HOUSEHOLD DECISIONS AND EQUILIBRIUM EFFICIENCY

1 INTRODUCTION

With a few exceptions, economic theory as well as empirical research have treated households as if they were single consumers. As a practical matter, indeed, household expenditure data commonly used in empirical research may report the composition of households without disaggregating household consumption (expenditure) and factor supply (income) with respect to household members. Both from a normative and a positive perspective, this prevailing practice raises the question whether it makes any difference who participates in the market, households as entities or household members individually. Such considerations have attracted renewed attention after the widely acclaimed article by Chiappori (1988a) who presents a model of collective rationality of households as an alternative to the neoclassical model where households are treated like single consumers. See also the surveys by Bourguignon and Chiappori (1992) and Kapteyn and Koreman (1992).

The normative issue is optimality: Will competitive exchange among households as entities lead to a Pareto-optimal allocation? The answer is in the affirmative as long as each household makes an optimal choice subject to its budget constraint and, by doing so, exhausts its budget. Moreover, under the same assumptions, a corresponding core inclusion result can be derived, if a modified notion of the core reflects competition among households instead of individuals.

Non-optimal equilibrium allocations can occur even in economies consisting exclusively of one-person households, provided that some consumers possess satiation points in the interior of their budget sets whereas other consumers have non-satiated preferences and exhaust their budgets. With multi-person households rather than individuals participating in the market, this phenomenon is more likely, however. Namely, a household with negative intra-household externalities may have a bliss point despite the fact that each household member has monotonic preferences with respect to her individual consumption. Just imagine a household composed of two smokers. Each household member may individually prefer to always smoke more, since the additional nicotine intake more than compensates for the deterioration of air quality it causes. Nevertheless, the negative externalities due to air pollution can be such that the two smokers agree on an unconstrained "optimum" consumption for the household. Examples 3.3 – 3.5 below aim to capture such a situation.

It is not too surprising that certain externalities lead to sub-optimal equilibrium allocations. More importantly, we can identify externalities that do not hinder Pareto-optimality of equilibrium outcomes: Each household, by internalizing its intra-household externalities, furthers global efficiency.

The positive issue is individual decentralization: Does competitive exchange among households lead to outcomes that can also be attained via competitive exchange among individuals? In other words: Given a competitive equilibrium allocation with only households participating in the market, can this allocation also be attained as a competitive equilibrium allocation where the individual household members participate in the market — after being allotted suitable income or endowment shares? The answer is in the affirmative in the absence of any externalities and with standard monotonicity and smoothness conditions.

When intra-household externalities are present, individual decentralization of equilibrium outcomes among households is still possible in exceptional cases. But as a rule, individual market participants do not fully internalize intra-household externalities whereas a household does it by assumption.[1]

We set out to address both issues, optimality and individual decentralization, *in a model of a pure exchange economy*. Our findings with regard to individual decentralization are potentially helpful in answering the question whether tests can be designed which discriminate between equilibria among households and equilibria among individuals. In the next section, we describe the model and present general results for economies where intra-household externalities are absent or non-negative. Section 3 contains various examples with non-positive intra-household externalities. In Section 4, we summarize and assess our formal results. Elaborate proofs, of Propositions 2 and 3, are postponed until Section 5.

[1]Needless to say that if an allocation cannot be supported by an impersonal market price system, it may well be supportable by means of personalized prices in the sense of Lindahl.

2 MODEL AND MAIN RESULTS

We consider a pure exchange economy composed of finitely many households $h = 1, \dots, H$. The commodity space is $\mathbb{R}^\ell$ with $\ell \geq 1$. Household h is endowed with a commodity bundle $\omega_h \in \mathbb{R}^\ell, \omega_h > 0$.

Each household h consists of finitely many members $i = hm$ with $m = 1, \dots, M(h), M(h) \geq 1$. Put $I = \{hm : h = 1, \dots, H; m = 1, \dots, M(h)\}$. A generic individual $i = hm \in I$ has:

- consumption set $X_i = \mathbb{R}^\ell_+$;
- preferences $\succeq_i$ on the allocation space $\mathcal{X} \equiv \prod_{j \in I} X_j$ represented by a utility function $U_i : \mathcal{X} \longrightarrow \mathbb{R}$.

Let $\mathbf{x} = (x_i), \mathbf{y} = (y_i)$ denote generic elements of $\mathcal{X}$. For $h = 1, \dots, H$, define $\mathcal{X}_h = \prod_{n=1}^{M(h)} X_{hn}$ with generic elements $\mathbf{x_h} = (x_{h1}, \dots, x_{hM(h)})$. If $\mathbf{x} \in \mathcal{X}$ is an allocation, then for $h = 1, \dots, H$, household consumption is $\mathbf{x_h} = (x_{h1}, \dots, x_{hM(h)}) \in \mathcal{X}_h$. For the economy with social endowment $\omega = \sum_h \omega_h$ and consumers $i = hm$ $(h = 1, \dots, H; m = 1, \dots, M(h))$,
a **Pareto-optimal allocation (PO)** is defined in the standard fashion:

$\mathbf{x} = (x_i) \in \mathcal{X}$ is a **Pareto-optimal allocation**, if

(i) $\sum_i x_i = \omega$;

(ii) there is no $\mathbf{y} = (y_i) \in \mathcal{X}$ with

$\sum_i y_i = \omega$;
$U_i(y) \geq U_i(x)$ for all i;
$U_i(y) > U_i(x)$ for some i.

The first welfare theorem asserts that any competitive equilibrium allocation in the sense of Walras is Pareto-optimal. Here we allow for the possibility that instead of individual members, households act collectively on the market. We shall assume **efficient bargaining within households**. The latter means that a household h chooses an allocation at the Pareto frontier of its budget set, i.e. an element of its efficient budget set $EB_h(p)$ as defined below.

We shall from now on restrict attention to the case where consumption externalities, if any, exist only between members of the same household. That is,

(E1) Intra-Household Externalities: $U_i(\mathbf{x}) = U_i(\mathbf{x_h})$
for $i = hm$, $\mathbf{x} \in \mathcal{X}$.

We shall sometimes pay special attention to the case of no externalities, i.e.

(E2) Absence of Externalities: $U_i(\mathbf{x}) = U_i(x_i)$
for $i = hm$, $\mathbf{x} = (x_i) \in \mathcal{X}$.

Now consider a household h and a price system $p \in \mathrm{IR}^{\ell}$. For

$$\mathbf{x_h} = (x_{h1}, \ldots, x_{hM(h)}) \in \mathcal{X}_h,$$

denote

$$p * \mathbf{x_h} = p \cdot \left(\sum_{m=1}^{M(h)} x_{hm} \right).$$

Then h's budget set is defined as

$$B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq p \cdot \omega_h\}.$$

We define the **efficient budget set** $EB_h(p)$ by:

$\mathbf{x_h} = (x_{h1}, \ldots, x_{hM(h)}) \in EB_h(p)$ IF AND ONLY IF

1. $\mathbf{x_h} \in B_h(p)$ and
2. there is no $\mathbf{y_h} \in B_h(p)$ such that

   $U_{hm}(\mathbf{y_h}) \geq U_{hm}(\mathbf{x_h})$ for all $m = 1, \ldots, M(h)$;
   $U_{hm}(\mathbf{y_h}) > U_{hm}(\mathbf{x_h})$ for some $m = 1, \ldots, M(h)$.

A **Competitive Equilibrium** (among households) is a price system p together with an allocation $\mathbf{x} = (x_i)$ satisfying

(1h) $\quad \mathbf{x_h} \in EB_h(p)$

for $h = 1, \ldots, H$ and

(2) $\quad \sum_i x_i = \omega.$

Thus in a competitive equilibrium, each household makes an efficient choice under its budget constraint and markets clear. Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household. Classical versions of the first welfare theorem are based on the crucial property that each consumer's demand lies on the consumer's budget set — which implies Walras' Law. This property follows from local non-satiation of consumer preferences. A sufficent condition for the latter is monotonicity of consumer preferences. With the possibility of multi-person households and intra-household externalities, the crucial property needs to be adapted. The modified property stipulates that each household's choice lies on the household's "budget line". It will be called budget exhaustion (BE). Condition (BE) makes the underlying argument appear extremely transparent, if not trivial. It should be emphasized therefore that (BE) follows from standard assumptions on the primitive data of the model: Monotonicity in own consumption (MON) and Non-Negative Externalities (NNE) combined yield (BE). To formulate the latter properties, we need some more notation. Let $i = hm \in I$ be any individual. Whereas x_{hm} denotes the individual's private consumption bundle, $x_{h,-m}$ stands for the consumption plans of all the other members of household h; that is, $x_{h,-m} = (x_{hn})_{n \neq m}$.

Further, consider any $L \in \mathbb{N}$, $a = (a_1, \ldots, a_L), b = (b_1, \ldots, b_L) \in \mathbb{R}^L$, and $f : \mathbb{R}^L \to \mathbb{R}$. By $a \geq b$, we mean $a_l \geq b_l$ for all $l = 1, \ldots, L$. By $a \gg b$ we mean $a_l > b_l$ for all l. Finally, $a > b$ stands for $a \geq b, a \neq b$. The function f is called non-decreasing, if for any $a, b \in \mathbb{R}^L$, $a \geq b$ implies $f(a) \geq f(b)$. It is increasing, if for any $a, b \in \mathbb{R}^L$, $a \gg b$ implies $f(a) > f(b)$. It is strictly increasing, if for any $a, b \in \mathbb{R}^L$, $a > b$ implies $f(a) > f(b)$. Now various properties of individual preferences can be unambiguously defined.

(BE) Budget Exhaustion: For each household $h = 1, \ldots, H$, any household consumption profile $\mathbf{x_h} \in \mathcal{X}_h$, and any price system $p \in \mathbb{R}^\ell$,

$$\boxed{\mathbf{x_h} \in EB_h(p) \Rightarrow p * \mathbf{x_h} = p \cdot \omega_h}$$

(MON) Monotonicity: $U_i(x_{hm}, x_{h,-m})$ is increasing in x_{hm} for all $i = hm \in I$.

Strict Monotonicity: $U_i(x_{hm}, x_{h,-m})$ is strictly increasing in x_{hm} for all $i = hm \in I$.

(NNE) Non-Negative Externalities: $U_i(x_{hm}, x_{h,-m})$ is non-decreasing in $x_{h,-m}$ for all $i = hm \in I$.

A routine argument establishes the first welfare theorem in our context. The first welfare theorem is also an immediate consequence of Proposition 3 below.

Proposition 1 (First Welfare Theorem) *Suppose (E1) and (BE).*
If $(p; \mathbf{x})$ is a competitive equilibrium, then $\mathbf{x}$ is a Pareto-optimal allocation.

Corollary 1 *Suppose (E1), (MON) and (NNE).*
If $(p; \mathbf{x})$ is a competitive equilibrium, then $\mathbf{x}$ is a Pareto-optimal allocation.

Corollary 2 *Suppose (E2) and (MON).*
If $(p; \mathbf{x})$ is a competitive equilibrium then $\mathbf{x}$ is a Pareto-optimal allocation.

The last result can be sharpened, if the first order approach can be employed.

Proposition 2 (Optimality and Decentralization) *Suppose (E2) and strict monotonicity of consumer preferences. Suppose further that for each $i \in I$, the utility function U_i is concave and in the interior of $\mathcal{X}_i$ differentiable.*
If $(p; \mathbf{x})$ is a competitive equilibrium with $\mathbf{x} = (x_i)_{i \in I} \gg 0$, then

(i) *$\mathbf{x}$ is a Pareto-optimal allocation and*

(ii) *$(p; \mathbf{x})$ is a competitive equilibrium of the economy where the market participants are the individually acting $i \in I$, trading from the endowments x_i.*

To the extent that the second welfare theorem applies to economies satisfying (E2) and (BE), competitive equilibrium allocations are Pareto-optimal and, hence, attainable as competitive equilibrium allocations where the $i \in I$ act individually. Proposition 2 qualifies this general decentralization result to the effect that equilibrium prices do not depend on who participates in the competitive market exchange, households or individual household members.

If (E1) and (BE), but not (E2) are satisfied, then competitive equilibrium allocations are still Pareto-optimal (Proposition 1). However, as a rule, they cannot be individually decentralized (Examples 2.1 and 3.2). So the assertion of Proposition 2(ii) need no longer hold.

The conclusion of Proposition 1 can be generalized to a *core inclusion* statement. To this end, we introduce the notion of the H-core (household core) which reflects the fact that only households are market participants.

Let $\mathcal{G}$ denote the family of non-empty subsets of $\{1, \ldots, H\}$. For $G \in \mathcal{G}$, define

$$C(G) := \{i \in I | i = hm \text{ for some } h \in G, m = 1, \ldots, M(h)\}.$$

$C(G)$ is the coalition consisting of all the constituents of all the households in G.

Definition 1 *$x \in \mathcal{X}$ belongs to the* **H-core***, if*

(i) *x is socially feasible, i.e. $\sum_i x_i = \omega$;*

(ii) *there is* **no** *$G \in \mathcal{G}$ and $(\mathbf{y_h})_{h \in G} \in \prod_{h \in G} \mathcal{X}_h$ with:*

1. *$u_i(\mathbf{y_h}) \geq u_i(\mathbf{x_h})$ for all $i = hm \in C(G)$.*
2. *$u_i(\mathbf{y_h}) > u_i(\mathbf{x_h})$ for some $i = hm \in C(G)$.*
3. *$\sum_{i \in C(G)} y_i = \sum_{h \in G} \omega_h$.*

Proposition 3 (H-Core Inclusion) *Suppose (E1) and (BE).*
If $(p; \mathbf{x})$ is a competitive equilibrium, then $\mathbf{x}$ belongs to the H-core.

To exemplify the various possibilities, we shall impose two further restrictions: aggregate welfare maximization by households (WM) and separable externalities (SEP). Examples 2.1 and 3.1 – 3.5 share these two properties.

For any household $h = 1, \ldots, H$ and $\mathbf{x_h} \in \mathcal{X}_h$, define the household's aggregate welfare W_h as

$$W_h(\mathbf{x_h}) = \sum_{m=1}^{M(h)} U_{hm}(\mathbf{x_h}).$$

(WM) Welfare Maximization: $\mathbf{x_h}$ maximizes W_h on $B_h(p)$
for $h = 1, \ldots, H$.

(SEP) Separable Externalities: $U_i(x_{hm}, x_{h,-m}) = u_i(x_{hm}) + \sum_{n \neq m} v_{i;hn}(x_{hn})$
for $i = hm \in I$.

Given (SEP), (MON) amounts to increasing functions $u_i, i \in I$, and (NNE) amounts to non-decreasing functions $v_{i;j}$ for $i, j \in I$ with $i = hm, j = hn, 1 \leq m, n \leq M(h), m \neq n$.

Example 2.1 [Pareto-Optimality Without Individual Decentralization]
Let $\ell = 2$, $H = 1$ and $M(1) = 2$. We label the two consumers simply $i = 1, 2$ with generic consumption bundles $(x_i, y_i) \in \mathbb{R}^2_+$. Let $\omega_h = \omega = (2, 3) \in \mathbb{R}^2$.
We assume (SEP) with

$u_i(x_i, y_i) = x_i y_i$,
$v_{1;2}(x_2, y_2) = x_2$,
$v_{2;1} \equiv 0$.

Then (MON) and (NNE) hold. We further assume (WM). Then (BE) holds. Clearly,
$(x_1^*, y_1^*) = (1, 2)$, $(x_2^*, y_2^*) = (1, 1)$ maximizes W_h on $B_h(p^*)$ where $p^* = (2, 1)$. Set
$\mathbf{x}^* = (x_i^*, y_i^*)_{i=1,2}$.
Then $(p^*; \mathbf{x}^*)$ is a competitive equilibrium and the allocation $\mathbf{x}^*$ is Pareto-optimal. But because of the homotheticity of $u_1 = u_2$, individual consumer demands are collinear and $\mathbf{x}^*$ cannot be individually decentralized. Furthermore, it can be shown that each competitive equilibrium among households $(p; \mathbf{x})$ is of the form $\mathbf{x} = \mathbf{x}^*$ and $p = t \cdot p^*$ with $t > 0$. Hence $\mathbf{x}^*$ is the only competitive equilibrium allocation among households. ••

3 NEGATIVE EXTERNALITIES

For an economy with negative intra-household externalities, the following four distinct scenarios are mutually exclusive, but by no means exhaustive:

1.) The assertion of Proposition 2 persists, i.e. if $(p; \mathbf{x})$ is a competitive equilibrium among households, then $\mathbf{x}$ is Pareto-optimal and can be individually decentralized using the prevaling price system p. This possibility is illustrated by Example 3.1.

2.) Competitive equilibrium allocations are Pareto-optimal, but cannot be individually decentralized (with any market price system). This possibility is illustrated by Example 3.2.

3.) Competitive equilibrium allocations fail to be Pareto-optimal, yet can be individually decentralized. Examples 3.3 and 3.4 present such cases.

4.) Competitive equilibrium allocations are not Pareto-optimal and cannot be individually decentralized. See Example 3.5.

Incidentally, our examples demonstrate that Pareto-optimality and individual decentralizability of competitive equilibrium allocations among household are independent properties.

The subsequent Examples 3.1 – 3.5 all satify (MON), (SEP), and (WM). They constitute instances of negative or, more accurately, Non-Positive Externalities (NPE). The latter property is symmetric to (NNE):

(NPE) Non-Positive Externalities: $U_i(x_{hm}, x_{h,-m})$ is non-increasing in $x_{h,-m}$ for all $i = hm \in I$.

Under (SEP), (NPE) amounts to non-increasing functions $v_{i;j}$ for $i, j \in I$ with $i = hm, j = hn, 1 \leq m, n \leq M(h), m \neq n$.

Example 3.1 [Pareto-Optimality and Individual Decentralization]
We assume (SEP), (MON), and (WM). Moreover, we assume coefficients $\alpha_{i;j}$ such that

$v_{i;j} = -\alpha_{i;j} u_j$.

Then for $h = 1, \ldots, H$:

$W_h = \sum_{m=1}^{M(h)} (1 - \sum_{n \neq m} \alpha_{hn;hm}) u_{hm}$.

Put $\beta_{hm} = 1 - \sum_{n \neq m} \alpha_{hn;hm}$.

If $\beta_{hm} > 0$ for some $m = 1, \ldots, M(h)$, then household h exhausts its budget. Suppose this holds true for all households. Then a competitive equilibrium allocation among households is Pareto-optimal and can be individually decentralized. ••

Example 3.2 [Pareto-Optimality Without Indidividual Decentralization]
Modify Example 2.1 as follows:

$\omega_h = \omega = (2, 3)$,

$v_{1;2}(x_2, y_2) = -x_2$,

so that (MON) and (NPE) hold. (WM) still implies (BE), since consumer 2 does not experience any externality. Then $(x_1, y_1) = (1, 1)$, $(x_2, y_2) = (1, 2)$ maximizes W_h on $B_h(p)$ where $p = (1, 1)$. This yields a competitive eqilibrium allocation for the household that is Pareto-optimal, yet cannot be individually decentralized for the reasons given in Example 2.1. Moreover, any competitive equilibrium for the household takes the form $p = (t, t)$ with $t > 0$, $\mathbf{x} = ((1,1);(1,2))$ so that $\mathbf{x} = ((1,1);(1,2))$ is the unique equilibrium allocation for the household. ••

Failure of the classical first welfare theorem regarding competitive equilibria with individual market participation (one-person households) occurs already, if there exist one consumer i with locally non-satiated preferences and a second consumer j with satiation consumption bundle x_j^* who is over-endowed, i.e. whose endowment ω_j satisfies $\omega_j \gg x_j^*$. In the presence of negative intra-household externalities, a household may have a bliss point despite the fact that (MON) holds, that is each household member has monotonic preferences with respect to her individual consumption. Examples 3.3 – 3.5 demonstrate the possibility of such a household bliss point and a resulting inefficiency. Example 3.3 exhibits an interior bliss point and a Pareto-improving transfer to a household not exposed to externalities. Example 3.5 presents instances of interior bliss points and Pareto-improving transfers from a household suffering from negative externalities to another household that is partially exposed to negative externalities. Example 3.4 presents instances of boundary bliss points and Pareto-improving transfers from households suffering from externalities to other households that may also be exposed to negative externalities.

Furthermore, Examples 3.3 – 3.5 provide cases where (BE) is violated. A corresponding notion of competitive equilibrium has to relax the social feasibility condition (2): $\sum_i x_i = \omega$. Accordingly, market clearing will be replaced by a free disposal assumption:

(3) $\sum_i x_i \leq \omega$.

Example 3.3 [Sub-Optimality and Individual Decentralization]
Let $\ell = 1$ and $H = 2$. There are three consumers, simply labelled $i = 1, 2, 3$ with generic consumption bundles $x_i \geq 0$. Consumers 1 and 2 form a household denoted h which satisfies (SEP) and (WM). Consumer 3 forms a second household denoted k. The utilities are

$u_i(x_i) = x_i$;

$v_{1;2}(x_2) = -x_2^2$;

$v_{2;1}(x_1) = -x_1^2$.

Hence household h has a global bliss point $\mathbf{x}_\mathbf{h}^* = (x_1, x_2) = (\frac{1}{2}, \frac{1}{2})$ wheras household k satisfies (MON) without experiencing any externalities. Now let $\omega_h = 2$ and $\omega_k = 1$ mT $p > 0$; $\mathbf{x} = ((\frac{1}{2}, \frac{1}{2}), 1)$ constitute the competitive equilibria among households — with free disposal. Whereas $\mathbf{x}$ can be individually decentralized, it is weakly Pareto-dominated by the feasible allocation $\mathbf{y} = ((\frac{1}{2}, \frac{1}{2}), 2)$. ••

Example 3.4 [Sub-Optimality and Individual Decentralization]
Consider the set-up of Example 3.1 with the following properties:

- There exists a household h with $\beta_{hm} > 0$ for some $m = 1, \ldots, M(h)$.
- There exists a household k with $\omega_k \gg 0$ and $\beta_{kn} < 0$ for all $n = 1, \ldots, M(k)$.
- For all consumers $i \in I$, $\beta_i \neq 0$.

Then a competitive equilibrium allocation $\mathbf{x}$ satisfies $x_k = 0$ for a household k as specified above, is sub-optimal, and can be individually decentralized. ••

Example 3.5 [Sub-Optimality Without Individual Decentralization]
Add to Example 3.2 a two-person household k with endowment $\omega_k = (4, 4)$ and bliss point $b_k = ((1, 1); (1, 1))$. Then $p = (t, t)$ with $t > 0$ remains a (free disposal) equilibrium price system, with equilibrium consumption $\mathbf{x_h} = ((1, 1); (1, 2))$ and $\mathbf{x_k} = ((1, 1); (1, 1))$. This equilibrium allocation among households is still unique. It is not Pareto-optimal and cannot be individually decentralized. ••

4 CONCLUDING REMARKS

4.1 Résumé

Our conclusions are immediate ones, once an adequate framework has been put into place. Concerning optimality, we find that, by and large, competitive equilibria among households yield Pareto-optimal allocations. Sub-optimality can occur, if individual consumer preferences are satiated or exhibit negative externalities. Notice, however, that the latter type of consumer characteristics would impede Pareto-optimal equilibrium outcomes even more, if individual consumers instead of households participated in the market.

Regarding individual decentralization, we find that competitive equilibria among households yield allocations that can be individually decentralized, provided standard monotonicity and smoothness properties hold and externalities are absent. In the presence of — positive or negative — externalities, individual decentralization of competitive equilibria among households is bound to fail — with rare exceptions.

4.2 Testable Restrictions

McElroy and Horney (1981), Horney and McElroy (1988), and McElroy (1990) have developed parametric models to discern testable properties of household consumption plans resulting from cooperative bargaining.[2] Recently, Snyder (1994) has proposed non-parametric tests based on revealed preference theory and quantifier elimination techniques to achieve two objectives:

(a) Determine whether a sample of price-quantity data could have been generated at all as equilibrium outcomes of some finite economy.

(b) Discriminate between data potentially generated by equilibria among households and those generated by equilibria among individuals.

Regarding (b), our theoretical insights suggest that one can distinguish between two types of models of finite pure exchange economies. A model is of the first type, if individual decentralization of equilibria among households is possible. Then empirical data drawn from such an economy would allow either interpretation: outcomes of equilibria among households and outcomes of equilibria among individuals. Consequently, certain tests designed to discriminate between those two kinds of outcomes would be rendered rather powerless, if not obsolete. The model economy belongs to the second type, if individual decentralization is impossible. In that case, tests designed to discriminate between equilibria among households and equilibria among individuals promise to have more bite.

4.3 Addendum: The Neoclassical Household

Like the literature, we distinguish between (I) equilibria among households and (II) equilibria among individuals. Apart from terminology and minor technical details, there exist no drastic conceptual

[2] See also the critique by Chiappori (1988b) and the reply by McElroy and Horney (1990).

differences with respect to (II). Regarding (I), there are noticeable differences in concepts and emphasis. Chiappori (1988a) and in the sequel Snyder (1994) have employed the general notion of the **neoclassical household** whereas McElroy and Horney (1981) and their subsequent work focus on the special case of **Nash-bargained household decisions**. A "neoclassical household" is a household h that has a household utility function

$$V_h : \mathcal{X}_h \longrightarrow \mathbb{R}$$

and maximizes V_h on $B_h(p)$ given any price system p. In contrast, our concept of an equilibrium $(p; \mathbf{x})$ among households requires

(1h) $\mathbf{x_h} \in EB_h(p)$

where the definition of an efficient budget set $EB_h(p)$ rests on individual utility functions $U_{hm},\ m = 1, \ldots, M(h)$. Suppose both a household utility function V_h and individual utility functions co-exist. Then a maximizer of V_h on $B_h(p)$ cannot be expected to belong to $EB_h(p)$ or to be individually decentralizable, unless the household utility function reflects individual welfare of household members. Individual and household welfare may be linked via a social welfare function for the household,

$$S_h : \mathbb{R}^{M(h)} \longrightarrow \mathbb{R}$$

which in turn determines

$$V_h(\mathbf{x_h}) \equiv S_h(U_{h1}(x_{h1}), \ldots, U_{hM(h)}(x_{hM(h)})).$$

If S_h is strictly increasing and the household satisfies (MON) and (NNE), then a maximizer of V_h on $B_h(p)$ belongs to $EB_h(p)$ and (BE) holds for this household. Furthermore, under the latter circumstances, the assertions of Propositions 1 – 3 hold true. Special cases of (in the relevant domain) strictly increasing social welfare functions for a household are a "Nash product", giving rise to a Nash-bargained household decision, and a "utility sum", giving rise to aggregate welfare maximization.

4.4 Further Qualifications

Our analysis is confined to a formal setting similar to that of our short list of references which constitutes the most closely related and relevant literature.[3] Thus we disregard the local public goods aspect of joint habitation and certain consumer durables, such as refrigerators, furnaces, microwaves, dishwashers, washing machines. Time and money savings due to joint shopping and, more generally, economies of joint household activities are ignored. On the other hand, conflict resolution within households is frictionless by assumption. In particular, time spent and resources expended on conflict resolution are neglected. Incorporating some of these omitted features, while intriguing and important, would exceed the scope and purpose of this inquiry.

[3] Let us point out, however, that most of the existing literature is concentrated on the particular case of one household with two members and relies on the assumption of "egoistic" household members, i.e. our assumption (E2).

5 PROOFS

PROOF OF PROPOSITION 2

Assume consumer preferences and utility representations as hypothesized. Let $(p;\mathbf{x})$ be a competitive equilibrium among households with $\mathbf{x} \gg 0$. Then (i) holds by Proposition 1.

Now consider any household $h = 1, \ldots, H$.

Since $\mathbf{x} \gg 0$, also $\mathbf{x_h} \gg 0$. Moreover, (i) implies that $\mathbf{x}$ is a Pareto-optimal allocation of the pure exchange economy consisting of all the members of household h with social endowment $\mathbf{e_h} \equiv \sum_{m=1}^{M(h)} x_{hm}$. Because of

◇ this optimality property,

◇ $\mathbf{x} \gg 0$, and

◇ the hypothesized properties of the utility functions,

there exist a vector $q \in \mathbb{R}^{\ell}_{++}$ and scalars $\gamma_{hm} > 0,\ m = 1, \ldots, M(h)$ such that $grad\ u_{hm}(x_{hm}) = \gamma_{hm} \cdot q$ for $m = 1, \ldots, M(h)$.

Suppose there is <u>no</u> $\mu_h > 0$ such that $q = \mu_h \cdot p$. Then for each $m = 1, \ldots, M(h)$, there exists a net trade $z_{hm} \in \mathbb{R}^{\ell}$ such that

$$x_{hm} + z_{hm} \in X_{hm},\ p \cdot z_{hm} \leq 0,\ u_{hm}(x_{hm} + z_{hm}) > u_{hm}(x_{hm}),$$

contradicting $\mathbf{x_h} \in EB_h(p)$. Since household h has been arbitrarily chosen, this shows that for each $i = hm \in I$, there exists $\lambda_i = \mu_h \cdot \gamma_{hm} > 0$ such that

$grad\ u_i(x_i) = \lambda_i \cdot p$.

Yet the latter identities are the first order conditions for an interior competitive equilibrium among consumers $i \in I$, with equilibrium price system p. Hence (ii). Q.E.D.

PROOF OF PROPOSITION 3

Assume (E1) and (BE). Let $(p; \mathbf{x})$ be a competitive equilibrium — among households.

Suppose $\mathbf{x}$ does <u>not</u> belong to the H-core. Then there exist a family of households $G \in \mathcal{G}$, a corresponding coalition $C(G)$, and a $(\mathbf{y_h})_{h \in G} \in \prod_{h \in G} \mathcal{X}_h$ such that

1. $u_i(\mathbf{y_h}) \geq u_i(\mathbf{x_h})$ for all $i = hm \in C(G)$.
2. $u_i(\mathbf{y_h}) > u_i(\mathbf{x_h})$ for some $i = hm \in C(G)$.
3. $\sum_{i \in C(G)} y_i = \sum_{h \in G} \omega_h$.

From 1. and 2. follows the existence of a household $h \in G$ such that $u_{hm}(\mathbf{y_h}) \geq u_{hm}(\mathbf{x_h})$ for all $m = 1, \ldots, M(h)$ and $u_{hm}(\mathbf{y_h}) > u_{hm}(\mathbf{x_h})$ for some $m = 1, \ldots, M(h)$.
Since $\mathbf{x_h} \in EB_h(p)$,

$p * \mathbf{y_h} > p \cdot \omega_h$

has to hold. In a similar way, the inequality $p * \mathbf{y_g} \geq p \cdot \omega_g$ for every $g \in G$
follows from 1. combined with (BE) and $\mathbf{x_g} \in EB_g(p)$. But then

$p \cdot \sum_{i \in C(G)} y_i = \sum_{h \in G} p * \mathbf{y_h}$
$> \sum_{h \in G} p \cdot \omega_h = p \cdot \sum_{h \in G} \omega_h,$

contradicting 3.. Hence, to the contrary, $\mathbf{x}$ belongs to the H-core. Q.E.D.

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