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Effects of Strategic Interactions on the Option Value of Waiting

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This paper considers an investment timing problem in a duopoly framework. The results of the seminal contribution by Fudenberg and Tirole (1985, RES) are extended by introduction of uncertainty. Three scenarios are identified. In the first scenario we have a preemption equilibrium with dispersed investment timing, while in the second scenario an equilibrium with joint investment prevails. In the third scenario preemption holds in case uncertainty is low, and joint investment is the Pareto dominating equilibrium if uncertainty is large.

From the theory of real options it is known that it is optimal to invest when the net present value exceeds the option value of waiting. In this paper we modify the *real options investment rule* by taking into account strategic interactions. Now the net present value must be compared with the so-called *strategic option value of waiting.* It can be shown that, compared to the option value of waiting in the monopoly case, the strategic option value of waiting is the same in the joint investment case and lower in the preemption equilibrium. In the latter case it can even occur that investing is optimal, while the net present value is negative.

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1 Introduction

In deciding whether or not to invest a firm is naturally concerned about *uncertainties* regarding future market conditions such as consumers' response to the product, costs of borrowing capital, hiring labor and using other inputs. Stressing this ongoing uncertainty of the economic environment in which investment decisions are made and the irreversibility of most investment decisions, Dixit and Pindyck (1996) provide a detailed exposition of a new approach to investment that recognizes the option value of waiting for better (but never complete) information. They argue that the decisive criterion for an investment project to be undertaken or not, should be that the resulting discounted cash flow stream exceeds the option value of waiting. In economic environments that are very uncertain this option value can be quite large, leading to a considerable opportunity cost of investment. In this way the option value of waiting drives a wedge between optimal investment behavior and investing according to the classical net present value criterion, which says that a firm must invest when the discounted cash flow stream exceeds zero. To distinguish from the well known financial options, the opportunities to acquire real assets are called *real options*.

Furthermore, an investment decision has *strategic* aspects since a firm has to take into account that its decision to go ahead with an investment project or not influences the behavior of its competitors which in turn affects the profitability of the investment. To study the effect of competition on optimal investment behavior Reinganum (1981) considered a duopoly with two identical firms, which both have the possibility to invest while the cost of investment falls over time. Despite the fact that the firms are identical, one of them is given the leader role beforehand. This implies that only this firm is allowed to invest first. The other firm is the follower which has the choice to invest at the same time as the leader or to invest later. As shown in Fudenberg and Tirole (1985), for this framework two scenarios can be identified. In one scenario the optimal outcome is joint adoption, i.e. both firms invest at the same time, while in the other scenario the follower invests later despite of the fact that the leader then obtains the highest payoff. Fudenberg and Tirole (1985) mainly consider the more realistic case of endogenous firm roles, meaning that beforehand it is not known which firm will be the leader. In the scenario where the leader obtains the largest payoff there is an incentive to try to invest earlier than its competitor, i.e. both firms will try to preempt each other. Fudenberg and Tirole show that this leads to a preemption equilibrium with dispersed adoption times and rent equalization.

This paper considers a framework with two identical firms which both have the possibility to make an investment that increases their payoff. By how much this payoff is raised is not known beforehand, since the future market conditions for the firm's products are uncertain. Both firms operate on the same output market which implies that the investment decision of one firm affects the payoff of the other firm. By analyzing this model uncertainty is combined with strategic aspects, so that the research streams just mentioned are unified. We identify three scenarios. In the first scenario a preemption equilibrium occurs, where the moments of investment of both firms are dispersed. In the second scenario the outcome is that the firms simultaneously invest at the moment that demand is relatively large. We call this a collusive equilibrium. The first scenario particularly holds when first mover advantages are large. In the third scenario it turns out that in economic environments with low uncertainty the preemption equilibrium is applied, while with large uncertainty both firms invest together at the moment that demand is large. This is understandable since the option value of waiting rises with uncertainty. Then opportunity costs of investment are large so that the output market conditions must compensate for this when the firm invests.

Furthermore we find that, compared to the monopoly situation, the demand trigger value is lower for the first investor in the preemption equilibrium. Hence, in order to be able to preempt its rival, the firm is satisfied with lower revenue at the moment it invests. Therefore, the discounted cash flow stream of the investment, which equals the strategic option value of waiting, is lower than the option value of waiting that prevails in a monopoly situation. On the other hand, the demand trigger value in the collusive case is higher than in the monopoly case. The reason is that the market has to be shared by two firms. It turns out that in the collusive case the strategic option value of waiting exactly equals the option value of waiting of the monopoly case.

Finally we compare our analysis with the still few contributions that include the real option framework in multiple firm models. Doing this we were able to make a methodological point. In the preemption equilibrium, situations occur where it is optimal for one firm to invest, but at the same time investment is not beneficial if both firms decide to do so. Nevertheless, since the firms are identical there is a possibility that still both firms invest at the same time, which leads to a low payoff for both of them. Following Fudenberg and Tirole (1985) (see also Simon (1987a,b)) we obtain that such a coordination failure can occur with positive probability at moments of time where the leader's payoff is strictly larger than the follower's payoff. Most contributions in this area, such as Grenadier (1996), Dutta *et al.* (1995), and Weeds (1999), make unsatisfactory assumptions with the aim to be able to ignore the possibility of simultaneous investment at points of time that this is not optimal[1].

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[1] Grenadier assumes that "if each tries to build first, one will randomly (i.e., through the toss of

2 The Model

Both firms have the possibility to make an irreversible investment which results in a higher profit flow. A possible interpretation is that both firms have the possibility to adopt a new technology which after adoption increases the firm's profit. We assume that the firms are risk neutral, value maximizing and discount with constant factor r. The sunk cost is constant and equals $I\,(>0)$. Future profits are of an yet unknown size. When we denote one firm by i, the other firm is denoted by j. At time t the profit flow of firm i equals

$$Y(t)\,D(N_i, N_j)\,, \tag{1}$$

where, for $k \in \{i, j\}$:

$$N_k = \begin{cases} 0 & \text{if firm } k \text{ has not invested,} \\ 1 & \text{if firm } k \text{ has invested.} \end{cases} \tag{2}$$

In order to incorporate uncertainty, $Y(t)$ follows a geometric Brownian motion process:

$$dY(t) = \alpha Y(t)\,dt + \sigma Y(t)\,dW(t)\,, \tag{3}$$

$$Y(0) = y, \tag{4}$$

where $0 < \alpha < r$ and the $W(t)$'s are independently and identically distributed according to a normal distribution with mean zero and variance dt.[2] Keeping in mind that (i) the irreversible investment increases the profit flow and (ii) the firm obtains higher profits if the

a coin) win the race" (Grenadier, 1996, pp. 1656-1657), while on p.568 of Dutta et al. (1995) it is assumed that "If both i and j attempt to enter at any period t, then only one of them succeeds in doing so".

[2] In Fudenberg and Tirole (1985) investing sooner is more expensive since the investment expenditure decreases over time. Similarly, in this model $\alpha > 0$ implies that the expected profitability of investment increases over time.

competitor is weak (thus not having invested (yet)), the following restrictions on $D(N_i, N_j)$[3] are implied:

$$D(1,0) > D(1,1) > D(0,0) > D(0,1), \tag{5}$$

Further we assume that there is a first mover advantage to investment:

$$D(1,0) - D(0,0) > D(1,1) - D(0,1). \tag{6}$$

The aim of this paper is to study effects of strategic interactions on the option value of waiting, and thus on the speed of investment.

In Section 3 we solve the investment problem if there is only one firm active. This will give the benchmark result. The duopoly model is solved in Section 4. In Section 5 comparisons are made with related contributions. Section 6 concludes.

3 Monopoly

In this section we assume that there is and will be only one firm active on the output market. We use the solution of this model as a benchmark for the results of the duopoly model. Whenever the firm has not invested its value equals[4]

$$V(Y) = \begin{cases} A_1 Y^{\beta_1} + \frac{Y D(0,0)}{r-\alpha} & \text{if } Y < Y_M, \\ \frac{Y D(1,0)}{r-\alpha} - I & \text{if } Y \geq Y_M. \end{cases} \tag{7}$$

Here it is optimal for the firm to invest when $Y \geq Y_M$, otherwise the firm waits with investing. Equation (7) is derived by solving the optimal stopping problem with use of Itô's lemma. Expressions for the investment threshold Y_M and the constant A_1 are found

[3] Contrary to Nielsen (1999) we only consider the case in which strategic interaction results in negative externalities.

[4] From here on we omit the time dependence of Y, whenever confusion is not possible.

by exploiting the so called value matching and smooth pasting conditions. See Dixit and Pindyck (1996) for a rigorous explanation of these matters.

The β_1 in equation (7) is the positive root of the following quadratic equation

$$\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{8}$$

The investment threshold, which is the point at which the firm is indifferent between investing and not investing, and the constant are given by

$$Y_M = \frac{\beta_1}{\beta_1 - 1}\frac{(r-\alpha)\, I}{D\,(1,0) - D\,(0,0)}, \tag{9}$$

$$A_1 = \frac{Y_M^{1-\beta_1}}{\beta_1}\frac{D\,(1,0) - D\,(0,0)}{r-\alpha}. \tag{10}$$

The optimal investment strategy of the firm is to invest at time T_M, where

$$T_M = \inf\left(t|\, Y\,(t) \geq Y_M\right). \tag{11}$$

When Y is below the threshold value Y_M the value of the firm consists of two parts (see expression (7)). The first part resembles the value of the option to invest and the second part is the expected value of the firm if the firm never invests. The option value rises with uncertainty (β_1 is decreasing in σ and note (10)), thus uncertainty creates value for the firm. The implication is that the investment threshold also rises with uncertainty, so that the firm's willingness to invest decreases with uncertainty. Intuitively this can be understood by noting that under large uncertainty it is more valuable to wait for new information about the profitability of an investment before undertaking it. As stressed in Dixit and Pindyck (1996) the difference between the traditional net present value method and the real options approach to investment problems is completely captured in the factor $\frac{\beta_1}{\beta_1-1}\,(>1)$, that occurs in the threshold value (see (9)). The net present value would be equal to zero if the firm would invest when $Y = \frac{(r-\alpha)I}{D(1,0)-D(0,0)}$. Investing when $Y = Y_M$ thus gives a positive net present value:

$$\frac{Y_M D(1,0)}{r-\alpha} - I - \frac{Y_M D(0,0)}{r-\alpha} = \frac{I}{\beta_1 - 1} > 0. \tag{12}$$

From the theory of financial options we know that it is only optimal to exercise an option if it is sufficiently deep *in the money*, whereas the net present value method prescribes to exercise the investment option when it is *at the money*.

4 Duopoly

In this section we extend the model of Section 3 by adding one firm. We solve the model in which both firms are initially active on the output market. This distinguishes our model from Smets (1991) (see also Dixit and Pindyck (1996, Chapter 9)), where the firms do not produce initially. Then a firm enters a new market at the moment that it invests. Note that this new market model is retrieved by setting

$$D(0,0) = D(0,1) = 0. \tag{13}$$

We compare our results to those of the new market model in Section 5.

We call the firm that invests first the leader, and the other firm is the follower. The model is solved backwards. First we derive the optimal investment decision for the follower, and using that we derive the optimal investment strategy for the leader. In Subsection 4.3 the optimal collusive outcome is derived. The analysis of the first three subsections is used in Subsection 4.4, where we characterize the possible equilibria. In the last subsection we describe the properties of the equilibria and compare them with the outcomes of the monopoly model.

4.1 Follower

For the moment let us assume that the leader has invested. Then the value of the follower is given by

$$F(Y) = \begin{cases} B_1 Y^{\beta_1} + \frac{YD(0,1)}{r-\alpha} & \text{if } Y < Y_F, \\ \frac{YD(1,1)}{r-\alpha} - I & \text{if } Y \geq Y_F, \end{cases}. \tag{14}$$

The threshold Y_F is defined in the same fashion as Y_M: it is the point at which the follower is indifferent between investing and not investing. When Y is smaller than Y_F the value of the follower equals the value of the option to invest, $B_1 Y^{\beta_1}$, plus the value of never investing, $\frac{YD(0,1)}{r-\alpha}$. Solving the value matching and smooth pasting conditions gives

$$B_1 = \frac{Y_F^{1-\beta_1}}{\beta_1} \frac{D(1,1) - D(0,1)}{r-\alpha}, \tag{15}$$

$$Y_F = \frac{\beta_1}{\beta_1 - 1} \frac{(r-\alpha) I}{D(1,1) - D(0,1)}. \tag{16}$$

Due to equation (5) the last two expressions are strictly positive. It is optimal for the follower to invest at time T_F, where

$$T_F = \inf \left(t | \, Y(t) \geq Y_F \right). \tag{17}$$

4.2 Leader

The expected value of the leader, after investing at time $t < T_F$, equals

$$L(Y(t)) = E\left[\int_{\tau=t}^{T_F} Y(\tau) D(1,0) e^{-r\tau} d\tau - I + \int_{\tau=T_F}^{\infty} Y(\tau) D(1,1) e^{-r\tau} d\tau \right]. \tag{18}$$

Working out the expectation (see Dixit and Pindyck (1996), appendix to Chapter 9, for details) gives

$$L(Y) = \frac{YD(1,0)}{r-\alpha} - I + \left(\frac{Y}{Y_F}\right)^{\beta_1} \frac{Y_F (D(1,1) - D(1,0))}{r-\alpha}. \tag{19}$$

If the leader invests when $Y(t) \geq Y_F$, the follower will invest too, so that the leader's expected value equals the value of joint investment, denoted by $M(Y)$:

$$M(Y) = \frac{YD(1,1)}{r-\alpha} - I. \tag{20}$$

4.3 Collusive Investment

Here we assume that the firms invest simultaneously at time T_θ, where

$$T_\theta = \inf\left(t | Y(t) \geq \theta\right). \tag{21}$$

The expected value of each firm at time $t < T_\theta$ equals

$$C(Y(t),\theta) = E\left[\int_{\tau=t}^{T_\theta} Y(\tau) D(0,0) e^{-r\tau} d\tau + \int_{\tau=T_\theta}^{\infty} Y(\tau) D(1,1) e^{-r\tau} d\tau - Ie^{-rT_\theta}\right]. \tag{22}$$

Thus

$$C(Y,\theta) = \begin{cases} \frac{YD(0,0)}{r-\alpha} + \left(\frac{Y}{\theta}\right)^{\beta_1}\left(\frac{\theta(D(1,1)-D(0,0))}{r-\alpha} - I\right) & \text{if } Y < \theta, \\ \frac{YD(1,1)}{r-\alpha} - I & \text{if } Y \geq \theta. \end{cases} \tag{23}$$

Note that $M(Y) = C(Y,Y)$. The optimal joint investment time T_C equals

$$T_C = \inf\left(t | Y(t) \geq Y_C\right), \tag{24}$$

where Y_C is given by (analogous to (9)):

$$\begin{aligned} Y_C &= \arg\max_{\theta > Y} C(Y,\theta) \\ &= \frac{\beta_1}{\beta_1 - 1}\frac{(r-\alpha)I}{D(1,1) - D(0,0)}. \end{aligned} \tag{25}$$

4.4 Equilibria

It turns out to be convenient to distinguish between the following two cases. In the first case there exists an Y such that there are incentives to become the leader. With other words, for such an Y the leader's payoff, $L(Y)$, which can be obtained after investing right away, exceeds the collusive payoff which the firm obtains when it waits with investment until

Y reaches Y_C, at which both firms invest simultaneously. This implies that

$$\exists Y \in (0, Y_F) \text{ such that } L(Y) > C(Y, Y_C). \tag{26}$$

In the second case there does not exist such an Y so that

$$L(Y) < C(Y, Y_C) \text{ for all } Y \in (0, Y_F). \tag{27}$$

Since the firms are identical, no reason can be found why they should behave differently. Therefore, we concentrate on equilibria that are supported by symmetric strategies. We use Fudenberg and Tirole (1985)'s perfect equilibrium concept for timing games. They argue that in this kind of games a strategy can not be represented by a single distribution function. It is necessary to be able to distinguish between types of atoms. Therefore the closed loop strategy of firm i consists of a collection of simple strategies: $\left(G_i^t(\cdot), p_i^t(\cdot)\right)_{t\geq 0}$. The time index t denotes the starting time of the game. $G_i^t(s)$ is the probability that firm i has invested by some time s given that the other firm has not invested. The function $p_i^t(s)$ measures the intensity of atoms on the interval $[s, s+ds]$. By definition (see Fudenberg and Tirole (1985)) a positive $p_i^t(s)$ implies that a firm is sure to invest by time s, i.e. $G_i^t(s) = 1$.

Next we give an interpretation of the $p_i^t(s)$ function. Forget for a moment the dependence on t. Let τ_i be the smallest point in time at which $p_i(s)$ is positive: $\tau_i = \inf\{s | p_i(s) > 0\}$ and define τ to be equal to $\tau := \min(\tau_1, \tau_2)$. From the definition we know for sure that at least one firm has invested by time τ.

The function value $p_1(\tau)$ ($p_2(\tau)$) should be interpreted as the probability that firm 1 (2) chooses row (column) 1 in the matrix game of which the payoffs are depicted in Figure 1. Playing the game costs no time and if player 1 chooses row 2 and player 2 column 2 the game is repeated. If necessary the game will be repeated infinitely often.

		firm 2	
		$p_2(\tau)$	$1-p_2(\tau)$
firm 1	$p_1(\tau)$	(M(Y(τ)),M(Y(τ)))	(L(Y(τ)),F(Y(τ)))
	$1-p_1(\tau)$	(F(Y(τ)),L(Y(τ)))	repeat game

Figure 1. Payoffs (first for firm 1 and second for firm 2) and strategies of matrix game played at time τ.

In our model the firms will use the same strategy, so that $p^t(s) = p_i^t(s) = p_j^t(s)$. Then the probability that firm i is the only firm that invests at time τ, $\Pr(\text{one})$, equals

$$\Pr(\text{one}) = p^t(\tau)\left(1 - p^t(\tau)\right) + \left(1 - p^t(\tau)\right)\left(1 - p^t(\tau)\right)\Pr(\text{one}),$$

which gives

$$\Pr(\text{one}) = \frac{1 - p^t(\tau)}{2 - p^t(\tau)}. \tag{28}$$

For the probability that both firms invest at τ, $\Pr(\text{two})$, we get

$$\Pr(\text{two}) = p^t(\tau)\,p^t(\tau) + \left(1 - p^t(\tau)\right)\left(1 - p^t(\tau)\right)\Pr(\text{two}),$$

so that

$$\Pr(\text{two}) = \frac{p^t(\tau)}{2 - p^t(\tau)}. \tag{29}$$

Thus firm i invests while firm j does not invest with probability $\frac{1-p^t(t)}{2-p^t(t)}$, with the same probability firm j invests while firm i does not invest, and with probability $\frac{p^t(t)}{2-p^t(t)}$ both firms invest at the same time. Consequently, if $p^t(\tau) = 0$ we have

$$\Pr(\text{one}) = \frac{1}{2}, \tag{30}$$

$$\Pr(\text{two}) = 0. \tag{31}$$

4.4.1 First Case: Preemption

For a moment assume that one firm, say firm i, has been given the leader role beforehand, thus firm j can only decide to invest after firm i has done so. The optimal investment time for the leader in the first case, thus where expression (26) holds, is denoted by

$$T_L := \inf \left(Y\left(t\right) \geq Y_L \right), \tag{32}$$

where

$$Y_L = \frac{\beta_1}{\beta_1 - 1} \frac{\left(r - \alpha\right) I}{D\left(1,0\right) - D\left(0,0\right)}. \tag{33}$$

The threshold Y_L[5] is derived by solving the value matching and smooth pasting conditions that result from the leader's optimal stopping problem. As $D\left(1,0\right)$ increases it is more attractive to be the first investor so that Y_L, and thus the expected value of T_L, decreases. In Appendix A we prove the following proposition.

Proposition 1

It holds that

$$L\left(Y_L\right) > F\left(Y_L\right). \tag{34}$$

Now let us drop the assumption that one firm is given the leader role beforehand. Then the implication of Proposition 1 is that each firm wants to be the only one to invest at time T_L. A firm will try to preempt its competitor by investing at time $T_L - \varepsilon$, since it knows that the other firm would like to be the first to invest at time T_L. But then the other firm will

[5] Note that Y_L is equal to Y_M. The reason is that for $Y \in (0, Y_F)$ the leader's decision has no effect on the optimal reply of the follower. Therefore the leader acts as if there is no follower, and thus behaves like a monopolist.

try to invest at time $T_L - 2\varepsilon$. This process of preemption stops at time T_P, where

$$T_P = \inf\left(t | Y(t) \geq Y_P\right), \tag{35}$$

in which Y_P is implicitly given by the equality

$$L(Y_P) = F(Y_P).$$

Before time T_P there are no incentives to become leader, since for $t < T_P$ the follower payoff exceeds the leader payoff. This is because $t < T_P$ implies that $Y < Y_P$, which in turn implies that $F(Y) > L(Y)$ due to the fact that Y_P is unique as stated in the following proposition. The proof can be found in Appendix B.

Proposition 2

There exists an unique value for Y, which we denote by Y_P, such that

$$L(Y_P) = F(Y_P) \text{ and } 0 < Y_P < Y_F. \tag{36}$$

For this first case the payoff curves are depicted in Figure 2. The investment opportunity is worthless for Y equal to 0. Therefore at $Y = 0$ the leader (L) and joint investment (M) value equal minus the investment cost and the follower (F) and collusive value (C) equal zero. M is a linear increasing function of Y (see equation (20)). As we know the follower has the choice between investing at the same time as the leader or to wait. Since the optimal follower action on the interval $(0, Y_F)$ is to wait, the follower curve is situated above the joint investment curve on that interval. From Subsection 4.2 we know that the leader, follower and joint investment curve coincide with each other for Y larger or equal than Y_F. Due to the existence and uniqueness of Y_P (see Proposition 2), the leader curve crosses the follower curve once on the interval $(0, Y_F)$ (at Y_P). Since (26) holds here, the leader curve also crosses the collusive curve somewhere on the interval $(0, Y_F)$. For Y larger or equal than Y_C

the collusive curve coincides with the other three curves.

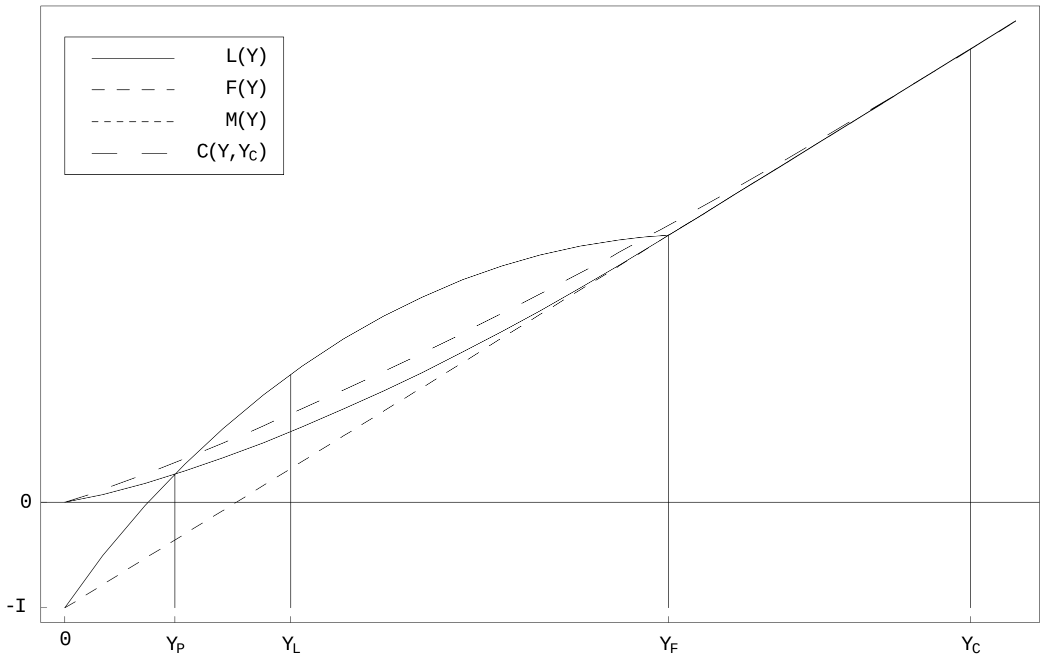

Figure 2. First Case: Preemption

The equilibrium strategy of firm $i \in \{1, 2\}$ equals

$$G_i^t(s) = G(s) = \begin{cases} 0 & \text{if } s < T_P, \\ 1 & \text{if } s \geq T_P, \end{cases} \tag{37}$$

$$p_i^t(s) = p(s) = \begin{cases} 0 & \text{if } s < T_P, \\ \frac{L(Y(s)) - F(Y(s))}{L(Y(s)) - M(Y(s))} & \text{if } T_P \leq s < T_F, \\ 1 & \text{if } s \geq T_F. \end{cases} \tag{38}$$

For $T_P \leq s < T_F$ the optimal $p(s)$ function is found by maximizing the firm's payoff in the matrix game described above. For a moment omit the time dependence and denote firm i's payoff in the matrix game by R_i. Then

$$R_i(p_i, p_j) = \frac{p_i p_j M + p_i(1 - p_j) L + (1 - p_i) p_j F}{p_i p_j + p_i(1 - p_j) + (1 - p_i) p_j}. \tag{39}$$

Rewriting the first order condition of maximization of (39) with respect to p_i gives

$$p_j = \frac{L - F}{L - M}. \tag{40}$$

Since we only consider symmetrical strategies we impose that $p_i = p_j$, so that

$$p_i = \frac{L - F}{L - M}. \tag{41}$$

The equilibrium outcome depends on the value $y\,(= Y\,(0))$. Three regions have to be distinguished.

The first region is defined by $y \leq Y_P$. There are two possible equilibrium outcomes. In the first outcome firm 1 is the leader and invests at time T_P and firm 2 is the follower and invests at time T_F. The second outcome is the symmetric counterpart: firm 2 is the leader and invests at time T_P and firm 1 is the follower and invests at time T_F. Since at time T_P it holds that $Y = Y_P$, it can be obtained from (36) and (38) that $p\,(T_P) = 0$. Due to (30) it can be concluded that each outcome occurs with probability one-half. Furthermore, from (31) we get that the probability that both firms invest simultaneously is zero. Due to (36), it follows that the expected value of each firm equals $F\,(Y_P)$.

In the second region it holds that $Y_P < y < Y_F$. There are three possible outcomes. Since L exceeds F in case $Y \in (Y_P, Y_F)$, it can be obtained from (38) that $p\,(0) > 0$. Due to (28) we know that with probability $\frac{1-p(0)}{2-p(0)}$ firm i invests at time 0 and firm j invests at time T_F, where $i, j \in \{1, 2\}$ and $i \neq j$. Expression (29) implies that the firms invest simultaneously at time 0 with probability $\frac{p(0)}{2-p(0)}$, leaving them with a low value of $M\,(y)\,(< F\,(y))$. The expected payoff of each firm thus equals

$$\frac{1 - p\,(0)}{2 - p\,(0)}\,(L\,(y) + F\,(y)) + \frac{p\,(0)}{2 - p\,(0)}M\,(y) = F\,(y)\,,$$

where the equality sign follows from (38). Since there are first mover advantages in this region, each firm is willing to invest with positive probability. However, this implies that the probability of simultaneous investment, leading to a low payoff $M(y)$, is also positive. Since the firms are both assumed to be risk neutral, they will fix the probability of investment such that their expected value equals $F(y)$, which is also their payoff if they let the other firm invest first.

When y is in the third region $[Y_F, \infty)$, the outcome exhibits joint investment at time 0. The value of each firm is $M(y)$.

4.4.2 Second Case: Collusion

In the second case expression (27) holds, which leads to Figure 3. There turn out to be an infinite number of symmetric equilibrium strategies, which can be divided into two classes. The first class consists of the strategy described above (see equations (37)-(38)). The second class consists of strategies where firms invest simultaneously. They have the following form ($i \in \{1, 2\}$):

$$G_i^t(s) = G(s) = \begin{cases} 0 & \text{if } s < T^*, \\ 1 & \text{if } s \geq T^*, \end{cases} \tag{42}$$

$$p_i^t(s) = p(s) = \begin{cases} 0 & \text{if } s < T^*, \\ 1 & \text{if } s \geq T^*, \end{cases} \tag{43}$$

where $T^* \in [T_S, T_C]$ and

$$T_S = \inf(t | Y(t) \geq Y_S),$$

$$Y_S = \min(\theta | C(Y, \theta) \geq L(Y) \text{ for all } Y \geq 0).$$

From (24) it can be concluded that the equilibrium that is supported by the strategies (42)-(43) with $T^* = T_C$ is the Pareto dominant equilibrium and therefore the most reasonable outcome in this case. In what follows we assume that the Pareto dominant equilibrium is in-

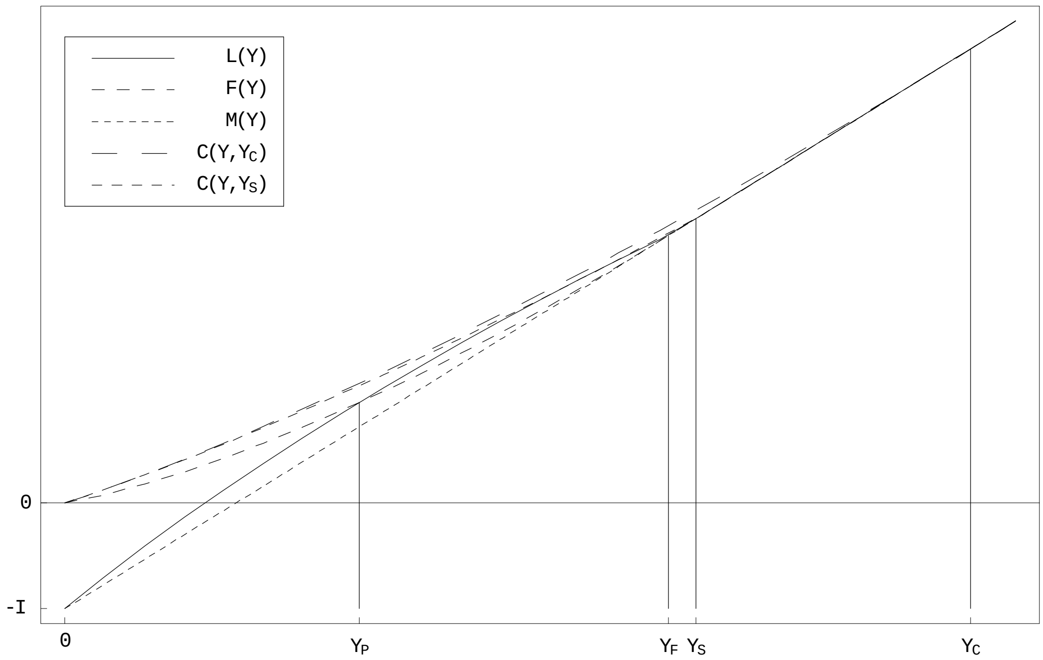

Figure 3. Second Case: Collusion

deed the outcome in the second case[6]. For this equilibrium it holds that there is simultaneous investment at time T_C. The expected value of each firm equals $C(y, Y(T_C))$.

4.5 Properties

The following proposition states when which case applies. See Appendix C for a proof.

Proposition 3

Define

$$f(\beta_1) = \beta_1 \left(\frac{D(1,0) - D(1,1)}{D(1,1) - D(0,1)} \right) + \left(\frac{D(1,1) - D(0,0)}{D(1,1) - D(0,1)} \right)^{\beta_1}, \quad (44)$$

$$g(\beta_1) = \left(\frac{D(1,0) - D(0,0)}{D(1,1) - D(0,1)} \right)^{\beta_1}. \quad (45)$$

[6] This would have been the only equilibrium if we would have applied the setup described in Simon (1987a).

Whenever the following inequality holds the equilibrium is of the preemption type and otherwise of the collusion type:

$$f(\beta_1) < g(\beta_1). \tag{46}$$

Proposition 3 implies that the equilibrium is always of the preemption type, no matter the value of β_1 and thus the degree of uncertainty, if $D(1,0)$ is large enough, i.e. if the incentives to become leader are large enough. If $D(1,0)$ is relatively small, the incentives to become leader almost vanish and the collusive equilibrium turns up.

Note that condition (46) is independent of the value of the investment cost I (as long as it is strictly positive). This for the reason that changing the investment cost only changes the absolute values of the investment triggers, and therefore the value functions, but not the relative values.

The following proposition, that is proved in Appendix D, states the effect of β_1 on the type of the equilibrium.

Proposition 4

There are three different scenarios:

(i) If $g'(1) \geq f'(1)$ the equilibrium is always of the preemption type.

(ii) If $f'(1) > g'(1)$ and $f\left(\frac{r}{\alpha}\right) \geq g\left(\frac{r}{\alpha}\right)$ the equilibrium is always of the collusion type.

(iii) If $f'(1) > g'(1)$ and $f\left(\frac{r}{\alpha}\right) < g\left(\frac{r}{\alpha}\right)$ the equilibrium is of the collusion type for relatively low values of β_1 and of the preemption type for relatively high values of β_1.

Propositions 3 and 4 are visualized in Figure 4. In that figure we have plotted the function $g(\beta_1)$, the boundary between the preemption case and collusion case (Proposition 3), and for each possible scenario the corresponding $f(\beta_1)$ function (Proposition 4).

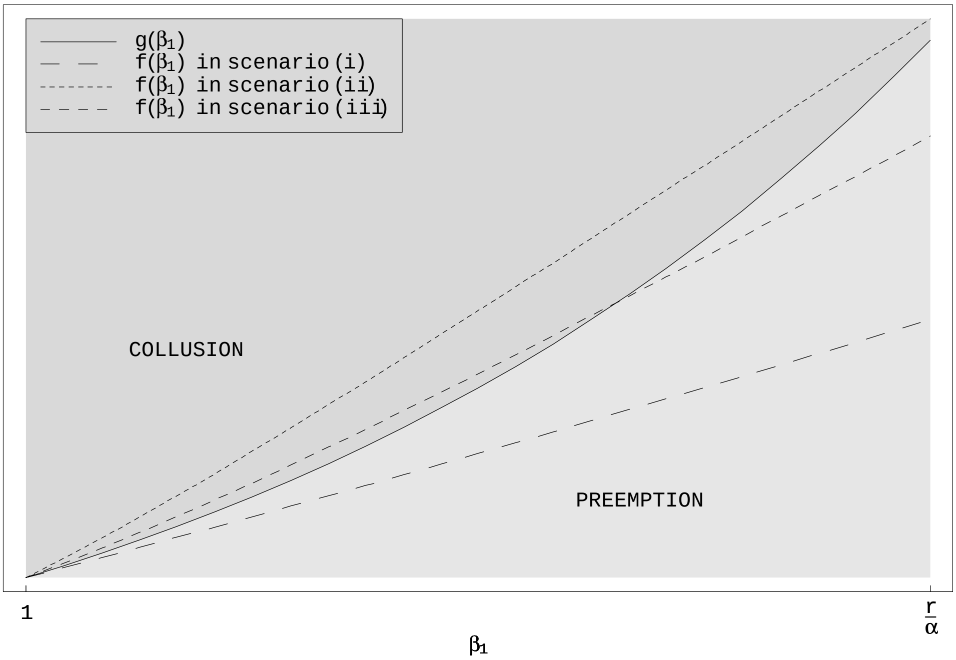

Figure 4. Possible scenarios.

In scenario (i) the first mover advantage is that large that the preemption equilibrium will always turn up. The opposite is going on in scenario (ii), where the first mover advantage is that low that the equilibrium where both firms invest jointly at a later point in time is the Pareto-dominant equilibrium.

Hence, only in scenario (iii) the type of equilibrium depends on β_1. The economic implications are stated in the following corollary to Proposition 4. The proof can be found in Appendix E.

Corollary 1

In scenario (iii) the equilibrium is of the collusion (preemption) type for high (low) values of σ *and* α *and low (high) values of* r.

Here, it is natural that the preemption equilibrium arises if there is not much uncertainty (low σ), the α is low, or the interest rate is high, since then the value of waiting is low. The contrary holds in very uncertain (high σ) economic environments, or environments where the α is high, or the interest rate is low. Then the value of waiting is large, which implies that investing faces high opportunity costs. This makes a preemption strategy unattractive.

Proposition 5 compares the investment thresholds of the duopoly model with the investment threshold of the monopoly model. The proof is given in Appendix F.

PROPOSITION 5

For every parameter configuration it holds that

$$Y_P \leq Y_M < Y_C. \tag{47}$$

Proposition 5 implies that the speed of investment increases (decreases) if strategic interactions result in a preemption (collusive) equilibrium. The following proposition states that the investment thresholds are decreasing functions of β_1 (for a proof see Appendix G).

PROPOSITION 6

The investment thresholds Y_P, Y_L, Y_M, Y_F, and Y_C are decreasing in β_1.

We can conclude that uncertainty delays investment. In scenarios (i) and (ii) investment is delayed because the investment thresholds rise with uncertainty. Increasing the uncertainty in scenario (iii) not only rises the investment thresholds, but may also lead to a change of a preemption equilibrium (with relative low investment thresholds) into a collusion equilibrium (with relative high investment thresholds).

In the real options literature it is argued that an investment should be undertaken when the net present value exceeds the option value of waiting. For models with strategic interactions this investment rule should be changed: investing is optimal when the net present value exceeds the *strategic option value of waiting*. The strategic option value of waiting incorporates the money value of the strategic interactions in the option value of waiting. The following proposition compares the strategic option value of waiting in the duopoly case with the option value of waiting in the monopoly case. The proof is given in Appendix H.

Proposition 7

Compared to the option value of waiting in the monopoly case (at Y_M, see (12)),

$$\frac{I}{\beta_1 - 1},$$

the strategic option value of waiting is

(i) smaller in the preemption case (at Y_P);

(ii) the same in the collusive case (at Y_C).

When strategic interactions lead to a preemption equilibrium, it is even possible that the firms make an investment with a negative net present value! Then the strategic option value of waiting is negative. For example, take the following parameter values $D(1,0) = 10$, $D(1,1) = 4$, $D(0,0) = 2$, $D(0,1) = 1$, $r = 0.10$, $\alpha = 0.05$, and $I = 10$. For these parameters equation (46) is always satisfied so that the equilibrium is always of the preemption type. The net present value of investment at Y_P equals $L(Y_P) - \frac{Y_P D(0,0)}{r-\alpha}$. In the left part of Figure 5 this net present value is plotted as function of σ. For sake of comparison, in the right part the corresponding net present value of investment for the monopolist is presented.

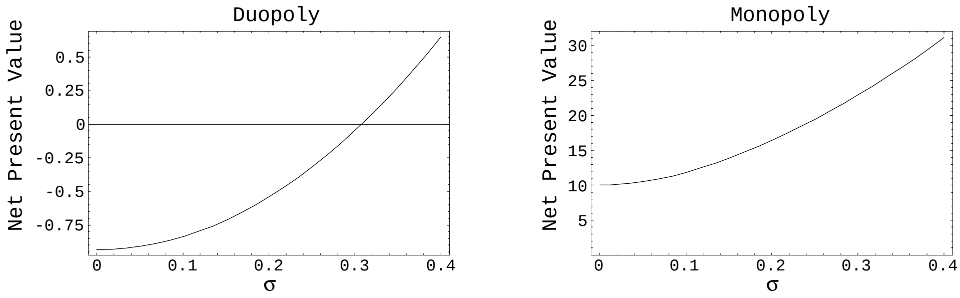

Figure 5. Net present value of investment of duopolist (monopolist) at Y_P (Y_M) as function of σ.

As in the monopoly case, the option value of waiting still increases with uncertainty in the duopoly model. From Figure 5 we conclude that for low (but realistic) values of σ ($\sigma < 0.308$) strategic interactions lead to a negative strategic option value of waiting. Thus the strategic interactions force the firms to make an investment with a negative net present value.

5 Existing Literature

In this section we confront our results with the existing literature. Our model is an extension of the Smets (1991) model described in Dixit and Pindyck (1996, Chapter 9). Contrary to that model we also allow that before the moment of investment the firms are already active on the output market on which they compete. Nielsen (1999) showed that in the Smets (1991) model competition on the output market decreases the option value of waiting and therefore duopolistic firms will invest earlier than monopolistic firms. Remember that for $D(0,0) = D(0,1) = 0$ our model is the Smets (1991) model. It can be shown that in this case equation (46) is always satisfied[7], which implies that the equilibrium is always of the preemption type in the Smets (1991) model. Thus Nielsen (1999)'s result is a consequence

[7] This follows from the fact that for $x := \frac{D(1,0)}{D(1,1)} > 1$ we have that $\beta_1 x - (\beta_1 - 1) < x^{\beta_1}$.

of the initial conditions on the output market in the sense that if both firms are initially active on the output market, Nielsen (1999)'s result does not hold anymore in general. From Proposition 5 we conclude that there are two possibilities. In the case where a collusion equilibrium is the most reasonable outcome, strategic interactions result in delayment of investment by the firms. In the case where the only equilibrium is of the preemption type, competition quickens investment if we compare the moment of investment of the leader in the duopoly to the monopolist.

In the new market model the optimal investment threshold for the follower and the optimal joint investment threshold coincide (cf. (16) and (25)). Therefore, due to (14) and (23) it follows that the follower and the collusive investment curve coincide, which implies that there can not be a second case in the new market model. The economic reason for these two thresholds to coincide is the fact that the investment timing of the leader does not affect the follower's profit flow in a new market model, whereas in our model the follower's profit flow decreases from $YD(0,0)$ to $YD(0,1)$ at the moment the leader invests. Thus, in our model the follower will invest earlier to recapture market share from the leader. That is why we have $Y_F < Y_C$.

At present, only a few contributions deal with the effect of strategic interactions on the option value of waiting associated with investments under uncertainty. However, in these papers the coordination problem is avoided, and thus not treated in the way we did in Section 4. For instance, Weeds (1999) implicitly makes the unsatisfactory assumption that only one firm will succeed in investing in case there is an incentive to be the first firm to invest and it is only optimal for one firm to invest. There are two reasons for her assumption to be unsatisfactory: (1) Weeds (1999) imposes the firms to be equal and (2) the firms can invest simultaneously if it is optimal for both. Note that this assumption, although explicitly, is also made in Nielsen (1999), Grenadier (1996) and Dutta *et al.* (1995).

The reason for our outcomes to be more realistic is as follows. When there is an incentive to be the first to invest ($L > F > M$) both firms are willing to take a risk and since they are both assumed to be risk neutral they will risk so much that their expected value equals F, which equals their payoff if they allow the other firm to invest first. Employing the results of Section 4 learns that in this case both firms set $p = \frac{L-F}{L-M}$, and that there is a positive probability $\frac{p}{2-p}$ that both firms invest exactly at the same time, leaving them with the low payoff M.

Dixit and Pindyck (1996, p. 313) claim that in the Smets (1991) model, the probability that both firms invest simultaneously, while it is only optimal for one firm to invest, is always zero. From the above argumentation it should be clear by now that this claim is not correct[8].

6 Conclusion

This paper brings together two streams of literature: investment under competition (Reinganum (1981), Fudenberg and Tirole (1985)) and investment under uncertainty (Dixit and Pindyck (1996)). Here in this conclusion section we focus on the question how introduction of uncertainty changes the results derived for the deterministic duopoly framework of Fudenberg and Tirole (1985). In Fudenberg and Tirole (1985) it was obtained that under large

[8] To correct another point, consider page 314 of Dixit and Pindyck (1996). First, note that their threshold Y_2 is equal to our threshold Y_F and their threshold Y_3 equals our threshold Y_C. Now, we know that in the new market model we have that $Y_C = Y_F$ so that Y_3 is equal to Y_2 in their model and not, as they claim, greater than Y_2.

first mover advantages a preemption equilibrium with dispersed adoption timings resulted, while otherwise a joint adoption equilibrium is the Pareto-dominant outcome.

After introduction of uncertainty the firm's investment timing problem has to deal with the option value of waiting: when a firm makes an irreversible investment expenditure, it exercises its option to invest. It gives up the possibility of waiting for new information to arrive that might affect the desirability or timing of the expenditure (Dixit and Pindyck (1996)). It is clear that a huge option value of waiting, which arises in highly uncertain economic environments, results in a considerable delayment of investment. On the other hand, in the preemption equilibrium of Fudenberg and Tirole (1985) it is imperative for a firm to invest quickly and thereby preempt investment by potential competitors.

Our paper brings these contrary forces together and it turns out that our results relate to those of Fudenberg and Tirole (1985) in the following way. Whenever Fudenberg and Tirole concluded that joint adoption is the Pareto-dominant outcome, this also holds for our model. Also, if first mover advantages are sufficiently large, for both models the preemption equilibrium results, but in the stochastic case for both firms the investment timing is delayed by the option value of waiting. Finally, if first mover advantages are a bit lower, but still high enough for the preemption equilibrium to prevail in the deterministic framework of Fudenberg and Tirole (1985), introduction of sufficiently large uncertainty results in a joint adoption equilibrium that Pareto-dominates all other equilibria. This brings us to the conclusion that introduction of uncertainty reduces the number of scenarios under which the preemption equilibrium is the optimal outcome.

Appendix

A Proof of Proposition 1

Proof of Proposition 1

Define the function ϕ *as follows*

$$\phi(Y) := L(Y) - F(Y). \tag{48}$$

Then we have to prove that

$$\phi(Y_L) > 0. \tag{49}$$

For $Y \in [0, Y_F]$ *the value of the follower, in case the leader has already invested, can be expressed by*

$$F(Y) = \frac{YD(0,1)}{r-\alpha} + \left(\frac{Y}{Y_F}\right)^{\beta_1}\left(\frac{Y_F(D(1,1)-D(0,1))}{r-\alpha} - I\right).$$

For $Y \in [0, Y_F]$ *the function* $\phi(Y)$ *equals*

$$\begin{aligned}\phi(Y) &= L(Y) - F(Y)\\ &= \frac{YD(1,0)}{r-\alpha} - I + \left(\frac{Y}{Y_F}\right)^{\beta_1}\frac{Y_F(D(1,1)-D(1,0))}{r-\alpha}\\ &\quad - \frac{YD(0,1)}{r-\alpha} - \left(\frac{Y}{Y_F}\right)^{\beta_1}\left(\frac{Y_F(D(1,1)-D(0,1))}{r-\alpha} - I\right)\\ &= \frac{Y(D(1,0)-D(0,1))}{r-\alpha} - I - \left(\frac{Y}{Y_F}\right)^{\beta_1}\left(\frac{Y_F(D(1,0)-D(0,1))}{r-\alpha} - I\right).\end{aligned} \tag{50}$$

Substitution of equation (33) into (50) gives

$$\begin{aligned}\phi(Y_L) = I\Bigg[&\frac{\beta_1}{\beta_1-1}\frac{D(1,0)-D(0,1)}{D(1,0)-D(0,0)} - 1\\ &- \left(\frac{D(1,1)-D(0,1)}{D(1,0)-D(0,0)}\right)^{\beta_1}\left(\frac{\beta_1}{\beta_1-1}\frac{D(1,0)-D(0,1)}{D(1,1)-D(0,1)} - 1\right)\Bigg].\end{aligned} \tag{51}$$

Define

$$a = \frac{D(1,0)-D(0,1)}{D(1,0)-D(0,0)} > 1, \tag{52}$$

$$b = \frac{D(1,1)-D(0,1)}{D(1,0)-D(0,0)} < 1. \tag{53}$$

The inequalities hold due to equations (5) and (6). After substitution of (52) and (53) into (51) it is obtained that

$$\frac{(\beta_1 - 1)\,\phi(Y_L)}{I} = h(a,b) = \beta_1 a - (\beta_1 - 1) - \beta_1 a b^{\beta_1 - 1} + (\beta_1 - 1)\, b^{\beta_1}. \tag{54}$$

The proposition is proved if we show that $f(a,b) > 0$ *for all* $a \in (1,\infty)$ *and* $b \in (0,1)$. *This holds since*

$$\frac{\partial h(a,b)}{\partial a} = \beta_1 - \beta_1 b^{\beta_1 - 1} > 0, \tag{55}$$

$$\frac{\partial h(a,b)}{\partial b} = -(\beta_1 - 1)\,\beta_1 a b^{\beta_1 - 2} + \beta_1 (\beta_1 - 1)\, b^{\beta_1} < 0, \tag{56}$$

$$h(1,1) = 0. \tag{57}$$

B Proof of Proposition 2

Proposition 2 is a direct result of the following lemma.

Lemma 1

Define the function ϕ *as follows*

$$\phi(Y) := L(Y) - F(Y). \tag{58}$$

Then it holds that

$$\phi(0) < 0, \tag{59}$$

$$\phi(Y_F) = 0, \tag{60}$$

$$\left.\frac{\partial \phi(Y)}{\partial Y}\right|_{Y=Y_F} < 0, \tag{61}$$

$$\frac{\partial^2 \phi(Y)}{\partial Y^2} \leq 0 \text{ for all } Y \geq 0. \tag{62}$$

Proof of Lemma 1

For $Y \in [0, Y_F]$ the function $\phi(Y)$ equals (see Appendix A):

$$\phi(Y) = \frac{Y(D(1,0) - D(0,1))}{r-\alpha} - I - \left(\frac{Y}{Y_F}\right)^{\beta_1} \left(\frac{Y_F(D(1,0) - D(0,1))}{r-\alpha} - I\right). \tag{63}$$

Expressions (59) and (60) follow directly after setting $Y = 0$ and $Y = Y_F$, respectively in equation (63).

The first derivative of $\phi(Y)$ equals

$$\frac{\partial \phi(Y)}{\partial Y} = \frac{D(1,0) - D(0,1)}{r-\alpha} - \beta_1 \frac{Y^{\beta_1 - 1}}{Y_F^{\beta_1}} \left(\frac{Y_F(D(1,0) - D(0,1))}{r-\alpha} - I\right). \tag{64}$$

Setting $Y = Y_F$ in equation (64) gives

$$\left.\frac{\partial \phi(Y)}{\partial Y}\right|_{Y=Y_F} = \frac{D(1,0) - D(0,1)}{r-\alpha} - \beta_1 \frac{1}{Y_F} \left(\frac{Y_F(D(1,0) - D(0,1))}{r-\alpha} - I\right). \tag{65}$$

Substitution of equation (16) and rearranging gives

$$\left.\frac{\partial \phi(Y)}{\partial Y}\right|_{Y=Y_F} = -(\beta_1 - 1)\left(\frac{D(1,0) - D(1,1)}{r-\alpha}\right) < 0, \tag{66}$$

which confirms (61). The second derivative of $\phi(Y)$ is given by

$$\begin{aligned}\frac{\partial^2 \phi(Y)}{\partial Y^2} &= -\beta_1(\beta_1 - 1)\frac{Y^{\beta_2 - 1}}{Y_F^{\beta_1}} \left(\frac{Y_F(D(1,0) - D(0,1))}{r-\alpha} - I\right) \\ &= -\beta_1(\beta_1 - 1)\frac{Y^{\beta_2 - 1}}{Y_F^{\beta_1}} \left(\left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{D(1,0) - D(0,1)}{D(1,1) - D(0,1)}\right) - 1\right) I. \end{aligned} \tag{67}$$

Expression (62) follows from equation (67) since $\beta_1 > 1$ and $D(1,0) > D(1,1)$.

C Proof of Proposition 3

Define the function $\gamma(Y)$ as follows

$$\begin{aligned}\gamma(Y) &= C(Y, Y_C) - L(Y) \\ &= \frac{Y(D(0,0) - D(1,0))}{r-\alpha} + I + (H_1(Y_C) - E_1) Y^{\beta_1}. \end{aligned} \tag{68}$$

Expression (68) is derived using the following equations (cf. (19), (23), (25)):

$$C(Y, Y_C) = \frac{YD(0,0)}{r-\alpha} + H_1(Y_C)Y^{\beta_1}, \tag{69}$$

$$H_1(Y_C) = \frac{Y_C^{1-\beta_1}}{\beta_1}\left(\frac{D(1,1)-D(0,0)}{r-\alpha}\right) > 0, \tag{70}$$

$$L(Y) = \frac{YD(1,0)}{r-\alpha} - I + E_1Y^{\beta_1}, \tag{71}$$

$$E_1 = Y_F^{1-\beta_1}\left(\frac{D(1,1)-D(1,0)}{r-\alpha}\right) < 0. \tag{72}$$

Whenever there exists an $Y \in (Y_P, Y_F)$ such that $\gamma(Y) < 0$, the first case applies. If $\gamma(Y) \geq 0$ for all $Y \in (Y_P, Y_F)$ we are in the second case.

Lemma 2

The following properties hold:

$$\gamma(Y_P) > 0, \tag{73}$$

$$\gamma(Y_F) > 0, \tag{74}$$

$$\frac{\partial^2\gamma(Y)}{\partial Y^2} > 0. \tag{75}$$

In the proof of Lemma 2 we use the following lemma.

Lemma 3

For $0 < Y \leq Y_F$ it holds that

$$C(Y, Y_C) > F(Y). \tag{76}$$

Proof of Lemma 3

It is obvious that $C(Y, Y_F) > F(Y)$ for $Y \in (0, Y_F)$, since $D(0,0) > D(0,1)$. And by definition it holds that $C(Y, Y_C) \geq C(Y, Y_F)$.

Proof of Lemma 2

Properties (73) and (74) follow from Lemma 3 together with

$$L(Y_P) = F(Y_P), \tag{77}$$

$$L(Y_F) = F(Y_F). \tag{78}$$

The second derivative of $\gamma(Y)$ *is equal to*

$$\frac{\partial^2 \gamma(Y)}{\partial Y^2} = (\beta_1 - 1)\beta_1 (H_1(Y_C) - E_1) Y^{\beta_1 - 2}. \tag{79}$$

Remembering that $E_1 < 0$ *gives equation (75).*

Lemma 2 implies that there exists an $Y \in (Y_P, Y_F)$ such that $\gamma(Y) < 0$ if and only if the minimum of the function γ is negative and reached somewhere between Y_P and Y_F. Lemma 4 derives a condition for the minimum of γ to be negative and Lemma 5 proves that the minimum is reached in the interval $(0, Y_F)$.

Lemma 4

It holds that

$$\min_{Y \geq 0} \gamma(Y) < 0, \tag{80}$$

if and only if

$$f(\beta_1) < g(\beta_1). \tag{81}$$

Proof of Lemma 4

The first derivative of $\gamma(Y)$ *is given by*

$$\frac{\partial \gamma(Y)}{\partial Y} = \frac{D(0,0) - D(1,0)}{r - \alpha} + \beta_1 (H_1(Y_C) - E_1) Y^{\beta_1 - 1}. \tag{82}$$

The solution of

$$\frac{\partial \gamma(Y)}{\partial Y} = 0 \tag{83}$$

equals

$$Y^* = \left(\frac{D(1,0) - D(0,0)}{\beta_1 (H_1(Y_C) - E_1)(r-\alpha)}\right)^{\frac{1}{\beta_1 - 1}} > 0. \tag{84}$$

The minimum (expression (75) implies that $\gamma(Y^*)$ is an unique minimum) of γ equals

$$\gamma(Y^*) = I + (D(1,0) - D(0,0))^{\frac{\beta_1}{\beta_1-1}} (H_1(Y_C) - E_1)^{\frac{1}{1-\beta_1}} (r-\alpha)^{\frac{\beta_1}{1-\beta_1}} \left(\beta_1^{\frac{\beta_1}{1-\beta_1}} - \beta_1^{\frac{1}{1-\beta_1}}\right). \tag{85}$$

The minimum is negative if and only if (substitute (70) and (72) in (85) and rewrite)

$$I\left[1 - \left[\left(\frac{D(1,1) - D(0,0)}{D(1,0) - D(0,0)}\right)^{\beta_1} + \beta_1 \left(\frac{D(1,0) - D(1,1)}{D(1,1) - D(0,1)}\right)\left(\frac{D(1,1) - D(0,1)}{D(1,0) - D(0,0)}\right)^{\beta_1}\right]^{\frac{1}{1-\beta_1}}\right]$$
$$< 0$$

Dividing by I and rewriting gives

$$\left[\left(\frac{D(1,1) - D(0,0)}{D(1,0) - D(0,0)}\right)^{\beta_1} + \beta_1 \left(\frac{D(1,0) - D(1,1)}{D(1,1) - D(0,1)}\right)\left(\frac{D(1,1) - D(0,1)}{D(1,0) - D(0,0)}\right)^{\beta_1}\right]^{\frac{1}{1-\beta_1}} > 1.$$

Rearranging gives

$$\beta_1 \left(\frac{D(1,0) - D(1,1)}{D(1,1) - D(0,1)}\right) + \left(\frac{D(1,1) - D(0,0)}{D(1,1) - D(0,1)}\right)^{\beta_1} < \left(\frac{D(1,0) - D(0,0)}{D(1,1) - D(0,1)}\right)^{\beta_1}. \tag{86}$$

Substitution of (44) and (45) in (86) gives (81).

Lemma 5

It holds that

$$0 < Y^* \leq Y_F, \tag{87}$$

where the equality sign only holds for $\beta_1 = 1$.

Proof of Lemma 5

From (84) we have that $Y^* > 0$. Substitution of equations (72) and (70) in (84) gives

$$Y^* = Y_F \left[\frac{D(1,0) - D(0,0)}{(D(1,1) - D(0,1))^{1-\beta_1} (D(1,1) - D(0,0))^{\beta_1} + \beta_1 (D(1,0) - D(1,1))}\right]^{\frac{1}{\beta_1 - 1}}.$$

Thus

$$Y^* \leq Y_F$$

if and only if

$$\left[\frac{D(1,0) - D(0,0)}{(D(1,1) - D(0,1))^{1-\beta_1}(D(1,1) - D(0,0))^{\beta_1} + \beta_1(D(1,0) - D(1,1))}\right]^{\frac{1}{\beta_1 - 1}} \leq 1.$$

Rewriting gives

$$\frac{D(1,0) - D(0,0)}{D(1,1) - D(0,1)} \leq \beta_1\left(\frac{D(1,0) - D(1,1)}{D(1,1) - D(0,1)}\right) + \left(\frac{D(1,1) - D(0,0)}{D(1,1) - D(0,1)}\right)^{\beta_1}. \tag{88}$$

Combining (44) with (88) gives

$$f(1) \leq f(\beta_1). \tag{89}$$

Define

$$x := \frac{D(1,0) - D(0,0)}{D(1,1) - D(0,1)} > 1,$$
$$y := \frac{D(1,1) - D(0,0)}{D(1,1) - D(0,1)} < 1.$$

Using these two definitions we have for $\beta_1 \geq 1$ *:*

$$f(\beta_1) = \beta_1(x - y) + y^{\beta_1},$$

$$f'(\beta_1) = x - y + y^{\beta_1}\log(y),$$

$$f''(\beta_1) = y^{\beta_1}(\log(y))^2 > 0.$$

It turns out that f *is strictly increasing, because for* $0 < y < 1$ *we have*

$$x > 1 > y - y\log(y) \geq y - y^{\beta_1}\log(y).$$

Thus equation (89) holds, the equality sign only holds for $\beta_1 = 1$, *and thereby the lemma.*

Combining Lemma's 4 and 5 gives Proposition 3.

D Proof of Proposition 4

First we prove the following lemma.

Lemma 6

If

$$f'(\beta_1^*) \leq g'(\beta_1^*), \tag{90}$$

it holds that for all $\beta_1 \in (\beta_1^*, \infty)$ *that:*

$$f'(\beta_1) < g'(\beta_1). \tag{91}$$

Proof of Lemma 6

Define

$$x := \frac{D(1,0) - D(0,0)}{D(1,1) - D(0,1)}, \tag{92}$$

$$y := \frac{D(1,1) - D(0,0)}{D(1,1) - D(0,1)}. \tag{93}$$

Then it holds that $0 < y < 1 < x$ *and*

$$f'(\beta_1) = x - y + y^{\beta_1} \log(y) > 0, \tag{94}$$

$$g'(\beta_1) = x^{\beta_1} \log(x) > 0. \tag{95}$$

The proof of $f'(\beta_1)$ *being positive is given in Appendix C. The second and third derivative of* f *and* g *are given by*

$$f''(\beta_1) = y^{\beta_1} (\log(y))^2 > 0, \tag{96}$$

$$f'''(\beta_1) = y^{\beta_1} (\log(y))^3 < 0, \tag{97}$$

$$g''(\beta_1) = x^{\beta_1} (\log(x))^2 > 0, \tag{98}$$

$$g'''(\beta_1) = x^{\beta_1} (\log(x))^3 > 0. \tag{99}$$

First consider the case where $f'(1) > g'(1)$. *Due to equations (96)-(99) we know that* f'' *is positive and decreasing and* g'' *is positive and increasing so that there exists a unique* β_1^* *for which* $f'(\beta_1^*) = g'(\beta_1^*)$ *and* $f'(\beta_1) < g'(\beta_1)$ *for all* $\beta_1 > \beta_1^*$.

When $f'(1) \leq g'(1)$ *we have to prove that for all* $\beta_1 > 1$ *it holds that* $g'(\beta_1) > f'(\beta_1)$.

This is certainly true when $f''(\beta_1) < g''(\beta_1)$ *for all* $\beta_1 \geq 1$. *Due to equations (97) and (99) it is sufficient to prove that* $f''(1) < g''(1)$. *Thus we have to prove that for* $0 < y < 1 < x$

$$f'(1) \leq g'(1) \quad \Longleftrightarrow \quad x - y + y \log(y) \leq x \log(x), \tag{100}$$

implies

$$f''(1) < g''(1) \quad \Longleftrightarrow \quad y (\log(y))^2 < x (\log(x))^2. \tag{101}$$

Using the transformation $u = \log(x)$ *and* $v = \log(y)$ *gives that for* $u > 0$ *and* $v < 0$

$$e^u (u-1) - e^v (v-1) \geq 0, \tag{102}$$

has to imply that

$$e^u u^2 - e^v v^2 > 0. \tag{103}$$

Consider the (u, v) *plane. Now equation (102) holds for a combination of values of* u *and* v *on and above the curve* $e^u (1-u) = e^v (1-v)$ *and equation (103) holds for* u *and* v *values above the curve* $e^v v^2 = e^u u^2$. *The lemma holds because the curve* $e^u (1-u) = e^v (1-v)$ *is situated above the curve* $e^v v^2 = e^u u^2$. *This is the case because the curves intersect at* $(0,0)$, *and the differential* $\frac{du}{dv}$ *of the first curve is smaller than the corresponding differential of the second curve since:*

$$\frac{ve^v}{ue^u} < \frac{(v^2+2v)\,e^v}{(u^2+2u)\,e^u},$$
$$1 > \frac{v+2}{u+2}.$$

For a visualization see Figure 6.

Proof of Proposition 4

Note that for $\alpha > 0$ *the relevant* β_1 *interval is* $\left(1, \frac{r}{\alpha}\right)$, *since*

$$\lim_{\sigma\to\infty} \beta_1(\sigma) = 1, \tag{104}$$

$$\lim_{\sigma\to 0} \beta_1(\sigma) = \frac{r}{\alpha}. \tag{105}$$

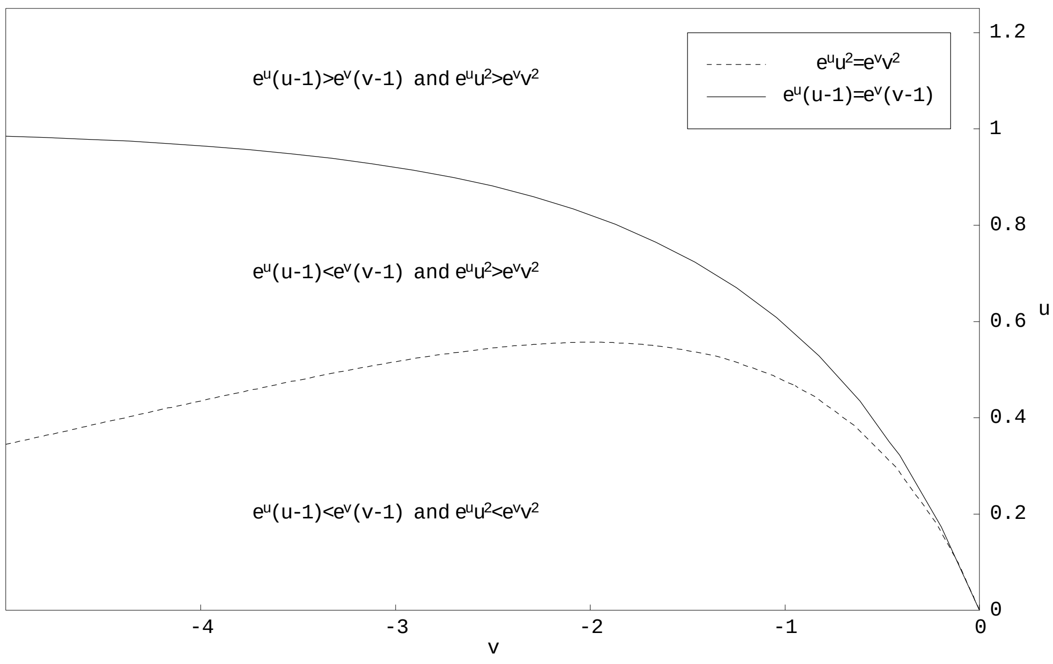

Figure 6. The curves $e^u u^2 = e^v v^2$ and $e^u (u-1) = e^v (v-1)$ for $u > 0$ and $v < 0$.

It holds that

$$f(1) = g(1) = x. \tag{106}$$

From (94)-(96) and (98) we know that f and g are convex and increasing in β_1. Further, Lemma 6 implies that only the following cases can occur (see also Figure 4):

(i) If $g'(1) \geq f'(1)$ equation (46) is satisfied for all $\beta_1 \in \left(1, \frac{r}{\alpha}\right)$, so that the equilibrium is always of the preemption type.

(ii) If $f'(1) > g'(1)$ and $f\left(\frac{r}{\alpha}\right) \geq g\left(\frac{r}{\alpha}\right)$ equation (46) is never satisfied for a $\beta_1 \in \left(1, \frac{r}{\alpha}\right)$, thus the equilibrium is always of the collusion type.

(iii) If $f'(1) > g'(1)$ and $f\left(\frac{r}{\alpha}\right) < g\left(\frac{r}{\alpha}\right)$ equation (46) is satisfied for high values of β_1 and not satisfied for low values of β_1.

E Proof of Corollary 1

Proof of Corollary 1

Please remember the quadratic equation (8):

$$Q(\beta_1) = \frac{1}{2}\sigma^2\beta_1(\beta_1 - 1) + \alpha\beta - r = 0,$$

It holds that

$$\frac{\partial Q}{\partial \beta_1}\frac{\partial \beta_1}{\partial \sigma} + \frac{\partial Q}{\partial \sigma} = 0,$$

Since $\frac{\partial Q}{\partial \beta_1} > 0$ *and* $\frac{\partial Q}{\partial \sigma} > 0$ *we have that* $\frac{\partial \beta_1}{\partial \sigma} < 0$. *In the same way we can show that* $\frac{\partial \beta_1}{\partial \alpha} < 0$ *and* $\frac{\partial \beta_1}{\partial r} > 0$ *(see also Dixit and Pindyck (1996, p. 144)).*

F Proof of Proposition 5

Proof of Proposition 5

Since $D(1,0) > D(1,1)$ *we know that* $Y_M < Y_C$. *Substitution of equation (9) in equation (58) yields*

$$\begin{aligned} \phi(Y_M) = {} & I\left[\left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{D(1,0) - D(0,1)}{D(1,0) - D(0,0)} - 1\right] \\ & - I\left(\frac{D(1,1) - D(0,1)}{D(1,0) - D(0,0)}\right)^{\beta_1}\left[\left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{D(1,0) - D(0,1)}{D(1,1) - D(0,1)} - 1\right]. \end{aligned} \tag{107}$$

Substitution of the following definitions

$$\xi = \frac{D(1,0) - D(0,1)}{D(1,0) - D(0,0)} > 1, \tag{108}$$

$$\chi = \frac{D(1,1) - D(0,1)}{D(1,0) - D(0,0)} \leq 1, \tag{109}$$

gives

$$\phi(Y_M) = I\left[\left(\frac{\beta_1}{\beta_1 - 1}\right)\xi\left(1 - \chi^{\beta_1 - 1}\right) - 1 + \chi^{\beta_1}\right]. \tag{110}$$

Differentiating with respect to χ *gives*

$$\begin{aligned} \frac{\partial \phi(Y_M)}{\partial \chi} &= I\left[-\beta_1\xi\chi^{\beta_1 - 2} + \beta_1\chi^{\beta_1 - 1}\right] \\ &= -I\beta_1\chi^{\beta_1 - 2}(\xi - \chi) < 0. \end{aligned} \tag{111}$$

This implies that $\phi(Y_M)$ is decreasing in χ. Since for $\chi = 1$ we know that $\phi(Y_M) = 0$, we have $\phi(Y_M) \geq 0$. Therefore $Y_P \leq Y_M$.

G Proof of Proposition 6

Proof of Proposition 6

The thresholds, Y_L, Y_M, Y_F and Y_C are decreasing in β_1 since

$$\frac{\partial \frac{\beta_1}{\beta_1-1}}{\partial \beta_1} = -\frac{1}{(\beta_1-1)^2} < 0. \tag{112}$$

Hence, the only thing that is left to prove is that Y_P decreases with β_1. To do so define for $Y \in [0, Y_F]$ and $\beta_1 \in [0, \infty)$ (cf. (48)):

$$\begin{aligned}\phi(Y, \beta_1) :&= L(Y) - F(Y)\\ &= \frac{Y(D(1,0) - D(0,1))}{r-\alpha} - I + \left(\frac{Y}{Y_F}\right)^{\beta_1}\left(\frac{Y_F(D(0,1) - D(1,0))}{r-\alpha} + I\right).\end{aligned} \tag{113}$$

From the definition of $Y_P(\beta_1)$ we know that

$$\phi(Y_P(\beta_1), \beta_1) = 0. \tag{114}$$

Differentiating (114) to β_1 gives

$$\left.\frac{\partial \phi(Y, \beta_1)}{\partial \beta_1}\right|_{Y=Y_P(\beta_1)} + \left.\frac{\partial \phi(Y, \beta_1)}{\partial Y}\right|_{Y=Y_P(\beta_1)} \frac{\partial Y_P(\beta_1)}{\partial \beta_1} = 0. \tag{115}$$

Hence, to say something about the sign of $\frac{\partial Y_P(\beta_1)}{\partial \beta_1}$, we need to determine the signs of $\left.\frac{\partial \phi(Y,\beta_1)}{\partial \beta_1}\right|_{Y=Y_P(\beta_1)}$ and $\left.\frac{\partial \phi(Y,\beta_1)}{\partial Y}\right|_{Y=Y_P(\beta_1)}$. From Lemma 1 we already know that

$$\left.\frac{\partial \phi(Y, \beta_1)}{\partial Y}\right|_{Y=Y_P(\beta_1)} > 0. \tag{116}$$

Now let us concentrate at $\left.\frac{\partial \phi(Y,\beta_1)}{\partial \beta_1}\right|_{Y=Y_P(\beta_1)}$. To do so, first substitute equation (16) in (113), which gives

$$\begin{aligned}\phi(Y, \beta_1) =& \frac{Y(D(1,0) - D(0,1))}{r-\alpha} - I\\ &+ \frac{I\left(Y\frac{\beta_1-1}{\beta_1}\frac{D(1,1)-D(0,1)}{(r-\alpha)I}\right)^{\beta_1}}{(\beta_1-1)(D(1,1) - D(0,1))}((\beta_1 - 1)D(1,1) - \beta_1 D(1,0) + D(0,1)).\end{aligned} \tag{117}$$

From (117) it is obtained that

$$\frac{\partial \phi(Y,\beta_1)}{\partial \beta_1} = \frac{I\left(Y\frac{\beta_1-1}{\beta_1}\frac{D(1,1)-D(0,1)}{(r-\alpha)I}\right)^{\beta_1}}{(\beta_1-1)(D(1,1)-D(0,1))}\Bigg(D(1,1)-D(1,0) \tag{118}$$

$$+ \quad \log\left(Y\frac{\beta_1-1}{\beta_1}\frac{D(1,1)-D(0,1)}{(r-\alpha)I}\right)((\beta_1-1)D(1,1)-\beta_1 D(1,0)+D(0,1))\Bigg).$$

Define

$$\overline{Y}(\beta_1) := \frac{\beta_1}{\beta_1-1}\frac{(r-\alpha)I}{D(1,0)-D(0,1)}. \tag{119}$$

Substitution of (119) into (117) gives

$$\phi\left(\overline{Y}(\beta_1),\beta_1\right) = \frac{I}{\beta_1-1}\left[1+\left(\frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)}\right)^{\beta_1}\left(\beta_1-1-\beta_1\left(\frac{D(1,0)-D(0,1)}{D(1,1)-D(0,1)}\right)\right)\right]$$

$$= \frac{I}{\beta_1-1}\left[1+x^{\beta_1}\left(\beta_1-1-\frac{\beta_1}{x}\right)\right] > 0, \tag{120}$$

where

$$0 < x := \frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)} < 1.$$

Lemma 1 and equation (120) imply that

$$Y_P(\beta_1) < \overline{Y}(\beta_1). \tag{121}$$

From (118) we conclude that $\frac{\partial \phi(Y,\beta_1)}{\partial \beta_1} > 0$ *for sufficiently low values of* Y.

It holds that

$$\left.\frac{\partial \phi(Y,\beta_1)}{\partial \beta_1}\right|_{Y=\overline{Y}(\beta_1)} > 0, \tag{122}$$

if and only if

$$\omega(\beta_1) := D(1,1)-D(1,0)$$

$$+\log\left(\frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)}\right)((\beta_1-1)D(1,1)-\beta_1 D(1,0)+D(0,1)) > 0. \tag{123}$$

To prove this we first note that the function $\omega(\beta_1)$ *is increasing in* β_1 :

$$\frac{\partial \omega(\beta_1)}{\partial \beta_1} = \log\left(\frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)}\right)(D(1,1)-D(1,0)) > 0.$$

Furthermore $\omega(1) > 0$, *since*

$$\log\left(\frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)}\right) < \frac{D(1,1)-D(0,1)}{D(1,0)-D(0,1)} - 1,$$

so that (123) is valid. Thus equation (122) holds. Now, from (121) and (122) we have

$$\left.\frac{\partial \phi(Y, \beta_1)}{\partial \beta_1}\right|_{Y=Y_P(\beta_1)} > 0. \tag{124}$$

Finally, from (115), (116), and (124) it can be concluded that

$$\frac{\partial Y_P(\beta_1)}{\partial \beta_1} < 0.$$

H Proof of Proposition 7

Proof of Proposition 7

The option value of waiting in the monopoly case is given by equation (12). At the moment of investment in the preemption case, the strategic option value of waiting equals

$$\begin{aligned} L(Y_P) - \frac{Y_P D(0,0)}{r-\alpha} &< \frac{Y_P D(1,0)}{r-\alpha} - I - \frac{Y_P D(0,0)}{r-\alpha} \\ &< \frac{Y_M (D(1,0) - D(0,0))}{r-\alpha} - I \\ &= \frac{I}{\beta_1 - 1}. \end{aligned}$$

In the collusive case we have

$$\begin{aligned} C(Y_C, Y_C) - \frac{Y_C D(0,0)}{r-\alpha} &= \frac{Y_C D(1,1)}{r-\alpha} - I - \frac{Y_C D(0,0)}{r-\alpha} \\ &= \frac{I}{\beta_1 - 1}. \end{aligned}$$

References

Dixit, A. K. and R. S. Pindyck (1996). *Investment Under Uncertainty,* second printing. Princeton University Press, Princeton, New Jersey.

Dutta, P. K., S. Lach, and A. Rustichini (1995). Better late than early: Vertical differentiation in the adoption of a new technology. *Journal of Economics and Management Strategy*, 4, 563–589.

Fudenberg, D. and J. Tirole (1985). Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies*, 52, 383–401.

GRENADIER, S. R. (1996). The strategic exercise of options: Development cascades and overbuilding in real estate markets. *Journal of Finance*, 51, 1653–1679.

NIELSEN, M. J. (1999). Competition and irreversible investments. Working Paper, University of Copenhagen, Institute of Economics, Studiestraede 6, 1455 Kbh K, Denmark.

REINGANUM, J. F. (1981). On the diffusion of new technology: A game theoretic approach. *The Review of Economic Studies*, 48, 395–405.

SIMON, L. K. (1987a). Basic timing games. *Working Paper 8745*, Department of Economics, University of California Berkeley, Berkeley, California.

SIMON, L. K. (1987b). A multistage duel in continuous time. *Working Paper 8757*, Department of Economics, University of California Berkeley, Berkeley, California.

SMETS, F. (1991). Exporting versus FDI: The effect of uncertainty, irreversibilities and strategic interactions. Working Paper, Yale University.

WEEDS, H. (1999). Strategic delay in a real options model of R&D competition. University of Cambridge, Fitzwilliam College, Cambridge CB3 0DG, UK.