

Bets and bids: favorite-longshot bias and winner's curse*

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Abstract

A well-documented anomaly in racetrack betting is that the expected return per dollar bet on a horse increases with the probability of the horse winning. This so-called "favorite-longshot bias" is at odds with the presumptions of market efficiency. We offer a new solution to this much-debated puzzle which is related to another famous anomaly. We show that the bias can be explained by the same behavioral assumption that underlies the well-known "winner's curse" in common value auctions.

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1. Introduction

A prime concern of economists is the efficiency of markets. Do markets allocate resources to their most valued use? One of the issues at stake here, is the extent to which market prices aggregate and reflect all relevant information. For example, when traders are expected value maximizers, (weak) market efficiency requires that the expected return on investments should be the same across different assets. In another (stronger) form, efficiency requires that no asset offers a positive expected return.

Unfortunately, many asset markets offer a less than ideal setting to test (the various forms of) market efficiency. One of the difficulties is that many securities are infinitely lived and that the "true" underlying value of an asset does not reveal itself at any point in time to be compared with its price. Therefore, to test efficiency, economists have turned to institutions which are less important in itself, but which offer a more promising setting for empirical inquiry. One such institution is the market for racetrack bettings. Comforting in some sense, is the finding that these betting markets exhibit a relatively high degree of efficiency. Market odds are good predictors of winning chances. Nevertheless, there is one robust finding which is incongruous with market efficiency: the favorite-longshot bias (for a brief exposition, see Thaler and Ziemba, 1988). The implied winning probabilities of the market odds underestimate the winning chances of horses with a high winning probability (favorites) and overestimate those with a low winning probability (the longshots). As a consequence, the expected return of a dollar bet is not equal across bets and there are bets (on the favorite) with a positive expected return.

Several explanations for this anomaly have been offered. Some argue that betters are locally *risk seeking*, as a consequence of which the usual preference over the return's variance is reversed (Quandt, 1986). An explanation, in line with *prospect theory* (Kahneman and Tversky, 1979), is that people tend to overestimate small probabilities. Thaler and Ziemba (1988) suggest an explanation along the lines of *mental accounting*, where bets are partially determined by an

individual's win-loss account of preceding bets. Finally, *insider trading* has been hinted at as a potential source for biases (Shin, 1992). We do not wish to discuss these explanations at length or argue with their potential validity. Suffice it to say that the issue is far from settled. What we wish to do, however, is offer a new explanation, resting on a behavioral assumption that has proved successful in explaining another well-documented anomaly: the winner's curse.¹

The *winner's curse* is the phenomenon that the winning bidder in an auction for an item with a strong common value component, often stands to incur a loss (see, e.g., McAfee and McMillan, 1987). The driving force behind this phenomenon is a judgmental failure which leads to a kind of adverse selection. Bidders do not take into account that the mere fact of winning the auction reveals information about the item's value, namely, that the winning bidder's sample information of the item's worth is the maximum of these samples. Even if each bidder's sample information is an unbiased estimate of the item's value, it is a biased estimate conditional on the event that it is highest.

In the next section we present a simple static model of racetrack betting which shows that the type of judgmental failure observed in common value auctions, leads to the favorite-longshot bias in betting markets. For the intuition behind the result to be as transparent as possible, this model will have only two betting options, and no transaction costs. Section 3 discusses several extensions of the model.

¹ Elsewhere (Potters and Wit, 1995), we have shown that this behavioral assumption is also capable of explaining a less well-known anomaly, namely, the negative correlation between price bias and common value observed in experimental political stock markets.

2. A simple model of racetrack betting

Racetrack betting is similar to a common value auction in the sense that both bets and bids are mainly motivated, not by differences in taste, but by judgmental differences across individuals about the (expected) common value of the assets. The typical way to model a common value auction (e.g., Kagel and Levin, 1986) is by assuming that individual bidders are characterized by different pieces of information about the true common value. In the same way we proceed to model the betting market. In order to arrive at a simple closed form solution and to convey the intuition as plainly as possible, we will employ the simplest of possible settings. A discussion of extensions is delegated to the next section.

There are N betters, and each better has an endowment of E dollars. There are two possible bets (horses), h_1 and h_2 , and N_j denotes the amount of bets (dollars) on h_j ($j=1,2$). Under the American, so-called, parimutuel payoff system and in the absence of transaction costs, this implies that the gross payoff of a dollar bet on h_j is equal to $(N_1+N_2)/N_j$. The implied "price" of a bet on h_j can then be defined as $p_j := N_j/(N_1+N_2)$, for $j=1,2$. In racetrack terms, h_j has odds of $1/p_j-1$ to 1. The true probability of horse h_j winning the race is denoted by W_j , and we define $W := W_1 = 1 - W_2$, and $p := p_1 = 1-p_2$. To characterize the information possessed by the betters we make the following assumption.

ASSUMPTION 1

- (i) It is common knowledge that the winning probability W is a random draw from a uniform $(0,1)$ distribution. The actual value W is unknown to every better.
- (ii) Each better i ($i=1,2,\dots,N$) receives a private information signal s_i . This signal is a random draw from a uniform $(W-\varepsilon, W+\varepsilon)$ distribution, where $\varepsilon \in [0, 1/2]$ is a commonly known positive real number.

With this information structure every individual better possesses a small bit of information. A better knows that this information is an unbiased estimate of the winning probability, and that the signal is monotonically correlated with the winning probability. Low signals are supported by low winning probabilities and high signals indicate high winning probabilities. At the same time, the collection of betters has superior knowledge about the winning probability. If they could share their information, a large number of betters would be able to estimate the winning probability consistently.²

We introduce three behavioral assumptions. The first assumption concerns the motivation of betters. The second assumption describes betters' price-taking behavior. The third specifies betters' myopic information processing. Together they specify betters' strategies.

ASSUMPTION 2

Bettors are risk-neutral; they try to maximize expected (net) payoffs.

ASSUMPTION 3

Bettors are odds-takers; they ignore the consequences of their individual bets on the odds.

ASSUMPTION 4

Bettors base their expectations solely on their subjective posterior belief regarding the distribution of W .

Assumption 2 implies that if the actual value of W is known to all betters, the unique equilibrium price supported by the model is at $p = W$. In this case the odds reflect the true winning probabilities. Of course, in such a full information equilibrium, the number of bets on the horses is

² This description of the information structure is in line with Hayek's ideas (1945).

indeterminate, as each bet has zero expected return.³

Assumption 3 is common in Walrasian general equilibrium models. Bettors are assumed to disregard the effect of their individual bet on the odds. Implicitly, it assumes the number of bettors to be large.

Assumption 4 describes the judgmental failure which leads to the winner's curse in common value auctions. In such an auction, the highest bidder will receive the item but will incur a loss. He or she does not take account of the fact that the mere fact of winning the auction reveals information about the private information of other bidders.⁴ In the present model this assumption implies that bettors do not take account of the (equilibrium) mapping of other bettors' private information into bets and, hence, odds. The next theorem shows that the consequence of this assumption is that, as the number of bettors becomes large, the implied equilibrium price underestimates the winning probability if a horse has a high change of winning (a favorite), and overestimates this probability if a horse has a low probability of winning (a longshot).

³ According to Quandt's (1986) explanation of betting behavior - local risk seeking -, bettors would still strictly prefer to place bets in a full information equilibrium, since they like the payoff variance implied by betting.

⁴ In an auction for an item with a common value of W , a bidder with private signal s_i will base his bids on the expectation $E[W|s_i] = s_i$ if he falls prey to the judgement bias described by Assumption 3. A rational bidder who avoids the winner's curse, however, will base her bids on $E[W|s_i = \max_{k \in N} s_k] = s_i - \epsilon(N-1)/(N+1)$. See, e.g., Kagel and Levin (1986).

THEOREM

Under ASSUMPTIONS 1-4, and as the number of betters becomes large, the implied equilibrium price p for h_1 , given a winning probability W , converges in probability to:⁵

$$p^* = \frac{W+\varepsilon}{1+2\varepsilon}$$

Here, we will give an informal sketch of the proof. The formal proof is relegated to the Appendix.

By Assumption 1, for a better with signal s_i , the expected winning probability of h_1 is equal to s_i . Under Assumptions 2-4 then, a better with private signal s_i will bet on h_1 (h_2) if $s_i > (<)$ p . Hence, the total fraction of bets on h_1 will be equal to:

$$\frac{N_1}{N} = \frac{1}{N} \sum_{i=1}^N I_{[s_i, > p]}$$

where $I_{[\cdot]}$ is the indicator function. By the parimutuel payoff system, in equilibrium we must have that the implied price be equal to the fraction of bets on h_1 : $p = N_1/N$. As the number of betters goes to infinity this equilibrium equation requires:

$$p = 1 - G(p|W)$$

where $G(\cdot|W)$, the conditional distribution of private signals s_i , is uniform on $(W-\varepsilon, W+\varepsilon)$. Straightforward calculation shows that p^* given above is the unique solution to this equation.

Basically, the intuition behind the result is that betters tend to bet on the cheaper ticket, as they disregard the information contained in the (equilibrium) odds about the information possessed by other betters. To illustrate the market forces that drive the implied price away from the true winning probability, suppose that $W > 1/2$ (h_1 is the favorite). First, suppose that the implied price for h_1 correctly reflects the true winning probability: $p = W$. As s_i is an unbiased estimate of W , all

⁵ For simplicity we assume here that the winning probability lies in the interval $[2\varepsilon, 1-2\varepsilon]$. In the formal proof we also handle the 'border problems' that arise for winning probabilities outside this interval, but they mess up the presentation considerably.

betters with private signals $s_i > p$ would place their bet on h_1 , and betters with private signals $s_i < p$ would place their bet on h_2 . As *all* betters have unbiased sample information, the proportion of betters in both groups would tend to be equal if the number of betters is very large. However, with $N_1 = N_2$ the parimutuel payoff system would give an implied price of $p = 1/2 < W$, which contradicts the supposition $p = W$. Hence, at $p = W$ there is a downward pressure on the price. In turn, at $p = N_1/(N_1+N_2) = 1/2 < W$, betters with private signals $s_i > 1/2$, would tend to wager their bets on h_1 . As all betters have unbiased information and $W > 1/2$, the fraction of betters with $s_i > 1/2$ is larger than $1/2$. However, with $N_1 > N_2$ the parimutuel payoff system would give an implied price of $p = N_1/(N_1+N_2) > 1/2$. Hence, at $p = 1/2$, there is an upward pressure on the price. In sum, at $p = W$ there is a downward pressure on the price, and at $p = 1/2$ there is an upward pressure. Only at some price, $1/2 < p < W$, these pressures cancel. From the viewpoint of market efficiency, however, too few bets tend to be placed on the favorite and too many on the longshot in this equilibrium.

The implications of the simple model are quite impressive. It tells us that an infinitely large group of betters, who are privately given unbiased information and use this information in a Bayesian way, will bet in a such a way that the odds do not reflect the true winning probabilities almost surely, if the betters do not take account of the equilibrium correspondence between private information and betting odds (assumption 4). In case $W > 1/2$, the price p^* implied by the odds (h_1 has odds of $1/p^*-1$ to 1) is smaller than W and in case $W < 1/2$ the price is larger than W . The bets wagered give a correct reflection of the true winning probabilities, only if the winning chances of the horses are equal ($W = 1/2$). Hence, the theorem predicts a favorite-longshot bias, and, for fixed ϵ , a negative relation between the prediction bias p^*-W and the winning probability W . The more favorite or longshot a horse, the larger the prediction bias implied by the betting odds. These implications are fully borne out by empirical studies of racetrack betting markets (see, e.g., Thaler and Ziemba, 1988).

3. Discussion

The model in the preceding has been kept as simple as possible, in order to arrive at a neat closed form solution. Here, we will shortly discuss four possible extensions that we have examined.

Firstly, an extension of the model with transaction costs ("track take" and "breakage") is laborious, but does not qualitatively change the results. It appears, however, that under certain conditions a no-trade equilibrium results. This occurs if the transaction costs are relatively high and betters do not disagree sufficiently on the winning probability (ϵ small).

Secondly, we looked at models with more than two betting options (horses). It appears to be difficult to find analytical solutions for the equilibrium price vector. We have run several simulations, however, which all indicate that the favorite-longshot bias is not typical to two-horse races.

Thirdly, we examined several other (non-uniform) distributions of the private information signals. It appears that the implications of the model "survive" these other distributional assumptions. We do not have a formal proof that the favorite-longshot bias occurs under *all* distributional assumptions, but, definitely, it is not an anomaly of the uniform distribution.

Finally, we have examined the inclusion of a fraction of betters with "rational expectations", that is, a fraction of betters that does take account of the equilibrium mapping between private values and bets. It appears that it is possible to get a non-biased equilibrium ($p=W$) if this fraction of rational betters is large enough. Furthermore, for this non-biased equilibrium to exist, the fraction of rational betters must be larger, the further W moves away from $1/2$. Foremost, this extension shows that the favorite-longshot bias is not innate to the institutional setup of the racetrack betting market, but rests on the judgmental failure described by assumption 4.

In conclusion, whereas many of the previous explanations of the favorite-longshot bias rest on "motivational" considerations, our explanation rests on an "informational" consideration. It appears that, in a standard type Walrasian equilibrium model, the anomaly can be explained by the

same information-processing assumption that underlies the winner's curse in common value auctions.

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Appendix

PROOF OF THE THEOREM

Assumption 1 describes the information possessed by the betters. The prior belief with respect to the winning probability W is given by the density function $f(w)$,

$$f(w) = 1_{\{0 < w < 1\}}$$

The private signal s_i is an independent draw from a density $g(s|W)$:

$$g(s|W) = \frac{1}{2\varepsilon} 1_{\{W-\varepsilon < s < W+\varepsilon\}}.$$

Hence, for better i , the posterior belief about the common value can be described by

$$f(w|s_i) = c 1_{\{\max[0, s_i - \varepsilon], \min[1, s_i + \varepsilon]\}}$$

where c is a known constant. The posterior expectation about the winning probability, denoted by $w(s_i)$, is found to be an increasing function of the information signal s_i :

$$w(s_i) \equiv \mathcal{E}[W|s_i] = \frac{1}{2} \{\max[0, s_i - \varepsilon] + \min[1, s_i + \varepsilon]\}$$

Assumptions 2 and 4, enable us to write down the utility maximization problem for better i . In this problem $q_{i,1}$ and $q_{i,2}$ denote i 's demand for bets for horse 1 and 2, and B_1 and B_2 are the payoffs for a dollar bet on horse 1 and horse 2 respectively, where

$$B_1 = \begin{cases} 1 & \text{with probability } W \\ p & \text{with probability } 1-W \\ -1 & \end{cases}$$

and

$$B_2 = -\frac{p}{1-p} B_1.$$

Using this notation, the individual's maximization problem can be written as

$$\begin{aligned} \max \quad & \mathcal{E}\{q_{i,1} B_1 + q_{i,2} B_2\} \\ \text{s.t.} \quad & q_{i,1} + q_{i,2} \leq E. \end{aligned}$$

Using Assumption 4 one can rewrite this problem as:

$$\max \left\{ q_{i,1} \frac{w(s_i) - p}{p} + q_{i,2} \frac{p - w(s_i)}{1 - p} \right\}$$

$$s.t. \quad q_{i,1} + q_{i,2} \leq E.$$

The corner solutions of this problem give the individual demand correspondences

$$q_{i,1}(p) = \begin{cases} E & \text{if } p < w(s_i) \\ [0, E] & \text{if } p = w(s_i) \\ 0 & \text{if } p > w(s_i) \end{cases}$$

and

$$q_{i,2}(p) = E - q_{i,1}(p).$$

The aggregate demand correspondence for bets on horse 1 can be denoted by:

$$Q_1(p) \equiv \sum_{i=1}^N q_{i,1}(p) = E \sum_{i=1}^N \{ 1_{\{p < w(s_i)\}} + [0, 1] 1_{\{p = w(s_i)\}} \},$$

and aggregate demand for bets on horse 2 by:

$$Q_2(p) \equiv \sum_{i=1}^N q_{i,2}(p) = NE - Q_1(p)$$

By the parimutuel payoff system, the equilibrium price p^N must be equal to the fraction of bets on h_1 :

$$p^N = \frac{Q_1(p^N)}{Q_1(p^N) + Q_2(p^N)} \Leftrightarrow p^N = \frac{Q_1(p^N)}{NE} \Leftrightarrow Z(p^N) = 0.$$

where $Z(\cdot)$ is defined by

$$Z(p) = p - \frac{Q_1(p)}{NE}.$$

This price is uniquely defined, as $Z(\cdot)$ is strictly increasing in its argument, and $Z(0) < 0$ and $Z(1) > 0$. In order to establish the relation between the true winning probability and the equilibrium price, we will examine the equilibrium condition, conditional on $W = W^0$, where W^0 satisfies $0 < W^0 < 1$. Limit behavior can be established by using Kolmogorov's weak law of large numbers.

For a fixed p one can find:

$$\frac{Q_1(p)}{NE} | W^0 = \frac{1}{N} \sum_{i=1}^N \{1_{\{p < w(s_i)\}} + [0,1]1_{\{p = w(s_i)\}}\} | W^0 \xrightarrow{p} \text{Prob}(w(s_i) > p | W^0),$$

Thus,

$$Z(p) | W^0 \xrightarrow{p} p - \text{Prob}(w(s_i) > p | W^0).$$

Define p^* as the solution in p of

$$p - \text{Prob}(w(s_i) > p | W^0) = 0,$$

After tedious but straightforward calculations, it follows that the solution p^* can be written as

$$p^* = \begin{cases} \frac{\frac{1}{2}W^0 + \varepsilon}{1 + \varepsilon} & \text{for } W^0 < 2\varepsilon^2 \\ \frac{W^0 + \varepsilon}{1 + 2\varepsilon} & \text{for } 2\varepsilon^2 \leq W^0 \leq 1 - 2\varepsilon^2 \\ \frac{W^0 + 1}{2(1 + \varepsilon)} & \text{for } W^0 > 1 - 2\varepsilon^2 \end{cases}$$

To complete the proof, it is shown that p^N , conditional on $W = W^0$, converges in probability to p^* .

As $Z(p)$ is strictly increasing in p , this can be established by using the equivalence

$$p > p^N \Leftrightarrow Z(p) > 0.$$

Now, for every $\delta > 0$, one can find $\gamma_1, \gamma_2 > 0$ such that

$$\lim_{N \rightarrow \infty} \text{Prob}[p^N < p^* - \delta | W^0] = \lim_{N \rightarrow \infty} \text{Prob}[Z(p^* - \delta) > 0 | W^0] = \lim_{N \rightarrow \infty} \text{Prob}[Z(p^*) > \gamma_1 | W^0] = 0,$$

and

$$\lim_{N \rightarrow \infty} \text{Prob}[p^N > p^* + \delta | W^0] = \lim_{N \rightarrow \infty} \text{Prob}[Z(p^* + \delta) < 0 | W^0] = \lim_{N \rightarrow \infty} \text{Prob}[Z(p^*) < -\gamma_2 | W^0] = 0.$$

Combination of these expressions yields

$$\lim_{N \rightarrow \infty} \text{Prob}[|p^N - p^*| > \delta | W = W^0] = 0, \quad \forall \delta > 0,$$

which is equivalent to

$$p^N | W = W^0 \xrightarrow{p} p^*.$$

Q.E.D.

Appendix A. "tedious, but straightforward calculations" (not for publication)

The relation that determines p^* is:

$$p^* = \text{Prob}[w(s_i) > p^* | W^0]$$

If $2\varepsilon \leq W^0 \leq 1 - 2\varepsilon$, then $w(s_i) = s_i$. Thus

$$\text{Prob}[w(s_i) > p | W^0] = \frac{W^0 - p + \varepsilon}{2\varepsilon},$$

and

$$p^* = \frac{W^0 + \varepsilon}{1 + 2\varepsilon}$$

For $W^0 > 1 - 2\varepsilon$, we can use the equivalence

$$\begin{aligned} \text{Prob}[w(s_i) < x | W^0] &= \text{Prob}[s_i < x | s_i < 1 - \varepsilon, W^0] \text{Prob}[s_i < 1 - \varepsilon | W^0] + \\ &\quad \text{Prob}\left[\frac{1}{2}(s_i + 1 - \varepsilon) < x | s_i > 1 - \varepsilon, W^0\right] \text{Prob}[s_i > 1 - \varepsilon | W^0] \end{aligned}$$

to find

$$\text{Prob}[w(s_i) < x | W^0] = \begin{cases} 0 & x < W^0 - \varepsilon \\ \frac{x - W^0 + \varepsilon}{2\varepsilon} & W^0 - \varepsilon \leq x < 1 - \varepsilon \\ \frac{1 - W^0}{2\varepsilon} + \frac{x - 1 + \varepsilon}{\varepsilon} & 1 - \varepsilon \leq x < \frac{1}{2}(1 + W^0) \\ 1 & x > \frac{1}{2}(W^0 + 1) \end{cases}$$

If the unique fixed point p^* lies in the interval $[W^0 - \varepsilon, 1 - \varepsilon]$ then we find

$$p^* = \frac{W^0 + \varepsilon}{1 + 2\varepsilon}$$

where it is required that

$$W^0 - \varepsilon < \frac{W^0 + \varepsilon}{1 + 2\varepsilon} < 1 - \varepsilon \Leftrightarrow W^0 < 1 - 2\varepsilon^2.$$

If the unique fixed point p^* lies in the interval $[1 - \varepsilon, \frac{1}{2}(1 + W^0)]$ then we find

$$p^* = \frac{W^0 + 1}{2(1 + \varepsilon)}$$

where it is required that

$$1 - \varepsilon < \frac{\frac{1}{2}(W^0 + 1)}{1 + \varepsilon} < \frac{1}{2}(W^0 + 1) \Leftrightarrow W^0 > 1 - 2\varepsilon^2$$

For $W^0 < 2\varepsilon$, we can use the model's symmetry.