2.4}$$

is the price elasticity of demand. Solving this equality results in q^*. Then subsituting q^* into the inverse demand function generates the equilibrium price p^*. Figure 1 shows the situation for the monopoly firm graphically.

3 A Behavioral Firm Model

In this section we will develop a basic behavioral model of the firm based on the work of Cyert and March. As noted before, they introduced a pluralistic conception of the firm; it is a coalition of departments which cooperate, but each also pursuing its own objectives. First, let's discuss in a qualitative way how the different departments that constitute a fictitious firm are related to each other.

Assume that a large production firm consists of three departments (subcoalitions), viz. a production department (PRD), a sales department (SLD) and a central management department (CMD). For notational reasons we define these departments as player 1, 2, and 3, respectively. Each department k has its own goal $\tilde{\ell}_k$ and can make its own decisions up to a certain autonomous level. Furthermore, in order to keep the model in first instance tractable, we presume that each department aims at one goal and has one instrument available to reach its goal.

The CMD is aiming for a profit goal, PRD for a production cost goal and the SLD for a sales goal. The goals of these departments reflect the aspiration level of the department. The control variables determining the value of the goals are the decisions on price, production cost and sales effort. Furthermore, we introduce the idea of slack. Slack is the difference between the actual results and the result that is achieved if the department is fully efficient. Slack, therefore, represents X-inefficiency. We define slack in the PRD as the difference between the actual and minimum production cost given the level of output and slack in the SLD as the difference between the actual level of sales and the maximum level of sales that can be achieved at a given price level. Slack in the PRD and SLD means that with the same inputs the departments might achieve a better result by, for example, working harder, being more inventive etc.

Usually the aspiration levels of the PRD ans SLD include an amount of slack: production cost are accepted that are higher than a level that could be achieved with optimal planning, cost-accounting and monitoring. Sales are accepted that are lower than could be realized with optimal distribution effort. Slacks only induce a reaction from the side of the PRD and SLD if actual levels (of costs or sales) differ from the aspiration levels. If actual results are worse than the aspiration levels the department is under pressure to reduce its slack; if not under pressure slacks tend to increase.

In the long run the aspiration levels can change. If for a long period of time actual costs have been higher/lower than the aspiration level this is going to be viewed as the 'normal' situation and subsequently the aspiration level of cost increases or decreases. This implies that slacks included in the aspiration levels increase or decrease.

The CMD is supposed to coordinate the activities of the PRD, SLD and itself. However, information on slack in the departments is incomplete. In the model proposed in this paper the CMD can only affect profits directly by changing the price[1] The impact of CMD on PRD and SLD is only indirect by exercising pressure if actual cost and sales are below the corresponding aspiration levels and by changing the departments goals in the long run.

This paper is a first effort that tries to generalize the behavioral theory of the firm by translating it into a mathematical model. Several simplifications are necessary to make this feasible, but without loss of the essential ideas enumerated in section 1. The major simplifying assumptions are:

- Restriction to three departments (PRD, SLD, CMD) and later on to two departments (PRD, CMD);
- One objective and one control variable for each department;
- Fixed aspiration levels.

For each department $k = 1, \ldots, K$ we will examine the decision making procedure that it follows and hence how these decisions influence the associated payoffs. The modeling is based upon an aspirations framework which will first be discussed in subsection 3.1. Then the payoff functionals of the PRD, SLD and CMD will be outlined in the subsections 3.2, 3.3 and 3.4 respectively. Subsection 3.5 deals with the payoff functionals in relation to the external environment. Finally, 3.6 gives the monopoly outcome when using these specific functional forms of payoff functionals.

3.1 An Aspirations Framework

The modeling is based on a number of populations (departments) $k = 1, \ldots, K$ that can apply a finite number of strategies or actions $n = 1, \ldots, N$. The populations change

[1] But there is also imperfect information on how this will affect sales and total revenue.

their actions based upon a threshold value of realized payoffs just as in Cyert and March [1963]. That is, a strategy change is based upon aspirations and satisficing behavior. Models of satisficing behavior in economics have been introduced by [Simon 1955, 1957, 1959]. Economic applications of satisficing behavior are [Winter 1971] and [Nelson, Winter 1982]. Other models incorporating aspirations and satisficing can be found in [Bendor, Mookherjee, Ray 1994] and [Gilboa, Schmeidler, 1993, 1995, 1996].

Assume that each period t population k is aiming for an aspiration goal $\tilde{\ell}_k(t)$. In the evolutionary game literature that incorporates aspirations[2] players change their action when realized payoff is larger or less that the populations' or agents' aspiration level (cf. [Börgers, Sarin 1995, 1997] [Erev, Roth 1996] [Karandikar, Mookherjee, Ray, Vega-Redondo 1998]). In the firm model described below decision making is also in this line of research.

Firstly, the departments check if $\ell_k(t) = \tilde{\ell}_k(t)$. In general there will be a strategy switch if $\ell_k(t) \neq \tilde{\ell}_k(t)$. But as will see in the next section in the firm model, in specific situations a player might also take the goals of other players into account in order to make a justified 'economic' decision. Secondly, just as in the model of [Karandikar, Mookherjee, Ray, Vega-Redondo 1998] at every period t the population updates their aspiration level according to the convex function:

$$\tilde{\ell}_k(t+1) = \lambda_k \tilde{\ell}_k(t) + (1-\lambda_k)\ell_k(t), \tag{3.1}$$

where $\lambda_k \in [0,1]$.[3] The economic interpretation of (2.1) is that the department will decrease its aspiration level if it does not reach it. On the other hand, if the aspiration level is met then it will be increased. [Cyert, March 1963] assume that the adjustment of aspiration levels goes at a lower speed than compared to adjustment of control variables. This means values of λ_k close to 1. In this paper we assume a constant aspiration level: $\lambda_k = 1$.

3.2 Payoff Functionals Production Department

Cyert and March argue that in the steady state the level of production $q(t)$ is directly related to the sales goal one period earlier $\tilde{w}(t-1)$. However, because a steady state position does not often occur this might not be realistic to assume. Therefore, they argue that production depends also on changes in inventory of the final product and a so-called production-smoothing goal. However, in this paper we assume that the level of production equals the level of sales and therefore we can exclude the goals of inventory and production-smoothing.[4] We concentrate on average production costs $\tilde{c}$ as the objective of the PRD.[5] It is the responsibility of the PRD to prevent that production cost is too high or increases too much. The PRD has an aspiration level with regard to its cost per unit of output. If actual costs are below the aspiration level, costs tend to increase further due to an increase of slack. The difference between the actual and aspired cost level is defined as production slack z_q.

Given the goal and control variable the decision procedure is now defined in the following way. Firstly, at time $t = 1, 2, \ldots$ the PRD checks whether $\tilde{c}(t) = c(t)$. Given the outcome of this comparison a strategy switch in terms of an adjustment of z_q might occur.

[2] In this kind of literature, models with aspirations are also referred to as 'stimulus-response' models.

[3] The equation is one of the simplest forms of a so-called 'Mann's process'. More extended and sophisticated processes are also developed and can be found in e.g. Vasin and Ageev [1995].

[4] This is also due to the 'one goal-one instrument' setting. In a later version when we try to develop a 'multiple goal-multiple instrument' setting also inventory and production-smoothing might be included.

[5] Production costs are only a part of total costs $c(t) = \sum_{k=1}^{K} c_k(t)$. But for simplicity we assume that costs of production are the only costs.

The decision rule of the PRD follows is:

$$z_q(t+1) = \begin{cases} (1+\xi_{z_q})z_q(t) & \text{if } c(t) < \tilde{c}(t) \\ (1-\xi_{z_q})z_q(t) & \text{if } c(t) > \tilde{c}(t) \\ z_q(t) & \text{otherwise.} \end{cases} \tag{3.2}$$

where $0 < \xi_{z_q} \leq 1$. The economic interpretation is as follows. If the actual costs of production is below its aspiration level the PRD is not under pressure to improve its performance which, in turn, implies an increase of slack. Subsequently, costs will increase. If actual costs are higher than compared to its aspiration level, there is pressure to try to reduce costs. The PRD will look for possibilities to improve the cost position and save on inputs (given the level of production). In the behavioral theory adjustment of slack is more a short run process. That is, it is faster than adjustment of aspirations.

Finally, assuming a variable aspiration level, $\tilde{c}$ will be adjusted according to:

$$\tilde{c}(t+1) = \lambda_1\tilde{c}(t) + (1-\lambda_1)c(t), \tag{3.3}$$

where $0 \leq \lambda_1 \leq 1$. But in this paper we keep the aspiration level of profits fixed, i.e. $\lambda_1 = 1$. This concludes the decision procedure of the PRD.

3.3 Payoff Functionals Sales Department

Assume that the SLD is aiming for a sales goal $\tilde{w}(t)$, and that the control variable is sales slack z_w. The basic decision the SLD takes is an adjustment of z_w or not and is based on a comparision between $\tilde{w}(t)$ and the actual level of sales $w(t)$. At this stage it is important to realize that a potential conflict between the SLD and the CMD might exist. For example, if the sales goal is not met the SLD might request the CMD for a price cut. However, in turn, a price cut might worsen the position of the CMD in terms of the profit goal it pursues.

Every period t sales is calculated and it is checked whether $w_t = \tilde{w}_t$. Given the outcome the SLD applies one of the following three strategies:

$$z_w(t+1) = \begin{cases} (1+\xi_{z_w})z_w(t) & \text{if } w(t) > \tilde{w}(t) \\ (1-\xi_{z_w})z_w(t) & \text{if } w(t) < \tilde{w}(t) \\ z_w(t) & \text{otherwise.} \end{cases} \tag{3.4}$$

where $0 < \xi_{z_w} \leq 1$. Equation (3.4) states that if actual sales are above the aspiration level the selling efforts (e.g. sales promotion) and other inputs are lower or are less effective. Such an increase of slack in the form of an increase of 'on-the-job-leisure' implies that given the product price the volume of sales will decrease although there have not been exogenous changes in demand.[6]

Finally, the aspiration level of sales is adjusted:

$$\tilde{w}(t+1) = \lambda_2\tilde{w}(t) + (1-\lambda_2)w(t). \tag{3.5}$$

where $0 \leq \lambda_2 \leq 1$. Again we assume that $\lambda_2 = 1$.

[6]An increase of slack implies a decrease of productivity. For the PRD it is specified in terms of higher production cost for a given product; for the SLD as lower sales at a given level of sales cost. This implies that sales cost are defined as a component of total fixed cost.

3.4 Payoff Functionals Central Management

We assume that the price decision of the firm is taken by the CMD and is primarily determined by profit π. In general also sales performance, manufacturing costs, past price changes and the price behavior of competitors are important elements in the price decision making. But manufacturing costs and sales performance are already included in the decision making procedure of the PRD and SLD respectively. In this paper we model the decision making of the CMD of a firm that is a monopolist.

The CMD starts its decision tree every time period t by calculating manufacturing costs and profit. Given the firm's performance in terms of $p(t), c(t)$ and $w(t)$, the CMD calculates the actual profit level:

$$\pi(t) = [p(t) - c(t)]w(t), \tag{3.6}$$

where $p(t)$ and $c(t)$ represent the price and cost per unit of output. Then the CMD checks if realized profits $\pi(t)$ equal the aspiration level of profits $\tilde{\pi}(t)$. Subsequently, it determines $p(t+1)$:

$$p(t+1) = \begin{cases} (1+\xi_p)p(t) & \text{if } \pi(t) < \tilde{\pi}(t) \wedge w(t) > \tilde{w}(t) \\ (1-\xi_p)p(t) & \text{if } \pi(t) \neq \tilde{\pi}(t) \wedge w(t) < \tilde{w}(t) \\ p(t) & \text{otherwise} \end{cases} \tag{3.7}$$

where $0 < \xi_p \leq 1$. Equation (3.7) expresses a type of behavior of the CMD that, given its implerfect information, seems plausible. According to the first strategy the price will be increased if actual profits are below the aspiration level and a decrease is acceptable since sales are presently above the aspiration level.[7] Strategy 2 of decreasing price will be applied in two situations. First if $\pi(t) > \tilde{\pi}(t)$ and $w(t) < \tilde{w}(t)$. The CMD gives in to pressure from the SLD to take action to restore sales. It accepts the risk that lowering of the profit margin might have a negative impact on profits since $\pi(t) > \tilde{\pi}(t)$.[8] Second, a price decrease will also be accepted if presently $\pi(t) > \tilde{\pi}(t)$ and $w(t) < \tilde{w}(t)$. Decreasing the price is expected to increase sales. The impact on profits is uncertain. Profits might increase or might as well decrease. Strategy 3 - price is kept constant - will be followed if $\pi(t) \geq \tilde{\pi}(t)$ and $w(t) \geq \tilde{w}(t)$ Since the aspiration levels are realized there is no pressure from departments to experiment with the price.

Finally the CMD revises the aspiration level of profits:

$$\tilde{\pi}(t+1) = \lambda_3\tilde{\pi}(t) + (1-\lambda_3)\pi(t). \tag{3.8}$$

where $0 \leq \lambda_3 \leq 1$. Here it is assumed that $\lambda_3 = 1$.

3.5 Functional Form and the External Environment

In this section the aim is to show how some functions relate to the external environment in which the firm operates. Moreover, the purpose is to describe the functions in more detail.

The firm is linked to the external environment (market) by means of the demand function. Assume that the demand function in case of a perfectly efficient SLD, i.e. slack is zero, can be described as:

$$q(t) = w(t) = \alpha p(t)^{\beta} \qquad (\alpha > 0, \beta < -1). \tag{3.9}$$

[7] The strategy will be only succesful in raising profits if initially marginal revenue is below marginal cost ($MR < MC$). Because of imperfect information the CMD does not know whether this condition is fulfilled.

[8] Profits will increase if intially $MR > MC$.

Including slack, the demand function becomes:

$$w(t) = \alpha p(t)^{\beta} z_w(t) \tag{3.10}$$

Next specifying $z_w(t)$ we can rewrite (3.10):

$$w(t) = \alpha p(t)^{\beta}[1 - x_w(t)] \qquad (\beta < 0), \tag{3.11}$$

where $x_w(t)$ can be interpreted as an indicator of slack in the sales department. The higher x_w the lower is the level of effort of the department. Given the price this will affect since sales promotion is less effective.

Furthermore, assume that total cost $\bar{c}$ at time t are the sum of fixed and variable cost:

$$\bar{c}(t) = \theta + \eta q(t) \qquad (\theta, \eta > 0), \tag{3.12}$$

where θ are the fixed cost and η represents variable cost. Equation (3.12) are the efficient costs, i.e., the cost one necessarily needs to make in order to produce a certain amount of output. That is, there is no slack component. Including slack results in

$$\bar{c}(t) = [\theta + \eta q(t)] z_q(t), \tag{3.13}$$

Assuming $z_q = (1 + x_q)$, where $x_q \in [0, 1]$ is the percentage contribution of slack to total cost. Then rewriting (3.13) yields the final expression for the total cost function:

$$\bar{c}(t) = [\theta + \eta q(t)][1 + x_q(t)]. \tag{3.14}$$

Dividing by q gives the unit cost:

$$c(t) = \left(\frac{\theta}{q(t)} + \eta\right)[1 + x_q(t)] \tag{3.15}$$

Now given the level of $w(t), c(t)$ and $p(t)$ we can calculate profit as given by (3.6).

4 Continuous Time Framework

In this section we analytically derive an optimal control model of the firm as described in the previous section. The control variables, i.e., the instruments a department can apply in order to reach or maintain its goal, are z_q, z_w and p. The decision rules that can be adopted by the departments can evolve in three different directions. Allowing the control variables to move more free in a continuous setting provides a framework in which the model becomes more attractive in the sense of more flexibility. We will show how the discrete decision rule applied by the PRD can be transformed into a continuous setting.

Starting point is (3.2). Subtraction results in:

$$z_q(t+1) - z_q(t) = \begin{cases} \xi_{z_q} z_q(t) & \text{if } c_1(t) < \tilde{c}_1(t) \\ -\xi_{z_q} z_q(t) & \text{if } c_1(t) > \tilde{c}_1(t) \\ z_q(t) & \text{otherwise.} \end{cases} \tag{4.1}$$

Rewriting gives:

$$z_q(t+1) - z_q(t) = \xi_{z_q} z_q(t) u_q(t), \tag{4.2}$$

where

$$u_q(t) = \begin{cases} 1 & \text{if } c_1(t) < \tilde{c}_1(t) \\ -1 & \text{if } c_1(t) > \tilde{c}_1(t) \\ 0 & \text{otherwise.} \end{cases} \tag{4.3}$$

The first derivative of $z_q(t)$ with respect to time is now:

$$\dot{z}_q = \lim_{\Delta \to 0} \frac{z_q(t+\Delta) - z_q(t)}{\Delta} = \xi_{z_q} z_q(t) u_q(t), \qquad u_q(t) \in [-1, 1]. \tag{4.4}$$

The same routine can be applied to the control variables z_w and p. The time derivative of these two variables then are respectively:

$$\begin{aligned} \dot{z}_w &= \xi_{z_w} z_w(t) u_w(t) \\ \dot{p} &= \xi_p p(t) u_p(t), \end{aligned} \tag{4.5}$$

where both $u_w(t)$ and $u_p(t) \in [-1, 1]$. For convenience we redefine:

$$\begin{aligned} z_q &= 1 + x_1, \\ z_w &= 1 - x_2, \\ p &= x_3, \end{aligned}$$

where $x_3 \geq 0$ and both $x_1, x_2 \in [0, 1]$ represent the fraction of production slack and sales slack. Now lets also redefine $u_q = u_1, u_w = u_2, u_p = u_3, \xi_{z_q} = \xi_1, \xi_{z_w} = \xi_2$ and $\xi_p = \xi_3$. Moreover, define $s_1 = \tilde{c}, s_2 = \tilde{w}, s_3 = \tilde{\pi}$.

$$\dot{y}_k = \xi_k y_k u_k \qquad k = 1, 2, 3. \tag{4.6}$$

In turn, we can rewrite this system into:

$$\dot{x}_k = \varphi_k(x_k) u_k \qquad k = 1, 2, 3. \tag{4.7}$$

Equation (4.7) introduces a function $\varphi(x)$ in order to allow any functional form depending on x.

Now we have derived this control system we can also reconstruct a feedback system. In order to derive this, first the goals of each department will be transformed into a continuous time framework. The rule applied to (3.2) can also be used for the general updating function (3.1). First, subtraction yields:

$$\tilde{\ell}_k(t+1) - \tilde{\ell}(t) = (1 - \lambda_k)[\ell_k(t) - \tilde{\ell}_k(t)]. \tag{4.8}$$

From this we generate:

$$\dot{\tilde{\ell}} = \lim_{\Delta \to 0} \frac{\tilde{\ell}(t+\Delta) - \tilde{\ell}(t)}{\Delta} = (1 - \lambda_k)[\ell(t) - \tilde{\ell}(t)] \tag{4.9}$$

This rule can be applied to the 'updating' function of each department. For the PRD, SLD and CMD we get respectively:

$$\begin{aligned} \dot{\tilde{c}} &= (1 - \lambda_1)[c(t) - \tilde{c}(t)], \\ \dot{\tilde{w}} &= (1 - \lambda_2)[w(t) - \tilde{w}(t)], \\ \dot{\tilde{\pi}} &= (1 - \lambda_3)[\pi(t) - \tilde{\pi}(t)]. \end{aligned} \tag{4.10}$$

In general, u_k can be defined as:

$$u_k = \text{sgn}(\ell_k - \tilde{\ell}_k) = \text{sgn}\frac{\partial \mathcal{P}_k}{\partial x_k} = \arg\max_{|u_k| \leq 1} u_k(\ell_k - \tilde{\ell}_k). \tag{4.11}$$

Now we can write

$$(\ell_k - \tilde{\ell}_k) = \frac{\partial \mathcal{P}_k}{\partial x_k} \qquad k = 1, 2, 3. \tag{4.12}$$

The right-hand-side of (4.12) is not the exact payoff but represents the marginal payoff. Subsequently, the actual payoff function of department k is then determined by:

$$\mathcal{P}_k = \int (\ell_k - \tilde{\ell}_k) = \int \frac{\partial \mathcal{P}_k}{\partial x_k} dx_k. \qquad (4.13)$$

In the next section the payoff functions of the different departments will be calculated explicitly.

5 Analysis

In this section we will apply the model presented in section 4 to analyze the 'bhavioral firm' and answer the research questions formulated in section 1. Since this is a first approach we start with a simple 2-dimensional (2 departments) model instead of a 3-dimensional (3 departments) model and assuming that aspiration levels are fixed. It is assumed that a firm consists of a PRD and a CMD. As for the adjustment of slack and profits this has the following consequences. Adjustment of slack in production cost remains as outlined previously. Slack in the SLD is eliminated since the SLD does not exist in the 2-dimensional model. However, a fixed aspiration level for sales is included.

In subsection 5.1 the model is described in detail. In subsection 5.2 the 3-dimensional model is discussed.

5.1 2-Dimensional Firm Model

As stated in section 2 the PRD its aim is a certain fixed aspiration level of unit production cost $\tilde{c}$ which is controlled by production slack x,[9] The decision rule of the PRD is exactly the same as stated in (3.2). The continuous time equivalent is (4.5). Originally, in the 3-dimensional model the decision rule of the CMD is given in (3.7). However, since we are now dealing with a 2-dimensional model, i.e. excluding the sales department, w needs to be explicitly written as a function of p. The demand function is like given in (3.9). Now set $\delta = -\beta$ then the demand function becomes like:

$$w(p) = \frac{\alpha}{p^\delta} \qquad (\delta > 1). \qquad (5.1)$$

Given this, the discrete time decision rule for the CMD can now be written as follows:

$$p(t+1) = \begin{cases} (1+\xi_p)p(t) & \text{if } \pi(t) < \tilde{\pi}(t) \wedge \frac{\alpha}{p^\delta} > \tilde{w}(t) \\ (1-\xi_p)p(t) & \text{if } \pi(t) \neq \tilde{\pi}(t) \wedge \frac{\alpha}{p^\delta} < \tilde{w}(t) \\ p(t) & \text{otherwise} \end{cases} \qquad (5.2)$$

where $0 < \xi_p \leq 1$. Thus, when profit is below the corresponding aspiration level and sales are above the aspiration level then the price will be increased. When profit and sales are too low, the price level will be decreased. The price will also decrease if profit is higher than aspired but sales are too low. When both profit and sales are above the aspiration level the price will not be changed.

Now substituting the demand function (5.1) into in the unit cost function (3.15) we get:

$$\begin{aligned} c(p,x) &= \left(\frac{\theta}{\alpha}p^\delta + \eta\right)(1+x) \\ &= (\eta + bp^\delta)(1+x) \qquad (b = \frac{\theta}{\alpha}). \end{aligned} \qquad (5.3)$$

[9] Because the SLD is omitted in this model x instead of x_q is used.

Subsequently, substituting (5.1) and (5.3) into (3.6) yields the rewritten profit function:

$$\begin{aligned}\pi(p,x) &= \frac{\alpha[p-(\eta+bp^{\delta})(1+x)]}{p^{\delta}} \\ &= \alpha[p^{1-\delta}-(1+x)\eta p^{-\delta}-(1+x)b]. \end{aligned} \tag{5.4}$$

Before making the step to the dynamics of this system, we first calculate the static equilibrium in the neoclassical sense.

5.1.1 Static Equilibrium

The aim of the neoclassical firm is to maximize profits π assuming slack x to be zero. Given the functional forms as outlined above the first derivative of π with respect to p is:

$$\begin{aligned}\frac{\partial \pi}{\partial p} &= \alpha(1-\delta)p^{-\delta}+\alpha(1+x)\delta\eta p^{-\delta-1} \\ &= \alpha p^{-\delta}[(1-\delta)+\eta\delta(1+x)p^{-1}]. \end{aligned} \tag{5.5}$$

Solving (5.4) for $\delta\pi/\delta p = 0$ gives the optimal price:

$$p^* = \frac{\eta\delta(1+x)}{\delta-1}. \tag{5.6}$$

Now

$$\frac{\partial \pi}{\partial p} > 0 \iff p \leq p^*.$$

So, p^* is the unique maximizer to π. Substituting this into (5.1) yields production q^*:

$$q^* = \alpha\left(\frac{\eta\delta(1+x)}{\delta-1}\right)^{-\delta}. \tag{5.7}$$

For $x = 0$, i.e. slack is zero, the neoclassical equilibrium $p^*_{x=0}$ is obtained which equals:

$$p^*_{x=0} = \frac{\eta\delta}{\delta-1}$$

In order to have positive profits in the neoclassical sense we must have:

$$\pi(p^*_{x=0},0) > 0 \iff \left[\left(\frac{\alpha\delta}{\delta-1}\right)^{1-\delta} - \eta\left(\frac{\alpha\delta}{\delta-1}\right)^{-\delta} - b\right] > 0 \tag{5.8}$$

Then from this condition:

$$b < \left[\left(\frac{\alpha\delta}{\delta-1}\right)^{1-\delta} - \eta\left(\frac{\alpha\delta}{\delta-1}\right)^{-\delta}\right]. \tag{5.9}$$

5.1.2 Dynamics

Recall that aspiration levels $\tilde{c}$ and $\tilde{\pi}$ are fixed. As argued, the dynamics can be represented by (4.7). Applying this to slack x and price p the dynamics of these variables are respectively:

$$\begin{aligned}\dot{x} &= \varphi(x)\text{sgn}[\tilde{c}-c(p,x)], \\ \dot{p} &= \psi(p)g(p,x), \end{aligned} \tag{5.10}$$

where $\varphi(x), \psi(p) > 0$ and are differentiable. Moreover, $\varphi(0) = 0, \psi(0) = 0$. The function $g(p, x)$ is:

$$g(p,x) = \begin{cases} 1 & \text{if } \pi(p,x) < \tilde{\pi} \wedge p < \bar{p} \\ -1 & \text{if } \pi(p,x) \neq \tilde{\pi} \wedge p > \bar{p} \\ 0 & \text{otherwise} \end{cases} \tag{5.11}$$

where $\bar{p} = (\frac{\alpha}{w})^{\frac{1}{\delta}}$. Now assume that $x_0 = x(0) > 0$ and $p_0 = p(0) > 0$ then $x(t), p(t) > 0$ for all $t \geq 0$. The solution to (5.10) is understood in the Filippov's sense due to the discontinuous right-hand-side [Fillipov 1988].

To analyze the system of differential equations we make use of vector fields. To do so, so-called switching curves need to be derived. Switching curves are the curves where one of the coordinates (variables) changes its direction. In our case we thus derive two switching curves: one that determines the direction of x and one the direction of p.

Firstly, we analyze the dynamics of x. We have:

$$\begin{aligned} \dot{x} > 0 &\iff c(p,x) < \tilde{c} \\ &\iff (\eta + bp^{\delta})(1+x) < \tilde{c}. \end{aligned}$$

From this the $x-$switching curve $\mu(p)$ can be derived:

$$\mu(p) = \frac{\tilde{c}}{\eta + bp^{\delta}} - 1, \tag{5.12}$$

where $\mu(p) > 0.$ and $\lim_{p\to\infty} \mu(p) = -1$. The switching curve shows in which direction production slack x moves depending on the inital point it is relative to the curve. Above this curve $\dot{x} < 0$, below the curve $\dot{x} > 0$. The economic intuition behind this is that when actual production cost are above the aspiration level (above the $x-$switching curve), slack is reduced and hence actual production cost gradually declines. The reverse situation applies to a point under the $x-$switching curve. Next determine:

$$\frac{d\mu(p)}{dp} = \tilde{c}\left(-\frac{1}{(\eta + bp^{\delta})^2} b\delta p^{\delta-1}\right) > 0. \tag{5.13}$$

and

$$\begin{aligned} \frac{d^2\mu(p)}{dp^2} &= \tilde{c}\left(\frac{2(b\delta p^{\delta-1})^2}{(\eta+bp^{\delta})^3} - \frac{b\delta(\delta-1)p^{\delta-2}}{(\eta+bp^{\delta})^2}\right) \\ &= \frac{\tilde{c}b\delta p^{\delta-2}}{(\eta+bp^{\delta})^2}\left(\delta + 1 - \frac{2\eta\delta}{\eta+bp^{\delta}}\right). \end{aligned} \tag{5.14}$$

From (5.10) the point where $x = 0$ can be determined which equals:[10]

$$p_{x_0} = \left(\frac{\tilde{c} - \eta}{b}\right)^{\frac{1}{\delta}}. \tag{5.15}$$

Secondly, the dynamics of p. From (5.11) we see that:

$$\begin{aligned} \dot{p} > 0 &\iff \pi(p,x) < \tilde{\pi} \wedge p < \bar{p} \\ &\iff p^{1-\delta} - (1+x)\eta p^{-\delta} - (1+x)b < \frac{\tilde{\pi}}{\alpha} \wedge p < \bar{p}. \end{aligned}$$

[10] The point where the function $\mu(p)$ becomes convex can also be determined. This is where $\mu''(p) > 0$. This is the case if and only if $\eta + bp^{\delta} > \frac{2}{\delta+1} \Leftrightarrow p^{\delta} > \frac{\eta}{b}\left(\frac{2\delta}{\delta+1} - 1\right) \Leftrightarrow p > \left[\frac{\eta}{b}\left(\frac{\delta-1}{\delta+1}\right)\right]^{\frac{1}{\delta}}$.

Rewriting gives:

$$p^{1-\delta} - \frac{\pi}{\alpha} < (1+x)(\eta p^{-\delta} + b) \wedge p < \bar{p} \quad \Longleftrightarrow \quad \frac{\frac{1}{p^{\delta-1}} - \frac{\pi}{\alpha}}{\frac{\eta}{p^{-\delta}} + b} - 1 > x \wedge p < \bar{p}$$

Finally, the $p-$switching curve in the domain $p < \bar{p}$ is:

$$\nu(p) = \frac{p(\alpha - \tilde{\pi}p^{\delta-1})}{\alpha(\eta + bp^{\delta})} - 1. \tag{5.16}$$

where $\nu(p) > 0$. Above this curve $\dot{p} > 0$ and below the curve $\dot{p} = 0$. That is, above the curve actual profit is below the aspiration level. So, in order to raise profits the price level is increased. Recall that this is the case when $p < \bar{p}$, i.e. the sales goal is met. On the other hand, price is unchanged if one is in a situation below the curve; both the profit and sales goal is reached.

Furthermore,

$$\dot{p} < 0 \quad \Longleftrightarrow \quad p > \bar{p}, \tag{5.17}$$

which implies that the price decreases if and only if $p > \bar{p}$, i.e. actual sales are below the aspiration level. In this case sales are stimulated by means of a price cut. In this domain $p > \bar{p}$ the curve $\nu(p)$ does not play any role. Per definition:

$$\begin{aligned} \pi(p,x) &= [p - c(p,x)]w(p) < \tilde{\pi} \\ &\Leftrightarrow [p - c(p,0)(1+x)]w(p) < \tilde{\pi} \\ &\Leftrightarrow \underbrace{[p - c(p,0)]w(p)}_{\pi(p,0)} - c(p,0)x < \tilde{\pi} \\ &\Leftrightarrow x > \frac{\pi(p,0) - \tilde{\pi}}{c(p,0)} = \nu(p). \end{aligned} \tag{5.18}$$

But we can write (5.14) also as:

$$\nu(p) = \frac{p + \frac{\eta\tilde{\pi}}{\alpha b}}{\eta + bp^{\delta}} - \frac{\tilde{\pi}}{\alpha b} - 1. \tag{5.19}$$

Now the dynamics of x and p are explicitly determined, a set of equilibria can be derived. To derive this final set, first some subsets for each kind of dynamic of both x and p will be given. Regarding the dynamics of p the sets for $\dot{p} > 0, \dot{p} = 0$ and $\dot{p} < 0$ are respectively:

$$\begin{aligned} P^{+} &= \{(x,p) : 0 < p \leq \bar{p}, x > \nu(p), x \geq 0\} \\ P^{0} &= \{(x,p) : 0 < p \leq \bar{p}, 0 \leq x \leq \nu(p)\} \\ P^{-} &= \{(x,p) : p > \bar{p}, x \geq 0\} \end{aligned} \tag{5.20}$$

For the dynamics of x we get for $\dot{x} > 0, \dot{x} = 0$ and $\dot{x} < 0$ respectively:

$$\begin{aligned} X^{+} &= \{(x,p) : p > 0, 0 \leq x < \mu(p)\} \\ X^{0} &= \{(x,p) : p > 0, x \geq 0, x > \mu(p)\} \\ X^{-} &= \{(x,p) : x \geq 0, x = \mu(p)\} \end{aligned} \tag{5.21}$$

Then the set of equilibria, i.e. the attraction curve is:

$$A^{0} = P^{0} \cap X^{0}. \tag{5.22}$$

In the next subsection, the focus is on stationary points. In this subsection also some graphical illustrations of the dynamics of x and p for different values of $\bar{p}$ are given.

As an exercise we can calculate that $\nu(0) = -1$ and $\lim_{p\to\infty} \nu(p) = -\frac{\tilde{\pi}}{\alpha b} - 1$. Subsequently, the first derivative of μ with respect to p is:

$$\begin{aligned} \frac{d\nu(p)}{dp} &= \frac{\eta + bp^{\delta} - \left(p + \frac{\eta\tilde{\pi}}{\alpha b}\right) b\delta p^{\delta-1}}{(\eta + bp^{\delta})^2} \\ &= \frac{\eta + bp^{\delta} - b\delta p^{\delta} - \frac{\eta\tilde{\pi} b\delta p^{\delta-1}}{\alpha b}}{(\eta + bp^{\delta})^2} \\ &= \frac{\overbrace{\eta + b(1-\delta)p^{\delta} - \frac{\eta\tilde{\pi}\delta p^{\delta-1}}{\alpha}}^{\lambda(p)}}{(\eta + bp^{\delta})^2}. \end{aligned} \tag{5.23}$$

And we see that $\nu'(p) > 0$ if and only if $\lambda(p) > 0$. Moreover,

$$\begin{aligned} \lambda(0) &= a \\ \frac{d\lambda(p)}{dp} &= b\delta(1-\delta)p^{\delta-1} - \frac{\eta\tilde{\pi}\delta(\delta-1)p^{\delta-2}}{\alpha} < 0. \end{aligned}$$

Hence, either:

(i) $\nu'(p) \geq 0$ for all p, or
(ii) $\nu'(p) > 0$ for $p < p^-$ (ν grows) and
(iii) $\nu'(p) < 0$ for $p > p^-$ (ν declines).

But since $\lim_{p\to\infty} \nu(p) < -1 = \nu(0)$, *(i)* is not possible. Hence, we can derive the following proposition.

Proposition 5.1 *If $\nu(p) > 0$ for some $p > 0$, there is the single $p^- > 0$ and single $p^+ > p^-$ such that:*

$\nu(p^-) = \nu(p^+) = 0$
$\nu(p) < 0$ *for* $p < p^-$
$\nu(p) < 0$ *for* $p > p^+$
$\nu(p) > 0$ *for* $p^- < p < p^+$.

Moreover, $\nu'(p^-) > 0$ and $\nu'(p^+) < 0$.

Note that proposition 5.1 holds if and only if $\nu(\hat{p}) > 0$, where $\hat{p}$ is the maximum of ν determined by $\nu(\hat{p}) = 0$ or $\lambda(\hat{p}) = 0$.

In order to determine the point where the function switches from concave to convex, determine:

$$\begin{aligned} \frac{d^2\nu(p)}{dp^2} &= \frac{1}{(\eta + bp^{\delta})^2}\left(\lambda'(p) - \frac{2\lambda(p)\delta bp^{\delta-1}}{\eta + bp^{\delta}}\right) \\ &= \frac{1}{\eta + bp^{\delta}}\left(-\overbrace{(\eta b\delta(\delta-1) + 2\eta b\delta)}^{h_1} p^{\delta-1} - \overbrace{\eta^2\frac{\tilde{\pi}\delta(\delta-1)}{\alpha}}^{h_2} p^{\delta-2} + \right. \\ &\left. \overbrace{b^2\delta(\delta-1)}^{h_3} p^{2\delta-1} + \overbrace{\frac{\eta\tilde{\pi} b\delta[2\delta - (\delta-1)]}{\alpha}}^{h_4} p^{2\delta-2}\right) \end{aligned}$$

$$= \frac{p^{\delta-2}}{\eta + bp^{\delta}} \left(-h_1 p - h_2 + h_3 p^{\delta+1} + h_4 p^{\delta}\right). \tag{5.24}$$

Next, define

$$d(p) = -h_1 p - h_2 + h_3 p^{\delta+1} + h_4 p^{\delta}.$$

Now there exists a $\bar{p}$ such that:

(i)	$d(\bar{p}) = 0$	
(ii)	$d(\bar{p}) < 0 \Leftrightarrow \nu''(p) < 0$	$(p < \bar{p})$
(iii)	$d(\bar{p}) > 0 \Leftrightarrow \nu''(p) > 0$	$(p > \bar{p})$,

where *(ii)* shows the concave part of the trajectory of ν and *(iii)* the convex part. From (5.15) it follows that:

$$\frac{d\nu(p)}{d\tilde{\pi}} = \frac{1}{\alpha b} \left(\frac{\eta}{\eta + bp^{\delta}} - 1\right) < 0. \tag{5.25}$$

Thus, as $\tilde{\pi}$ grows, $\nu(p)$ goes down. Hence, proposition 5.1 holds for relatively small $\tilde{\pi}$ and is violated as $\tilde{\pi}$ is too high.

Lemma 5.1 *Let $\pi_{\otimes}$ be the maximum value for $\pi(p, 0)$, i.e. $\pi_{\otimes} = \pi(p^*_{\otimes}, 0)$. By (5.14) $\nu(p) \leq 0$ for all p if $\tilde{\pi} \geq \pi_{\otimes}$ (proposition 5.1 is violated) and holds if $\tilde{\pi} < \pi_{\otimes}$. Moreover, $\nu(p) > 0$ if and only if $\pi(p, 0) > \tilde{\pi}$. Hence the interval (p^-, p^+) where $\nu(p) > 0$ is contained in $(p^-_{\otimes}, p^+_{\otimes})$ where $\pi(p, 0) > 0$.*

Proof. Follows directly from (5.15).
□

5.1.3 Stationary points

Recall the dynamics $\dot{x}, \dot{p}$ as given in (5.9) and expressions of $\mu(p)$ and $\nu(p)$. Now: The stationary points can be found from:

1. $\mu(p) = x$ and ($\nu(p) = x$ or $\psi(p) = 0$)
2. $\nu(p) = x$ and ($\mu(p) = x$ or $\varphi(x) = 0$)

Assume the $x-$switching curve $\mu(p)$ and $p-$switching curve $\nu(p)$ intersect above the $p-$axis. Every intersection point is found from:

$$\mu(p) = \nu(p) \tag{5.26}$$

We assumed that (5.21) has at least one solution $p > 0$ such that $x = \mu(p) = \nu(p) > 0$. Equation (5.21) is an algebraic equation whose equivalent form is:

$$\tilde{c} = \frac{p(\alpha - \tilde{\pi} p^{\delta-1})}{\alpha}. \tag{5.27}$$

Rewriting yields the polynomial of degree δ :

$$-\frac{\tilde{\pi}}{\alpha} p^{\delta} + p - \tilde{c} = 0. \tag{5.28}$$

If δ is rational, $\delta = n/m \quad (n > m)$, then setting $y = p^{\frac{1}{m}}$, (5.23) becomes:

$$-\frac{\tilde{\pi}}{\alpha} y^n + y^m - \tilde{c} = 0. \tag{5.29}$$

This equation may have n roots. Hence, (5.24) may, generally, have n roots $p_1, \ldots, p_n > 0$ such that:

$$x_i = \mu(p_i) = \nu(p_i) > 0 \qquad (i = 1, \ldots, n).$$

If δ is irrational, the number of roots might be larger. But we will assume that δ is rational and that there are n roots. We will determine the rest points for the case where $\mu'(p_i) \neq \nu'(p_i)(i = 1, \ldots, n)$, which is obviously the general situation. The analysis will be carried out assuming that $n = 2$ and $m = 1$. A polynomial of degree 2 is obtained then. Moreover, it is assumed $0 = \nu(p_n) > \mu(p_n)$. This simply implies that at this point the function $\nu(p)$ lies above $\mu(p)$. Furthermore, as argued before $p^- < p_1 < \cdots < p_n < p^+$.

One of the stationary points is the point where $\nu(p) = x$ and $\varphi(x) = 0$. To see whether this point coincides with the neoclassical equilibrium price assume that the maximum aspiration level of profit cannot be larger than the maximum profit achieved according to the neoclassical theory. In the neoclassical theory the equilibrium price equals 2η. Substituting this into the profit function shows that the maximum profit is:

$$\pi^* = \frac{\alpha}{4\eta} - \theta. \tag{5.30}$$

Assume that this is also the maximum aspiration level of profits. Subsequently: $0 \leq \tilde{\pi} \leq \tilde{\pi}^*$. Substituting $\tilde{\pi}^*$ into $\nu(p)$ (5.14) yields:

$$\begin{aligned} \nu(p) &= \frac{p\left(\alpha - \left(\frac{\alpha}{4\eta} - \theta\right)p\right)}{\alpha\left(\eta + \frac{\theta}{\alpha}p^2\right)} - 1. \\ &= \frac{-\alpha p^2 + 4\alpha\eta p - 4\alpha\eta^2}{4\eta(\alpha\eta + \theta p^2)} \\ &= p^2 - 4\eta p + 4\eta. \end{aligned} \tag{5.31}$$

Solving $\nu(p) = 0$ gives $p^* = 2\eta$ which equals the neoclassical equilibrium price as depicted in (5.7). The following statement summarizes this result:

Proposition 5.2 *For $\tilde{\pi} = \tilde{\pi}^*$ and given the functional form of $w(p)$ and $c(q, x)$ as given in (3.9) and (3.15) respectively, the intersection point coincides with the neoclassical equilibrium price and equals 2η.*

Proof. See above.
□

For general $\tilde{\pi}$ the function $\nu(p)$ becomes:

$$\nu(p) = \left(\frac{\theta + \tilde{\pi}}{\alpha}\right)p^2 - p + \eta \tag{5.32}$$

Solving the function $\nu(p) = 0$ yields equilibrium prices:

$$p^* = \frac{1 \pm \sqrt{1 - 4\eta\left(\frac{\theta+\tilde{\pi}}{\alpha}\right)}}{2\left(\frac{\theta+\tilde{\pi}}{\alpha}\right)}. \tag{5.33}$$

Because it is assumed that:

$$\begin{aligned} 0 < \tilde{\pi} \leq \tilde{\pi}^* \quad &\Longleftrightarrow \quad 0 < \tilde{\pi} \leq \frac{\alpha}{4\eta} - \theta \\ &\Longleftrightarrow \quad 0 < \frac{\theta + \tilde{\pi}}{\alpha} \leq \frac{1}{4\eta}. \end{aligned} \tag{5.34}$$

Defining $\zeta = \frac{\theta+\tilde{\pi}}{\alpha}$ (5.27) is now:

$$\nu(p) = \zeta p^2 - p + \eta. \tag{5.35}$$

In (5.29) we derived that $0 < \zeta \leq \frac{1}{4\eta}$. Rewriting gives $0 < \gamma \leq 1$, where $\gamma = 4\zeta\eta$. Thus:

$$\tilde{\pi} \leq \tilde{\pi}^* \Leftrightarrow \gamma \leq 1.$$

Given this, the roots of $\nu(p)$ are:

$$p^* = \frac{2\eta\,(1 \pm \sqrt{1-\gamma})}{\gamma}. \tag{5.36}$$

Let's try to understand the dynamics of p in terms of a changing $\tilde{\pi}$. We see that d is linear with respect to $\tilde{\pi}$. The function $\nu(p) = 0$ yields two roots. First two functions $f_+(\gamma)$ and $f_-(\gamma)$ extracted from (5.31) will be introduced :

$$\begin{aligned} f_+(\gamma) &= \frac{1+\sqrt{1-\gamma}}{\gamma} \\ f_-(\gamma) &= \frac{1-\sqrt{1-\gamma}}{\gamma}. \end{aligned} \tag{5.37}$$

Differentiating generates:

$$\begin{aligned} \frac{df_+(\gamma)}{d\gamma} &= -\frac{1}{2\gamma\sqrt{1-\gamma}} - \frac{1+\sqrt{1-\gamma}}{\gamma^2} < 0 \\ \frac{df_-(\gamma)}{d\gamma} &= \frac{1}{2\gamma\sqrt{1-\gamma}} - \frac{1-\sqrt{1-\gamma}}{\gamma^2} > 0. \end{aligned} \tag{5.38}$$

In the first case ($f'_+(\gamma) < 0$), the price level decreases if $\tilde{\pi}$ increases, whereas in the second case ($f'_-(\gamma) < 0$) an increase of $\tilde{\pi}$ is positively related to p.

Recall that equation (5.31) shows the roots, i.e. the price levels for $\nu(p) = 0$. Compared with the neoclassical equilibrium the following proposition can be made.

Proposition 5.3 *Equation (5.31) yields the prices* p^*_- *and* p^*_+*. The price* $p^*_- < p^* < p^*_+$ *if and only if* $0 < \gamma < 1$*. The equilibrium prices coincide with the neoclassical equilibrium price, i.e.* $p^*_- = p^* = p^*_+$ *if and only if* $\gamma = 1$*.*

Proof. Per definition $0 < \gamma \leq 1$. If $\gamma = 1$ then $f_k(\gamma) = 1$ for $k = -, +$. Subsequently, the equilibrium price derived from (5.31) is:

$$p^* = \frac{2\eta\,(1 \pm \sqrt{1-\gamma})}{\gamma} = 2\eta f_k(\gamma) = 2\eta. \quad (k = -, +).$$

If $0 < \gamma < 1$ then $f_+(\gamma) > 1$ and $f_-(\gamma) < 1$. This implies that $p^*_- < p^* < p^*_+$.
□

The other stationary points can be determined by $\mu(p) = \nu(p)$. That is, solving (5.24) for $m = 1, n = 2$. The roots of this polynomial of degree 2 equal:

$$y = p^* = \frac{1 \pm \sqrt{1-4\phi\tilde{c}}}{2\phi} \qquad (\phi = \frac{\tilde{\pi}}{\alpha}). \tag{5.39}$$

Figure 2 illustrates graphically the possible equilibria. Curve A shows the function

$$\tilde{p} = \tilde{c} + \frac{\tilde{\pi}}{q}, \tag{5.40}$$

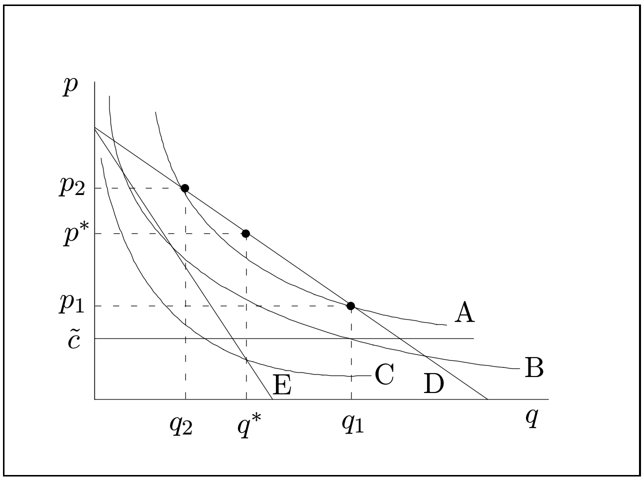

Figure 2: Equilibrium with fixed aspiration levels.

where $\tilde{\pi}/q$ is the aspired mark-up. The efficient average cost is represented by B. Furthermore, C, D and E are the efficient marginal cost, demand,[11] and efficient marginal revenue. Economic consistency requires that $\tilde{c} \geq c$ and $\tilde{\pi} \leq \pi^*$. Equation (5.40) then can be interpreted as the aspired price. The graph suggests that there can be two price levels where aspirations are satisfied: p_1, p_2. In the model the actual price p is on the demand curve $q = q(p)$. The interval $p_2 < p < p_1$ represents therefore the situations where actual cost c and/or actual profits π are higher than their corresponding aspiration levels. For $p > p_1$ and $p < p_2$ the opposite is true.

Finally, some graphical illustrations of the model described in this section are given. Three cases are distinguished based on different a varying sales goal. Figure 3 shows the switching curves and the vector fields when the sales goal is relatively low, i.e. $p_1 < p_2 < \bar{p}$ ($\bar{p}$ is far to the right). In this case a set of equilibria A^0 exist. Moreover, there is the equilibrium point B.

In the second case the sales goal is somewhat higher than in the first case, i.e. $p_1 < \bar{p} < p_2$. In this case again a set of equilibria exist. Figure 4 shows this set again denoted by A^0.

Finally, if $\bar{p} < p_1 <_2$ the system converges to the unique stable equilibrium B as shown in figure 5.

[11]Here it is assumed that demand is linear.

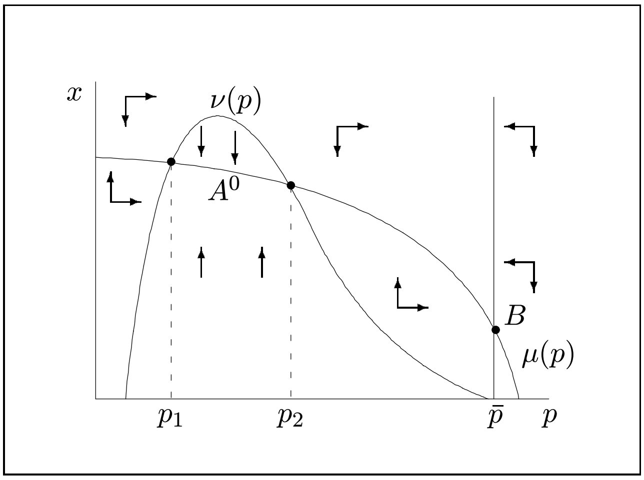

Figure 3: Switching curves and vector field dynamics for relatively low $\tilde{w}$.

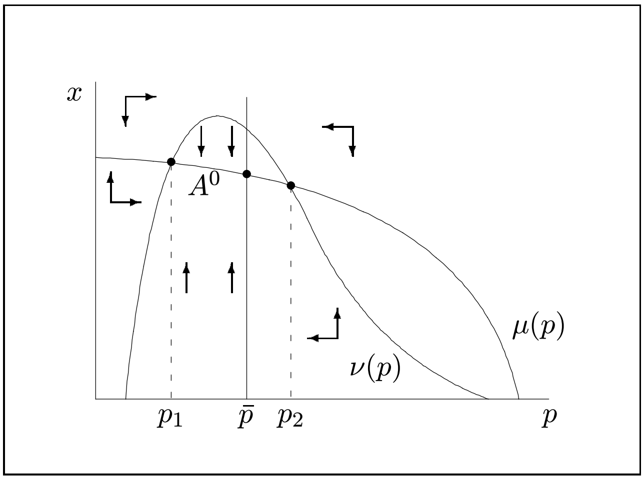

Figure 4: Switching curves and vector field dynamics for relatively normal $\tilde{w}$.

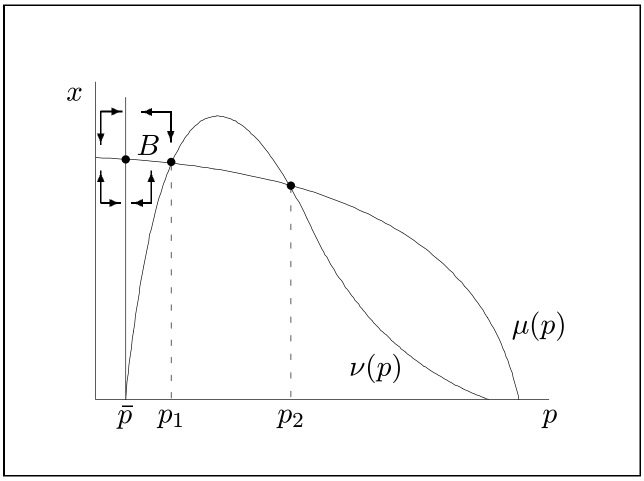

Figure 5: Switching curves and vector field dynamics for relatively high $\tilde{w}$.

5.2 3-Dimensional Firm Model

5.2.1 Outcome of the Modified Monopoly Case

In this section the final outcome of the monopoly is calculated given the functional forms previously stated. The difference with the equilibrium presented in section 1 is that there the equilibrium of a monopolist which is perfectly efficient (slack is zero) was analyzed. In this section the firm is X-inefficient. For simplicity we take a case where we start from given (fixed) levels of x_q and x_w. To formulate a reference case we first assume that the CMD maximizes profits accepting a certain aspiration level of slack on both production ($\tilde{x}_q$) and sales ($\tilde{x}_w$). First, the inverse demand curve is the inverse of (3.11):

$$p(t) = \left(\frac{q(t)}{\alpha(1-\tilde{x}_w)}\right)^{\frac{1}{\beta}}. \tag{5.41}$$

Revenue r equals the price of the product times the quantity produced:

$$r(t) = p(t)q(t) = \frac{q(t)^{\frac{1}{\beta}+1}}{[\alpha(1-\tilde{x}_w)]^{\frac{1}{\beta}}} \tag{5.42}$$

Differentiation of r with respect to q yields:

$$\frac{dr}{dq} = \left(\frac{1}{\beta}+1\right)\left(\frac{q(t)}{\alpha(1-\tilde{x}_w)}\right)^{\frac{1}{\beta}}. \tag{5.43}$$

Equation (2.3) states that β should be equal to the price elasticity of demand $\epsilon(q)$.

Corollary 5.1 *Suppose the demand function is of the form as in (3.9). Then $\epsilon(q)$ equals β.*

Proof.

$$\epsilon(q) = \frac{p}{q}\frac{dq}{dp} = \frac{p}{\alpha p^{\beta}}\alpha\beta p^{\beta-1} = \frac{p\alpha\beta p^{\beta-1}}{\alpha p^{\beta}} = p\beta p^{-1} = \beta.$$

□

The cost function of the monopolist is given by (3.15). Differentiation of this function with respect to q gives the marginal cost function:

$$\frac{dc}{dq} = \eta z_q = \eta(1 + \tilde{x}_q). \tag{5.44}$$

Solving $dr/dq = dc/dq$ gives q^* and subsequently substituting q^* into (5.24) yields p^*:

$$\begin{aligned} q^*(t) &= \alpha(1 - \tilde{x}_w)\left(\frac{\eta(1 + \tilde{x}_q)\beta}{1 + \beta}\right)^{\beta}, \\ p^*(t) &= \left(\frac{\eta(1 + \tilde{x}_q)\beta}{1 + \beta}\right)^{\frac{1}{\beta}}. \end{aligned} \tag{5.45}$$

The point (p^*, q^*) reflects the point where profit is at the maximum level.

5.2.2 Joint Payoff

In this section it is analyzed whether a so-called *potential* or joint payoff function can be constructed for the 3-dimensional case. Potential games have been introduced by [Monderer, Shapley 1996]. Learning in potential games can be found in [Ermoliev, Flåm 1997]. Before we can determine the existence of a potential, first the payoff functions of the three departments needs to be calculated as in (4.13).

First, the PRD. The target variable is production cost as derived in (3.15). Substituting (3.15) into (4.13) we get:

$$\begin{aligned} \mathcal{P}_1 &= \int (s_1 - c)dx_1 = \int \left(s_1 - \left(\frac{\theta}{\alpha x_3^{\beta}(1 - x_2)} + \eta\right)(1 + x_1)\right)dx_1 \\ &= s_1 x_1 - \left(\frac{\theta}{\alpha x_3^{\beta}(1 - x_2)} + \eta\right)\left(x_1 + \frac{x_1^2}{2}\right) + s_1 x_1 + F_1(x_2, x_3, s_1, s_2, s_3). \end{aligned} \tag{5.46}$$

Second, for the SLD the level of sales as given in (3.11) is important. Substituting this function into (4.13) yields:

$$\begin{aligned} \mathcal{P}_2 &= \int (w - s_2)dx_2 = \int \left(\alpha x_3^{\beta}(1 - x_2) - s_2\right)dx_2 \\ &= \alpha x_3^{\beta}(x_2 - \frac{x_2^2}{2}) - s_2 x_2 + F_2(x_1, x_3, s_1, s_2, s_3). \end{aligned} \tag{5.47}$$

Finally, substituting (3.6) into (4.13) we derive the payoff function of the CMD:

$$\begin{aligned} \mathcal{P}_3 &= \int (\pi - s_3)dx_3 = \int (p - c)w - s_3 dx_3 \\ &= \int \left(x_3 - \left(\frac{\theta}{\alpha x_3(1 - x_2)} + \eta\right)\alpha x_3(1 - x_2) - s_3\right)dx_3 \\ &= \frac{\alpha x_3^{\beta+2}(1 - x_2)}{\beta + 2} + \frac{\alpha \eta x_3^{\beta+1}(x_1 x_2 - x_1 + x_2 - 1)}{\beta + 1} - \\ &\quad x_3(\theta(1 + x_1) + s_3) + F_3(x_1, x_2, s_1, s_2, s_3). \end{aligned} \tag{5.48}$$

This leads to the following proposition.

Proposition 5.4 *Given the structure of $\mathcal{P}_1, \mathcal{P}_2$ and $\mathcal{P}_3$ there exists no potential $\mathcal{P}$ for this game such that $\mathcal{P} = \mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}_3$.*

Proof. If a potential $\mathcal{P} = \mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}_3$ exists, then $\frac{\partial^2 \mathcal{P}}{\partial x_i \partial x_j} = \frac{\partial^2 \mathcal{P}}{\partial x_j \partial x_i}$ for all k and $i \neq j$. Relating this to our firm model the first condition that needs to be satisfied is:

$$\frac{\partial^2 \mathcal{P}}{\partial x_1 \partial x_2} = \frac{\partial^2 \mathcal{P}}{\partial x_2 \partial x_1}.$$

Assume there is a function $\mathcal{P} = \mathcal{P}_1 = \mathcal{P}_2 = \mathcal{P}_3$. Then:

$$\mathcal{S}_1 = \frac{\partial \mathcal{P}}{\partial x_1} = \frac{\partial \mathcal{P}_1}{\partial x_1} = -\left(\frac{\theta}{\alpha x_3^{\beta}(1-x_2)} + \eta\right)(1+x_1) + s_1,$$

Now take the second partial derivative with respect to x_2 :

$$\frac{\partial^2 \mathcal{P}}{\partial x_1 \partial x_2} = \frac{\partial^2 \mathcal{P}_1}{\partial x_1 \partial x_2} = \frac{\partial \mathcal{S}_1}{\partial x_2} = -\frac{\theta(1+x_1)}{\alpha x_3^{\beta}(1-x_2)^2}.$$

Furthermore,

$$\mathcal{S}_2 = \frac{\partial \mathcal{P}}{\partial x_2} = \frac{\partial \mathcal{P}_2}{\partial x_2} = \alpha x_3^{\beta}(1-x_2) - s_2.$$

The second derivative of this function with respect to x_1 is:

$$\frac{\partial^2 \mathcal{P}}{\partial x_2 \partial x_1} = \frac{\partial^2 \mathcal{P}_2}{\partial x_2 \partial x_1} = \frac{\partial \mathcal{S}_2}{\partial x_1} = 0,$$

and we see that $\frac{\partial \mathcal{S}_1}{\partial x_2} \neq \frac{\partial \mathcal{S}_2}{\partial x_1}$. This already concludes the proof.

□

For the sake of completeness all second-order partial derivatives are calculated. The second equality that needs to hold is

$$\frac{\partial^2 \mathcal{P}}{\partial x_1 \partial x_3} = \frac{\partial^2 \mathcal{P}}{\partial x_3 \partial x_1}.$$

The first partial derivative we need is the same as (4.3). Now differentiating with respect to x_3 gives:

$$\frac{\partial^2 \mathcal{P}}{\partial x_1 \partial x_3} = \frac{\partial^2 \mathcal{P}_1}{\partial x_1 \partial x_3} = \frac{\partial \mathcal{S}_1}{\partial x_3} = \frac{\theta\beta(1+x_1)}{\alpha x_3^{\beta+1}(1-x_2)}$$

Next determine

$$\mathcal{S}_3 = \frac{\partial \mathcal{P}}{\partial x_3} = \frac{\partial \mathcal{P}_3}{\partial x_3} = \alpha\eta x_3^{\beta}(x_2 + x_1 x_2 - x_1 - 1) + \alpha x_3^{\beta+1}(1-x_2) - \theta(1+2x_1) - s_3.$$

Consequently,

$$\frac{\partial^2 \mathcal{P}}{\partial x_3 \partial x_1} = \frac{\partial^2 \mathcal{P}_3}{\partial x_3 \partial x_1} = \frac{\partial \mathcal{S}_3}{\partial x_1} = \alpha\eta x_3^{\beta}(x_2 - 1) - \theta.$$

So, also in this case we see that $\frac{\partial \mathcal{S}_1}{\partial x_3} \neq \frac{\partial \mathcal{S}_3}{\partial x_1}$. The final condition that needs to hold in our firm model is:

$$\frac{\partial^2 \mathcal{P}}{\partial x_2 \partial x_3} = \frac{\partial^2 \mathcal{P}}{\partial x_3 \partial x_2}.$$

First, calculate $\frac{\partial \mathcal{P}}{\partial x_2} = \frac{\partial \mathcal{P}_2}{\partial x_2}$. This is already shown in (4.5). Second,

$$\frac{\partial^2 \mathcal{P}}{\partial x_2 \partial x_3} = \frac{\partial^2 \mathcal{P}_2}{\partial x_2 \partial x_3} = \frac{\partial \mathcal{S}_2}{\partial x_3} = \alpha\beta x_3^{\beta-1}(1-x_2).$$

The next condition we need is $\frac{\partial \mathcal{P}}{\partial x_3} = \frac{\partial \mathcal{P}_3}{\partial x_3}$ which is given by (4.9). Finally it is easy to see that

$$\frac{\partial^2 \mathcal{P}}{\partial x_3 \partial x_2} = \frac{\partial^2 \mathcal{P}_3}{\partial x_3 \partial x_2} = \frac{\partial \mathcal{S}_3}{\partial x_2} = \alpha\eta x_3^{\beta}(1+x_1) - \alpha x_3^{\beta+1}.$$

and so we see that $\frac{\partial \mathcal{S}_2}{\partial x_3} \neq \frac{\partial \mathcal{S}_3}{\partial x_2}$. The calculations show that condition (4.1) does not hold. This implies that the game cannot be described by a general payoff function $\mathcal{P}$ and cannot serve as a a so-called 'potential' for this game. The missing of such a potential means that there are some differences in interest among the players. But this result is consistent with one of the assumptions of the behavioral theory of the firm which states that the firm is a set of subcoalitions that pursue their own 'local' interests. That is, in first instance they want to be locally optimal. We showed analytically that this assumption holds in our model. Furthermore, this implies that the constant term $F_k(\cdot)$ in the payoff functions $\mathcal{P}_k$ can be assumed to be zero.

6 Conclusions and Discussion

6.1 Conclusions

In this paper we developed a dynamical game theoretical model of a monopoly firm based on behavioral assumptions, i.e. satisficing behavior instead of maximizing behavior. A 2-dimensional (two departments) and 3-dimensional (three departments) model is distinguished. In the 2-dimensional model the firm consists of a production department and a central managament department. In the 3-dimensional model also a sales department is included. The PRD, SLD and CMD have a production cost goal, a sales goal and profit goal respectively. The PRD its instrument variable is slack on production, the SLD its instrument is slack on sales and the CMD its instrument variable is the price.

In the 2-dimensional case the sales goal is taken into account implicitly, whereas in the 3-dimensional case the sales goal is explicitly modeled. In the 2-dimensional model we distinguish three different cases:

1. Relatively low sales goal;
2. Relatively 'normal' sales goal;
3. Relatively high sales goal.

In all the three cases equilibria exist. In the first case, a set of equilibria exist and one other equilibrium point exist. In the second case only a set of equilibria exist. In the third case a unique stable equilibrium exist.

Furthermore, a comparison is made with the equilibrium outcome of the neoclassical monopoly. It is shown that the same outcome can be achieved if the profit goal equals the maximum profit level in neoclassical sense. However, for varying aspiration levels different equilibria might exist.

In the 3-dimensional model a sales department is explicitly modeled. It is analytically shown that in this case no joint payoff function, i.e. potential exists. This implies that interests differ among players (departments). This result is in line with assumption of the behavioral theory of the firm that the firm is a set of different players each pursuing their own goal.

6.2 Discussion

The modified monopoly model can be extended in several ways. First, the model might become more realistic to assume variable aspirations. One could start with a varying sales goal. Subsequently, aspiration levels of production cost and profit might be modeled as variables instead constants.

Second, one can try to include more than two or three departments. However, a drawback of this is that the mathematical analysis becomes much more difficult than in the two-dimensional (2 departments) case. That is, the system will consist of more than 2 differential equations which makes it harder to find an analytical solution. As a first step one could try to extend the 2-dimensional model by including a sales department.

Third, given the number of departments of the firm the model might be extended in terms of allowing competition. This implies modeling more than one firm in the market. It might be appropriate first to analyze a market with two firms. Moreover, one can model the firms assuming the same structure but with different parameters. Another extension might be to allow differences in structure between the firms. For example, one firm consists of two departments and the other of three departments.

Finally, we assumed that each department has one goal and one control variable at its disposal to reach their goal. It might be appropriate to analyze a model where certain departments are aiming for more than one goal and/or with the availability of more than one control variable.

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