Journal of Economic Dynamics and Control 14 (1990) 435–450. North-Holland

QUALITATIVE DYNAMICS AND CAUSALITY IN A KEYNESIAN MODEL

Ron BERNDSEN and Hennie DANIELS*

Tilburg University, 5000 LE Tilburg, The Netherlands

Received January 1989, final version received October 1989

In this paper we present a formalism to describe economic dynamics in a qualitative way. This formalism is a modification of an existing algorithm for qualitative simulation as proposed by Kuipers. It is demonstrated that the framework of qualitative dynamics can clarify economic reasoning without using any quantative data. Especially causal arguments that sometimes mysteriously occur when economists implicitly mix static and dynamic models, can be understood in a formal way. Furthermore, we bring together the lines of thought recently established in the field of artificial intelligence and the results of qualitative economics that can be found in earlier papers. A simple Keynesian model serves as an example throughout this text.

1. Introduction

Economic theory is largely concerned with economic modelling of complex systems. Economists are interested in the determination of the equilibrium values of the relevant parameters in static models or the characteristics of solutions to dynamic models such as stability issues. Economic models are composed of functional relationships between economic variables. They originate either from structural equations imposed by economic theory or relations in which variables are defined in terms of others, e.g., the definition of the balance of payments. The growing complexity of these models mirrors the tremendous increase in computer power available. The drawback of the increasing complexity is the intractability of the computer output. Both questions about the relevance and the explanation of the results are often difficult to answer [e.g., Royer and Ritschard (1984)].

Contrary to these developments, textbooks treat simple economic models and focus mainly on qualitative aspects and explanation. In the absence of precise quantitative data, it seems quite natural to perform qualitative reasoning. In moderately complex models, however, qualitative reasoning can be subtle and intuitive arguments are required to keep track of the steps in the

*We thank Johan M. Broek for many valuable discussions and the reviewer of this paper for comments on an earlier draft.

reasoning process. In fact, one sometimes encounters incomplete arguments, for example when feedback loops are part of a dynamic interpretation of a static model. Apart from these two approaches of, on the one hand, the elaborate numerical models and, on the other hand, the verbal and intuitive or semiformal models, there is a growing interest in formal *qualitative* models. The arguments to study formal qualitative models can be summarized as follows: the lack of consistent quantitative data, the wish to create formal procedures for tracing causal chains, the validation of the structure of quantitative models, and the description of structural change of economic models [cf. Fontela (1986), Royer and Ritschard (1984), Bourgine and Raiman (1986), Boutillier (1984)]. A reason of different nature is the proliferation of symbolic programming languages.

Samuelson (1947) was the first to consider qualitative static models. From then on many contributions have been made to the theory of qualitative comparative statics. A fairly extensive overview of the results can be found in Greenberg and Maybee (1981). In the last decade, researchers in AI have studied qualitative models of physical systems and electronic circuits [see Bobrow et al. (1984)]. Also in medical diagnosis qualitative models are being considered [e.g., Kuipers and Kassirer (1984)]. Some of the results obtained in qualitative reasoning correspond to earlier results in comparative statics. The similarity between the theory of confluences [de Kleer and Brown (1984)] and comparative statics [Samuelson (1947)] has been pointed out in Iwasaki and Simon (1986a).

The exploration of techniques developed in qualitative reasoning in economic theory are currently being studied [cf. Farley (1986), Bourgine and Raiman (1986), Pau (1986), Berndsen and Daniels (1988)]. These methods should fill the gap between the classical number crunching approach and verbal intuitive economic reasoning. The basic techniques in qualitative reasoning use causal modelling and constraint propagation. In section 2 we compare the formal notions of causality as described in Iwasaki and Simon (1986a, b) and de Kleer and Brown (1986). It is shown that the causality derived from static models by the methods of causal ordering and mythical causality does not reflect the intuitive notion of causality. One way to get around this problem is to consider dynamic models [see Iwasaki (1988)]. However, we believe that it is unsatisfactory to derive the causal structure from a somewhat arbitrary dynamic model. Therefore the explicit representation of causal relations in economic models is investigated.

Hicks (1979) considers two different kinds of causality: contemporaneous causality and sequential causality. In contemporaneous causality a variable A having a causal link to a variable B directly influences B and hence cause and effect occur in the same time period. In economics, sequential causality, takes place in two steps: a change in A leads to decisions based on it which in turn have effects on B. The decision-making is an intermediate stage in the causation taking place. There is always a time-lag between cause and effect in the case of sequential causality. In section 3 we consider the idea of an explicit representation of both notions of causality. This amounts to a dynamic qualitative model consisting of standard symbolic constraints originating from balance sheet equations and constraints representing contemporaneous causality. Relations between economic entities that correspond to sequential causality complete the model with so-called sequential causal constraints. Furthermore, we describe an algorithm for qualitative simulation based on constraint propagation. This algorithm is a modification of the algorithm QSIM [see Kuipers (1986)].

2. Causality

2.1. Causal ordering

To illustrate the notion of causal ordering, we start with a simple Keynesian model [see, e.g., Dennis (1981, ch. 4), Samuelson (1947, ch. 9)]:

$$f_1(Y, I) = 0, \qquad f_1 = Y - f(I), \qquad f' > 0, \tag{1}$$

$$f_2(I, r) = 0, \qquad f_2 = I - g(r), \qquad g' < 0, \tag{2}$$

$$f_3(Y, M_1) = 0, \qquad f_3 = M_1 - h(Y), \qquad h' > 0, \tag{3}$$

$$f_4(r, M_2) = 0, \qquad f_4 = M_2 - i(r), \qquad i' < 0, \tag{4}$$

$$M_d - M_1 - M_2 = 0, \tag{5}$$

$$M_d - M_s = 0, \tag{6}$$

$$M_s = c_1, \tag{7}$$

where

I = investment, M_d = total money demand,

Y = national income, M_1 = transactions money demand,

r = interest, M_2 = speculative money demand,

c_1 = constant > 0, M_s = money supply.

The theory of causal ordering can be found in Simon (1957). This technique derives a causal ordering among variables in a system of n equations and n

unknowns. The total causal ordering for the Keynesian model is as follows:

$$M_s \to M_d \to \{Y, I, r, M_1, M_2\}.$$

The only minimal complete subset of zero order is M_s and M_d is the derived structure of first order. The derived structure of second order consists of $\{Y, I, r, M_1, M_2\}$. The resulting ordering is correct if we keep in mind that the model describes equilibrium positions for a given value of the money supply. After a disturbance in the money supply, equilibrium is only restored when money supply equals money demand (money market equilibrium). Hence, the other variables change only after the total money demand has changed caused by a change in the money supply. This kind of explanations cannot be used to describe the trajectory from one equilibrium position to another. A similar example in physics has been given in Iwasaki (1988).

2.2. *Mythical causality*

De Kleer and Brown (1986) describe the qualitative increment between two equilibrium states by a set of confluences. The solution of this set of confluences can be found by constraint propagation. The static Keynesian model can be formulated in terms of confluences starting from the equations given in section 2.1:

$$\partial Y = \partial I, \tag{8}$$

$$\partial I = -\partial r, \tag{9}$$

$$\partial Y = \partial M_1, \tag{10}$$

$$-\partial r = \partial M_2, \tag{11}$$

$$\partial M_d = \partial M_1 + \partial M_2, \tag{12}$$

$$\partial M_d = \partial M_s. \tag{13}$$

A confluence is a constraint with qualitative derivatives of variables which can take on a value from the set $\{+, 0, -\}$. An assignment of a value to each variable in a set of confluences in such a way that all confluences are satisfied is called an 'interpretation'. Due to ambiguities, it is possible that a state has more than one interpretation.

The order in which the variables are determined is called mythical causality. It is called mythical because all the changes take place at the same instant. It is important to note that the differential ∂ is the differential with respect to M_s and not with respect to time [compare de Kleer and Brown (1986)]. The unique interpretation of the set of confluences after a money supply shock is given by

$$[\partial Y, \partial I, \partial r, \partial M_1, \partial M_2, \partial M_d, \partial M_s] = [+, +, -, +, +, +, +].$$

There are two ways to propagate the disturbance through the confluences which lead to two different so-called causal explanations:

$$\partial M_s = + \to \partial M_d = + \to \partial M_1 = + \to \partial Y = + \to \partial I = + \to$$
$$\partial r = - \to \partial M_2 = +,$$
$$\partial M_s = + \to \partial M_d = + \to \partial M_2 = + \to \partial r = - \to \partial I = + \to$$
$$\partial Y = + \to \partial M_1 = +.$$

Both explanations do not represent the intuitive notion of sequential causality. A clear difference between causal ordering and mythical causality is that mythical causality provides the signs of the changes of the variables after a disturbance. In general, the signs are not unique; in this case, however, they are. This can also be seen by applying a theorem of qualitative comparative statics. The confluences can be written in the following form:

$$\begin{pmatrix} \frac{\partial f_1}{\partial Y} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial r} \\ \frac{\partial f_2}{\partial Y} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial r} \\ \frac{\partial M_d}{\partial Y} & \frac{\partial M_d}{\partial I} & \frac{\partial M_d}{\partial r} \end{pmatrix} \begin{pmatrix} \frac{dY}{dM_s} \\ \frac{dI}{dM_s} \\ \frac{dr}{dM_s} \end{pmatrix} = \begin{pmatrix} -\frac{\partial f_1}{\partial M_s} \\ -\frac{\partial f_2}{\partial M_s} \\ \frac{\partial M_d}{\partial M_s} \end{pmatrix}, \tag{14}$$

which is equivalent to

$$\begin{pmatrix} + & - & 0 \\ 0 & + & + \\ + & 0 & - \end{pmatrix} \begin{pmatrix} \frac{dY}{dM_s} \\ \frac{dI}{dM_s} \\ \frac{dr}{dM_s} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ + \end{pmatrix}. \tag{15}$$

It can easily be seen [Maybee (1981)] that (15) is sign-solvable and the solution is given by

$$\left(\frac{dY}{dM_s} \; \frac{dI}{dM_s} \; \frac{dr}{dM_s} \right)^T = [+ \; + \; -]^T.$$

The kind of causality which appears in static models does not reflect the dynamic behaviour of a system. The reason, of course, is that in a static model no time elapses between cause and effect, i.e., all the effects are instantaneous. This only makes sense in case one variable is simply a definition in terms of the other variables (e.g., a balance-sheet equation). In other cases it is convenient to assume that cause precedes effect. The theory of causal ordering is extended to dynamical systems by Iwasaki (1988). The application to the Keynesian model is discussed in the next subsection.

2.3. Causal ordering in a mixed model

A mixed structure consists of static and dynamic equations. A mixed structure can be obtained from a static model by replacing one or more static equations with their dynamic counterparts, or from a dynamic model by replacing dynamic equations with corresponding static equations. Sometimes it is difficult to know which static equations should be altered into a dynamic equation. This should depend on the relative speed of adjustment of economic effects and the level of time-scale abstraction [cf. Boutillier (1984)]. In the static model [eqs. (1)–(7)] we alter the equation describing the money market and the equation representing the investment function. In the first case, the idea is that when the money market is in disequilibrium the interest rate is changing. In case of an excess supply (demand), the interest rate decreases (increases) [eq. (6′)]. Eq. (2) is replaced by the dynamic eq. (2′) because entrepreneurs have a desired level of investment that depends upon the interest rate and can differ from the actual level of investment. If the actual investment is lower (higher) than the desired investment, entrepreneurs increase (decrease) their investments. The dynamic equations are given by

$$f_2(I, r) = \dot{I}, \tag{2'}$$

$$M_d - M_s = \dot{r}. \tag{6'}$$

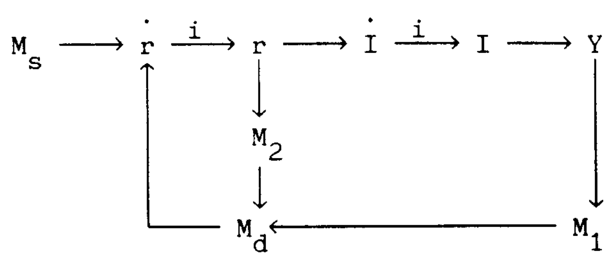

Fig. 1. Causal ordering in the mixed model.

The causal ordering of the mixed model in the sense of Iwasaki (1988) is depicted in fig. 1. The diagram represents the intuitive notion of causality in the Keynesian model. Although the causal ordering obtained from this model is correct, the translation of the verbal description of economic cause and effect into differential equations is somewhat arbitrary. Therefore, we propose an explicit representation of causality in the next section.

3. Qualitative modelling

In this section we describe a formalism for qualitative reasoning in economic systems and apply it to the Keynesian model. The formalism is an intermediate form of the method of qualitative simulation (QSIM) developed by Kuipers (1986) and the theory of confluences of de Kleer and Brown (1984). The main differences between these two methods and the approach taken here emerge from the fact that the former were designed to simulate the behaviour of physical systems. The differences are: intra-state behaviour as defined by de Kleer and Brown (1984) is not explicitly taken into account. In addition, we only consider fixed quantity spaces and uniform time intervals as opposed to quantity spaces containing arbitrary many landmark values [cf. Kuipers (1986)]. Furthermore, we incorporate so-called causal constraints which reflect sequential causality [Hicks (1979)] or 'Wiener–Granger' causality [Pierce and Haugh (1977)].

3.1. The formalism

An economic system S consists of a set of economic variables and a set of constraints. Time is represented by a totally ordered set T of n half-open intervals of fixed length:

$$T = \{[t_0, t_1\rangle, [t_1, t_2\rangle, \ldots, [t_{n-1}, t_n\rangle\},$$

or conveniently as

$$\{i_0, \ldots, i_{n-1}\}.$$

The economic interpretation of these fixed-length time intervals is that a time interval corresponds to an accounting period, e.g., a quarter or a year. However, the assumption of fixed-length time intervals is not crucial. The only thing that matters is that the time concept induces a partial ordering on the qualitative states of the system. [In Williams (1986) an event-based time representation is proposed such that the time in which the behaviour of a variable remains qualitatively the same, is mapped into a single time interval. This results in time intervals of different length.]

For every variable x_j of the economic system, two functions are defined $QVAL(x_j)$ and $QDIR(x_j)$. These functions denote respectively the qualitative value and the qualitative direction of x_j at a particular time interval:

$$QVAL(x_j)\colon T \to QSVAL_j, \qquad QDIR(x_j)\colon T \to QSDIR_j,$$

where $QSVAL_j$ and $QSDIR_j$ denote quantity spaces associated with x_j. A quantity space is a totally ordered finite set of symbolic values. Various quantity spaces have been investigated in the literature [cf. Kuipers (1986), de Kleer and Brown (1984), Raiman (1986)]. The quantity space for the qualitative value $QVAL$ may be different for every parameter in S. Usually, it contains a finite number of landmarks, which are interesting values for that parameter. Also all intervals between adjacent landmarks belong to this set. In the general case,

$$QSVAL_j = \{\langle l_l, l_1\rangle, l_1, \ldots, l_k, \langle l_k, l_u\rangle\},$$

where l_l and l_u are respectively the lower and upper limit of x_j that x_j cannot reach or pass. Since landmarks are just symbols, no arithmetic operators are defined. For brevity, we use the shorthand notation $l(i, j)$ denoting the landmark l_i if $i = j$ and the interval $\langle l_i, l_j\rangle$ if $i = j - 1$. The set of landmark values represents the granularity of the corresponding parameter in the model. For some parameters only $QDIR$'s are of importance. In that case $QSVAL$ consists of one single element λ which may denote $\langle -\infty, \infty\rangle$ or $\langle 0, \infty\rangle$. The quantity space for $QDIR$ is the same for every parameter: $QSDIR = \{-, 0, +\}$. The interpretation of the quantity space $QSDIR$ is that x_j is decreasing if $QDIR = -$, is steady if $QDIR = 0$, and is increasing if $QDIR = +$.

The qualitative state $QS(x_j, i_k)$ of a parameter x_j at $[t_k, t_{k+1}\rangle$ is defined as the pair $(QVAL(x_j, i_k), QDIR(x_j, i_k))$ which are, respectively, the qualitative value of x_j at t_k and the qualitative direction of x_j at $[t_k, t_{k+1}\rangle$. A qualitative state of the economic system S at $[t_k, t_{k+1}\rangle$ is the union of qualitative states of the parameters x_j. Thus,

$$QS(S, i_k) = QS(x_1, i_k), \ldots, QS(x_m, i_k).$$

A qualitative behaviour of a parameter x_j from $[t_k, t_{k+1}\rangle$ to $[t_{k+l}, t_{k+l+1}\rangle$ is a sequence of qualitative states:

$$QS(x_j, i_k), QS(x_j, i_{k+1}), \ldots, QS(x_j, i_{k+l}).$$

Accordingly, a qualitative behaviour of the system S from $[t_k, t_{k+1}\rangle$ to $[t_{k+l}, t_{k+l+1}\rangle$ is the corresponding sequence of qualitative states of S.

In qualitative simulation it is possible that a state at a given time interval has more than one successor state. In that case the simulation branches at the next time interval and each qualitative state is pursued separately. This results in multiple qualitative behaviours of the system S. The qualitative simulation of S is the tree of all qualitative behaviours of S from the starting point of the simulation $[t_0, t_1\rangle$ to the horizon $[t_{n-1}, t_n\rangle$.

Relations between parameters in S are expressed as constraints. Some constraints correspond to familiar mathematical operators, such as addition and differentiation, in a qualitative context. Other constraints define monotonic and causal relationships between parameters. A constraint is satisfied if the conditions corresponding to the constraint are met. The definition of the particular constraints represent the semantics of the economic relations. The constraints are defined in section 3.2.

3.2. *Example: The Keynesian model*

The Keynesian model can be reformulated into a constraint representation. In this representation there are seven parameters $\{C, I, Y, M_1, M_2, M_d, r\}$ and seven constraints. The quantity space $QSVAL$ for M_d is $\{\langle l_l, l_e\rangle, l_e, \langle l_e, l_u\rangle\}$ and the quantity space for the other parameters is $\{\lambda\}$ where λ stands for $[0, \infty\rangle$. The constraints are given by

$$ADD(C, I, Y), \tag{16}$$

$$ADD(M_1, M_2, M_d), \tag{17}$$

$$M^+(M_1, Y), \tag{18}$$

$$DERIV(r, M_d), \tag{19}$$

$$SC^+(Y, C), \tag{20}$$

$$SC^-(r, I), \tag{21}$$

$$SC^-(r, M_2). \tag{22}$$

The constraint (16) denotes the national accounting identity in a closed economy without a government ($Y = C + I$) and in (17) the total money demand is defined as the sum of M_1 and M_2. The relationship between M_1 and Y is given by a monotonicity constraint. This corresponds to the formal representation of contemporaneous causality. (20), (21), and (22) are constraints representing sequential causality. They impose a relation on the direction of change of the first parameter and the direction of change of the second parameter in the next time interval. In the SC^+ constraint both parameters point in the same direction, whereas in the SC^- constraint the directions are opposite. Constraint (19) reflects the adjustment mechanism of the money market.

In the following, the constraints are described formally.

3.2.1. ADD constraint

$ADD(a, b, c)$ defines the variable c as the qualitative sum of the variables a and b. Depending on the particular application at hand, it is possible to take both *QVAL* and *QDIR* into account or only *QDIR*. The former case applies only if for all parameters joined by an *ADD* constraint $QSVAL = \{\langle l_l, l_e\rangle, l_e, \langle l_e, l_u\rangle\}$. If the relative position of a variable with respect to l_e is taken into account, the quantity space can be written as $QSVAL = \{-, 0, +\}$. It is assumed that the *ADD* constraint holds for the tuple (0,0,0). A tuple of qualitative values of the variables a, b, and c satisfy the *ADD* constraint at $[t_k, t_{k+1}\rangle$ if

$$QVAL(a, i_k) \oplus QVAL(b, i_k) \cong QVAL(c, i_k),$$

where $\oplus$ (qualitative addition) and $\cong$ (weak equality sign) are defined by the following tables:

$\oplus$	+	−	0	?
+	+	?	+	?
−	?	−	−	?
0	+	−	0	?
?	?	?	?	?

$\cong$	+	−	0	?
+	T	F	F	T
−	F	T	F	T
0	F	F	T	T
?	T	T	T	T

The weak equality sign $\cong$ should be read as a two-place predicate. Here we will not go into details of qualitative algebra, the interested reader is referred to Dormoy and Raiman (1988) and Williams (1988).

Furthermore, the *ADD* constraint puts also a restriction on the *QDIR*'s of a, b, and c, which is equivalent to the restriction on the *QVAL*'s.

3.2.2. M^+ and M^- constraint

The monotonicity constraints $M^+(a, b)$ and $M^-(a, b)$ define a monotonic functional relationship between a and b. M^+ is appropriate if the relationship between a and b is monotonic and increasing. Conversely, if the relationship is decreasing M^- applies. The monotonicity constraint puts a restriction on the *QDIR*'s of a and b, namely for the M^+ constraint,

$$QDIR(a, i_k) = QDIR(b, i_k),$$

and similarly with a minus sign for M^-.

3.2.3. DERIV constraint

The derivative relation between two parameters is represented by the *DERIV* constraint. $DERIV(a, b)$ is satisfied at $[t_k, t_{k+1}\rangle$ iff the pair $(QDIR(a, i_k), QVAL(b, i_k))$ matches one of the entries in the table below:

$DERIV(a, b)$	$QDIR(a, i_k)$	$QVAL(b, i_k)$
	0	l_e
	+	$\langle l_e, l_u\rangle$
	–	$\langle l_l, l_e\rangle$

Note that a reference is made to the qualitative value of the second argument of the *DERIV* constraint. So, if the qualitative derivative relation $DERIV(a, b)$ holds, the quantity space of b must include at least three symbolic values $\{\langle l_l, l_e\rangle, l_e, \langle l_e, l_u\rangle\}$, where l_l and l_u are lower and upper limit of the parameter b.

3.2.4. SC^+ and SC^- constraint

The causal constraints $SC^+(a, b)$ and $SC^-(a, b)$ denote the relation of sequential causality between a and b. $SC^+(a, b)$ holds if a influences b positively. If the influence of a on b is negative, then $SC^-(a, b)$ holds. The constraint $SC^+(a, b)$ puts a restriction on $(QS(a, i_{k-1}), QS(b, i_k))$ as follows:

$$QDIR(a, i_{k-1}) = QDIR(b, i_k),$$

and similar with a minus sign for SC^-.

3.2.5. The simulation algorithm

The input for the simulation algorithm consists of:

- a set of parameters x_i $(i = 1, \ldots, m)$,
- a set of quantity spaces $QSVAL_i$ corresponding to x_i,
- a set of constraints expressing the relations between the parameters x_i,
- the initial conditions of the system at the starting period $[t_0, t_1\rangle$: $QS(S, i_0)$.

The output of the algorithm is the qualitative simulation of S, i.e., the tree of all qualitative behaviours of S originating from $QS(S, i_0)$ to the horizon i_n. The simulation starts with the creation of a list, containing states to be explored, called ACTIVE. Initially, ACTIVE consists of one state: $QS(S, i_0)$. Afterwards the algorithm repeatedly determines successor states from the first state in ACTIVE until ACTIVE is empty or the horizon of the simulation is reached. A successor state can be determined by applying the following steps:

(1) Determine for each parameter the possible transitions: D- or L-transitions depending on $QSVAL$ (The transitions are defined in table 1 and 2 below.)

(2) Constraint consistency filtering: determine for each constraint the combinations of transitions that satisfy the constraint.

(3) Pairwise consistency filtering: delete the combinations of transitions of adjacent constraints which do not agree on the transition of the parameters in common. This is called Waltz filtering.

(4) Global consistency filtering: generate global interpretations, i.e., an assignment of transitions to all parameters such that all constraints are satisfied simultaneously.

Table 1

D-transitions.

	$QDIR(x_i, i_{k-1}) \Rightarrow$	$QDIR(x_i, i_k)$
D_1	0	0
D_2	0	+
D_3	0	−
D_4	+	0
D_5	+	+
D_6	+	−
D_7	−	0
D_8	−	+
D_9	−	−

Table 2

L-transitions.[a]

	$QS(x_i, i_{k-1}) \Rightarrow$	$QS(x_i, i_k)$	
L_1	$\langle l(i, j), 0\rangle$	$\langle l(i, j), 0\rangle$	$j = i + 1$ or $j = i = 1$
L_2	$\langle l(i, j), 0\rangle$	$\langle l(i, j), +\rangle$	$j = i + 1$ or $j = i = 1$
L_3	$\langle l(i, j), 0\rangle$	$\langle l(i, j), -\rangle$	$j = i + 1$ or $j = i = 1$
L_4	$\langle l(i, j), +\rangle$	$\langle l(j, j+1), 0\rangle$	$j = i + 1$, $j < 2$ or $j = i = 1$
L_5	$\langle l(i, j), +\rangle$	$\langle l(j, j+1), +\rangle$	$j = i + 1$, $j < 2$ or $j = i = 1$
L_6	$\langle l(i, j), +\rangle$	$\langle l(j, j), +1), -\rangle$	$j = i + 1$, $j < 2$ or $j = i = 1$
L_7	$\langle l(i, j), +\rangle$	$\langle l(j, j), 0\rangle$	$j = i + 1$, $j < 2$
L_8	$\langle l(i, j), +\rangle$	$\langle l(j, j), +\rangle$	$j = i + 1$, $j < 2$
L_9	$\langle l(i, j), +\rangle$	$\langle l(j, j), -\rangle$	$j = i + 1$, $j < 2$
L_{10}	$\langle l(i, j), +\rangle$	$\langle l(i, j), 0\rangle$	$j = i + 1$
L_{11}	$\langle l(i, j), +\rangle$	$\langle l(i, j), +\rangle$	$j = i + 1$
L_{12}	$\langle l(i, j), +\rangle$	$\langle l(i, j), -\rangle$	$j = i + 1$

[a] $i = 0, 1$ and $j = 1, 2$ and the transitions L_{13}–L_{21} are defined symmetrically to L_4–L_{12}.

Table 3
Initial conditions.

i	x_i	$QSVAL_i$	$QS(x_i, i_0)$
1	C	$\{\lambda\}$	$\lambda, 0$
2	I	$\{\lambda\}$	$\lambda, 0$
3	M_1	$\{\lambda\}$	$\lambda, 0$
4	M_2	$\{\lambda\}$	$\lambda, 0$
5	M_d	$\{\langle l_l, l_e\rangle, l_e, \langle l_e, l_u\rangle\}$	$\langle l_l, l_e\rangle, 0$
6	r	$\{\lambda\}$	$\lambda, -$
7	Y	$\{\lambda\}$	$\lambda, 0$

(5) Apply global filters to the potential successor states and place remaining states on ACTIVE. The global filters are:
NO CHANGE: Mark a potential successor state as NO CHANGE if all the transitions of the parameters are in the set $\{D_1, D_5, D_9\}$ and $\{L_1, L_{11}, L_{21}\}$. Install a pointer to the immediate predecessor.
CYCLE: If a potential successor state is identical to one of its predecessors, except the immediate predecessor, mark the behaviour as cyclic and install a pointer from the current state to the identical predecessor.

If the quantity space for a particular parameter $QSVAL = \{\lambda\}$, then only transitions of the qualitative directions need to be taken into account, these transitions are called D-transitions. However, if the set of landmarks $QSVAL = \{\langle l_l, l_e\rangle, l_e, l_e, l_u\rangle\}$, so-called L-transitions apply.

3.3. Results of the simulation

The algorithm is applied to the Keynesian model. The parameters and initial conditions are given in table 3. The initial conditions correspond to a situation where a positive money supply stock is given at t_0. The landmark l_e in the quantity space of M_d corresponds with the value of the exogenous money supply after the money supply shock. The first transition is $QS(S, i_0) \Rightarrow QS(S, i_1)$. The initial state $QS(S, i_0)$ is placed on ACTIVE.

The first step of the algorithm is the determination of the possible transitions for each parameter. The result of step 1 is summarized below (the prefix D of D-transitions is omitted):

x_j	C	I	M_1	M_2	M_d	r	Y
	1	1	1	1	L_1	7	1
	2	2	2	2	L_2	8	2
	3	3	3	3	L_3	9	3

Table 4

Oscillating behaviour.

x_i	i_0	i_1	i_2	i_3	i_4	i_5	i_6
C	0	0	+	+	−	−	+
I	0	+	+	−	−	+	+
M_1	0	+	+	−	−	+	+
M_2	0	+	+	−	−	+	+
M_d	$\langle l_l, l_e\rangle$,0	$\langle l_l, l_e\rangle$,+	$\langle l_e, l_u\rangle$,+	$\langle l_e, l_u\rangle$,−	$\langle l_l, l_e\rangle$,−	$\langle l_l, l_e\rangle$,+	$\langle l_e, l_u\rangle$,+
r	−	−	+	+	−	−	+
Y	0	+	+	−	−	+	+

After the application of step 4 only one global interpretation is possible:

x_j	C	I	M_1	M_2	M_d	r	Y
	1	2	2	2	L_2	9	2

The transitions of the variables given in the global interpretation above determine $QS(S, i_1)$. One of the possible successors of $QS(S, i_1)$ initiates oscillating (cyclic) behaviour. This behaviour is shown in table 4.

Note that it is impossible to reach the equilibrium position where all $QDIR$'s = 0. In the predecessor state of equilibrium Y and r must have qualitative states:

$$QS(Y, [t_{k-1}, t_k\rangle) = \lambda, 0, \qquad QS(r, [t_{k-1}, t_k\rangle) = \lambda, 0.$$

This would imply that M_d is restricted to the transition L_1 from i_{k-1} to i_k. Therefore, M_d must remain steady on l_e. There are only three possible global qualitative states which satisfy these criteria. One of them is the equilibrium state itself. In the other two, C and I have opposite signs. However, in both cases, this would imply that I and M_2 do not agree on the same sign. Hence, there is no alternative for r at i_{k-2}.

Remark. The algorithm described here is implemented in LPA Prolog. Other examples, e.g., a dynamic three-sector macro model where the goods, money, and labour markets interact, have been tested. In Farley and Lin (1990) similar multiple-market models are analyzed. In their paper perturbations of equilibrium initiate a process of updating market states. These markets may interact through connections that are established by common variables. This analysis is quasi-static as a consequence of their market-clearing view of market adjustment, i.e., the first perturbation to reach a market determines the position of the new equilibrium. After simulation, new values are obtained from comparative static analysis. However, in general these final values are not unique. This is a well-known problem in qualitative models. One way to get around this problem is to classify the huge number of qualitative behaviours using clustering techniques. This is a topic of current research.

4. Conclusions

In economic reasoning causality plays an important role but it is mostly used in an implicit way. The notions of mythical causality and causal ordering, when applied to static models, often do not give a satisfactory explanation of causality. In dynamic models, the results of both methods may coincide with the intuitive notion of causality. However, it seems somewhat unnatural to describe economic phenomena in differential equations just to obtain a causal explanation of the effects resulting from a perturbation of the equilibrium state. It is shown that the explicit representation of causality can be considered as a part of the economic modelling process. The resulting structure of the economic system is characterized in terms of qualitative relations between the set of parameters. From the qualitative model one may derive the possible qualitative behaviours of the economic system by constraint propagation. Clearly, the analysis in this paper is only a first step and the method has to be refined considerably to treat realistic models.

References

Berndsen, R.J. and H.A.M. Daniels, 1988, Application of constraint propagation in monetary economics, in: Proceedings of expert systems & their applications (Avignon, France).

Bobrow, et al., 1984, Special issue on qualitative reasoning, Artificial Intelligence 24.

Bourgine, P. and O. Raiman, 1986, Economics as reasoning on a qualitative model, in: IFAC economics and artificial intelligence (Aix-en-Provence, France).

Boutillier, M., 1984, Reading macroeconomic models and building causal structures, in: J.P. Ancot, ed., Analysing the structure of econometric models (Martinus Nijhoff, The Hague).

Dennis, G., 1981, Monetary economics (Longman, London).

Dormoy, J. and O. Raiman, 1988, Assembling a device, in: Proceedings of the 7th national conference on AI (AAAI-88) (Saint Paul, MN).

Farley, A.M., 1986, Qualitative modeling of economic systems, in: IFAC economics and artificial intelligence (Aix-en-Provence, France).

Farley, A.M. and K.P. Lin, 1990, Qualitative reasoning in economics, Journal of Economic Dynamics and Control, this issue.

Fontela, E., 1986, Macro-economic forecasting and expert systems, in: IFAC economics and artificial intelligence (Aix-en-Provence, France).

Greenberg, H.J. and J.S. Maybee, eds., 1981, Computer-assisted analysis and model simplification (Academic Press, New York, NY).

Hicks, J., 1979, Causality in economics (Basil Blackwell, Oxford).

Iwasaki, Y. and H.A. Simon, 1986a, Causality in device behaviour, Artificial Intelligence 29.

Iwasaki, Y. and H.A. Simon, 1986b, Theories of causal ordering: Reply to de Kleer and Brown, Artificial Intelligence 29.

Iwasaki, Y., 1988, Causal ordering in a mixed structure, in: Proceedings of the 7th national conference on AI (AAAI-88) (Saint Paul, MN).
De Kleer, J. and J.S. Brown, 1984, A qualitative physics based on confluences, Artificial Intelligence 24.
De Kleer, J. and J.S. Brown, 1986, Theories of causal ordering, Artificial Intelligence 29.
Kuipers, B. and J. Kassirer, 1984, Causal reasoning in medicine: Analysis of a protocol, Cognitive Science 8.
Kuipers, B., 1986, Qualitative simulation, Artificial Intelligence 29.
Maybee, J.S., 1981, Sign solvability, in: H.J. Greenberg and J.S. Maybee, eds., Computer-assisted analysis and model simplification (Academic Press, New York, NY).
Pau, L., 1986, Inference of functional economic model relations from natural language analysis, in: L. Pau, ed., Artificial intelligence in economics and management (Elsevier/North-Holland, Amsterdam).
Pierce, D.A. and L.D. Haugh, 1977, Causality in temporal systems, Journal of Econometrics 5.
Raiman, O., 1986, Order of magnitude reasoning, in: Proceedings of the 5th national conference on AI (AAAI-86) (Philadelphia, PA).
Royer, D. and G. Ritschard, 1984, Qualitative structural analysis: Game or science?, in: J.P. Ancot, ed., 1984 Analysing the structure of econometric models (Martinus Nijhoff, The Hague).
Samuelson, P.A., 1947, Foundations of economic analysis (Harvard University Press, Cambridge, MA).
Simon, H.A., 1957, Models of man (Wiley, New York, NY).
Williams, B.C., 1986, Doing time: Putting qualitative reasoning on firmer ground, in: Proceedings of the 5th national conference on AI (AAAI-86) (Philadelphia, PA).
Williams, B.C., 1988, MINIMA, A symbolic approach to qualitative algebraic reasoning, in: Proceedings of the 7th national conference on AI (AAAI-88) (Saint Paul, MN).