On Adjusting the HP-Filter for the Frequency of Observations*

Morten O. Ravn
University of Southampton, Universitat Pompeu Fabra,
CEPR,
and Center for Non-Linear Modelling in Economics.
and
Harald Uhlig
CentER, Tilburg University
and CEPR

COMMENTS WELCOME

Keywords: HP-Filter, frequency of observation, real business cycles, output volatility, pre-war business cycles

JEL codes: C10, C22, C63, E32, F14

Fields: macroeconomics, econometrics

First draft: March 24th, 1997
This revision: May 29, 1997

*We thank Dave Backus for provision of data. The address of the authors are Morten O. Ravn, before September 1997: Dept. of Econ, Univ. of Southampton, Southampton S017 1BJ, UK, e-mail: mor@soton.ac.uk. Morten Ravn, after September 1997: Dept. of Econ.,Universitat Pompeu Fabra, Ramon Trias Fargas, 25-27, E-08005 Barcelona, Spain, e-mail: ravn@upf.es (probably). Harald Uhlig, CentER, Tilburg University, Postbus 90153, 5000 LE Tilburg, The Netherlands, e-mail: uhlig@kub.nl. Fax: +31-13-4663066. Phone: +31-13-4663104. Home page: http://cwis.kub.nl/~few5/center/STAFF/uhlig/home.htm.

Abstract

This paper studies how the HP-Filter should be adjusted, when changing the frequency of observations. The usual choices in the literature are to adjust the smoothing parameter by multiplying it with either the square of the observation frequency ratios or simply with the observation frequency. In contrast, the paper recommends to adjust the filter parameter by multiplying it with the fourth power of the observation frequency ratios. Based on this suggestion, some well-known comparisons of business cycles moments across countries and time periods are recomputed. In particular, we overturn a finding by Backus and Kehoe (1992) on the historical changes in output volatility and return instead to older conventional wisdom (Baily, 1978, Lucas, 1977): based on the new HP-Filter adjustment rule, output volatility turns out to have decreased after the Second World War.

1 Introduction

The Hodrick and Prescott (1980) filter (the HP-filter hereafter) has become the standard method for removing long-run movements from the data in the business cycle literature. The filter has been applied both to actual data in studies that attempt to document business cycle "facts" (see e.g. Backus and Kehoe, 1992, Blackburn and Ravn, 1992, Brandner and Neusser, 1992, Danthine and Donaldson, 1993, Fiorito and Kollintzas, 1994, and Kydland and Prescott, 1990) and in studies where artificial data from a model are compared with the actual data (see e.g. Backus, Kehoe and Kydland, 1992, Cooley and Hansen, 1989, Hansen, 1985, and Kydland and Prescott, 1982). This paper studies how one should adjust this filter to the frequency of data that one is working with. We show that the filter should be adjusted with the fourth power of the observation frequency ratios, and illustrate that this new adjustment rule changes some of the published business cycle facts.

There is a fairly large literature discussing and criticizing the HP-filter for a number of short-comings and undesirable properties (see e.g. Canova, 1994, 1997, Cogley and Nason, 1995, Harvey and Jaeger, 1993, King and Rebelo, 1992, or Söderlind, 1994). Among other things, this literature has argued that the filter might lead to spurious cycles if the data is difference stationary, that the filter might generate most of the cycles in the artificial data, that the filter is only optimal (in the sense of minimizing the mean square error) in special cases, and that it may produce extreme second-order properties in the detrended data. However, our reading of that literature is that none of these short-comings or undesirable properties are particularly compelling: the HP-filter has withstood the test of time and the fire of discussion remarkably well. Thus, it appears most likely that the HP-filter will remain the standard method for detrending in theoretically oriented work for a long time to come. For that reason, it seems particularly important to understand how the HP-filter should be adjusted when the frequency of the observations is changed. This is the purpose of this paper.

Most studies seem to use the standard value of 1600 for the smoothing parameter involved in the HP-filter at the quarterly frequency, but the literature seems to be divided over the issue of how to adjust the filter to the frequency of observations. Two different rules for adjusting the HP-filter to annual data has been suggested. On the one hand, Backus and Kehoe (1992), who study business cycle properties of a cross-section of countries using annual data, use a value of 100 for the smoothing parameter. Thus, their study implies that one should adjust the smoothing parameter to alternative frequencies by multiplying the standard value of 1600 at the quarterly frequency with the square of the alternative sampling frequency. On the other hand, Correia, Neves and Rebelo (1992) and Cooley and Ohanian (1991) use a value of

400 for annual data, thus implying to adjust the smoothing parameter linearly with the frequency of the data. In contrast to both branches of the literature, this paper recommends to adjust the filter parameter by multiplying it with the fourth power of the observation frequency ratios. For annual data, this implies a value of 6.25 for the smoothing parameter, while one obtains 129600 as the appropriate value for monthly data.

This rule is based on a number of different arguments. First, we analyze the issue in frequency domain and provide a visual comparison of the HP-filter transfer functions. For the proposed fourth-power adjustment, one gets a virtually perfect match, whereas wide gaps open for any other adjustment by integer powers of the frequency change. Secondly, we analyze the HP-filter transfer function analytically. More precisely, given that one conjectures that the rule for adjusting the smoothing parameter involves adjusting it with the sampling frequency raised to some power, say n, we show that n should be between 3.8 and 4. Third, we show more informally that the trends and cycles for data sampled at different frequencies (monthly and quarterly in our example) are very close to each other when using, say, a quadratic or a linear adjustment rule. This evidence is finally further supported with a Monte Carlo analysis of moments typically studied in the business cycle literature.

This leads one naturally to the question of whether this issue matters for the business cycle "facts". To investigate this we recompute some of the important results on the properties historical business cycles documented in Backus and Kehoe's (1992) study. We look at two of their more interesting results: (a) that output volatility was higher in the interwar period than in the postwar period while no such rule exists for a comparison of prewar fluctuations with postwar fluctuations, and (b) that prices changed from being mainly procyclical before World War II (WWII) to being mainly countercyclical thereafter. We find that when using the alternative value for the smoothing parameter advocated for here, the latter of these results remain but the former change. In particular, we find that output volatility generally has been lower in the postwar period than in both the prewar period and in the interwar period. This result is a return to the conventional wisdom of e.g. Baily (1978), Burns (1960), Lucas (1977), and Tobin (1980) that output volatility declined after WWII. Baily (1978) and Tobin (1980) took the decline in output volatility to imply that US economic policy since WWII, including the use of "..built-in and discretionary stabilization" (Tobin, 1980, p.48), had been successful in dampening macroeconomic fluctuations. Thus, our over-turning of Backus and Kehoe's (1992) result has important implications if interpreted along the lines of the successfulness of macroeconomic stabilization policy.

The remainder of the paper is organized as follows. In section 2 the HP-filter is defined and briefly discussed. In section 3, the adjustment problem is cast in frequency-domain language. Subsection 3.1 provides the visual comparison, whereas subsection 3.2 contains the analytics. Frequency-domain language provides a particularly natural framework to address the issue at hand. Nonetheless, we also provide a time-domain analysis of the problem in section 4. There, one has to resort to simulations, but one may get a better insight into the actual impact of the filter on a given series. Section 4.1 looks at one particular sample, whereas subsection 4.2 provides a more extensive Monte-Carlo analysis. In section 5, we recompute some facts stated in the literature about variability of certain macroeconomic time series: since these facts are based on annual rather than quarterly data, this recomputation is now necessary in light of our recommendation on how the HP-filter parameter should be adjusted when changing the frequency of the observations. Section 6 concludes.

2 The HP-Filter

The HP filter aims at removing a smooth trend τ_t from some given data y_t by solving

$$\min_{\tau_t} \sum_{t=1}^{T} \left((y_t - \tau_t)^2 + \lambda \left((\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1}) \right)^2 \right)$$

The residual $y_t - \tau_t$ is then commonly referred to as the "business cycle component". The HP-filter has been used extensively for a number of different purposes. First, a number of studies has applied it to actual data in order to establish "stylized facts" of macroeconomic fluctuations: several references were given in the introduction. Secondly, it has also been applied to study the shapes of business cycles: Sichel (1993), for example, uses the HP filter to analyze their asymmetry. Thirdly, the HP filter has been applied in quantitative business cycle theories when comparing artifical model data with actual data: again, a number of references were cited in the introduction. In these studies, it has become common practice to compare standard deviations and autocorrelations of filtered series coming from actual data vis-a-vis artificial data generated with some model.

The filter involves the smoothing parameter λ which penalizes the acceleration in the trend component relative to the business cycle component. Researchers typically set $\lambda = 1600$ when working with quarterly data. However, data does not always come at quarterly intervals. It may even be desirable to move to annual, monthly or some other time interval of observation instead. Thus, the question arises how the HP-filter should be adjusted for the frequency of observations? This question is the focus of this paper. More specifically, our aim is to check, how the HP-filter parameter λ should be changed, in case the frequency of observations is changed from, say, quarterly to monthly or annual data.

The point of view taken in this paper is that the value $\lambda = 1600$ for quarterly data is nothing but a definition of business cycles via the duration of its components:

movements of the data are defined to be of business-cycle or shorter nature, if the filter attributes them to the business cycle component $y_t - \tau_t$ rather than the long run component τ_t. The convention $\lambda = 1600$ generates business cycles components in line with older definitions, which view business cycles to last up to a few years, but not more. Thus, our measuring stick in judging a choice to be good is to keep attributing movements of the same duration to the business cycle component, regardless of the frequency of observations: if some cyclical movement in the data has a periodicity of four years, it should always (or never) be part of the business cycle component in just the same way, regardless of whether the data is observed at monthly, quarterly or annual frequency.

This point of view merits a bit of discussion, since other justifications for $\lambda = 1600$ have been given. Hodrick and Prescott (1980) favored the choice of $\lambda = 1600$ based on the argument that a 5 percent deviation from trend per quarter is relatively moderate as is an eighth percentage change in the trend component. They show that λ can be interpreted as the variance of the business cycle component divided by the variance of the acceleration in the trend component if the cycle component and the second difference of trend component are mean zero i.i.d. normally distributed variables. For Hodrick and Prescott's (1980) prior λ then follows as $5^2/(1/8)^2 = 1600$. Harvey and Jaeger (1994) state that attempts to *estimate* the smoothing parameter this way usually leads to very small values of the smoothing parameter because the maintained assumption of i.i.d. normally distributed cyclical and second differenced trend components are violated. The original Hodrick-Prescott (1980) justification is thus unlikely to be robust against the type of data used or the frequency at which it is sampled. Alternatively, λ can be thought of as a measure of fit or as a signal extraction coefficient and could in principle be estimated from the data by setting up the minimization problem as a signal extraction / prediction error decomposition (see Canova, 1997, for such an approach). But again, it is likely that the amount of information to be extracted from, say, monthly data differs from what can be extracted from annual data. We view these approaches as complementary but just technical in nature. Economically, the choice of $\lambda = 1600$ is one about defining as to what one views to be the length of business cycles: this is the point of view which we maintain throughout.

3 A Frequency-Domain Perspective.

First, we look at the comparison in the frequency domain. The frequency domain perspective allows us conveniently to talk about the durations of the cyclical pieces of the movements in the data, and hence to phrase the issue cleanly. King and Rebelo

(1993) have calculated the transfer function of the HP-filter and shown it to be given by[1]

$$h(\omega; \lambda) = \frac{4\lambda(1-\cos(\omega))^2}{1+4\lambda(1-\cos(\omega))^2} \tag{1}$$

A plot of this transfer function together with the plot of a high pass filter cutting off frequencies below $\omega \leq \pi/20$ can be seen in figure 1. As one can see, the two filters are rather similar: both attribute peak-to-peak cyclical movements of less than ten years of duration (and thus with a quarterly frequency of more than $\pi/20$) to be part of the business cycle component. Choosing different values for λ is comparable to choosing different values for the cut off point of the high pass filter. This figure thus makes sense out of our statement above, that the choice for $\lambda = 1600$ is nothing but a definition of business cycles via the duration length of its pieces.

With quarterly data, $\omega = \pi/2$ corresponds to a quarter. With annual data, $\omega = 2\pi$ corresponds to a quarter. Thus, let $h(\omega; \lambda_1)$ be the filter representation for quarterly data and let $h(\omega; \lambda_s)$ be the filter representation for an alternative sampling frequency s, where we let s be the ratio of the frequency of observation compared to quarterly data, i.e. $s = 1/4$ for annual data or $s = 3$ for monthly data. Then, ideally, we would like to have:

$$h(\omega; \lambda_1) \approx h(\omega/s; \lambda_s) \tag{2}$$

While this cannot hold exactly for all ω, it should hold at least approximately. In principle, one could choose some measure of distance between two functions, like e.g. the supremum metric, and find λ_s as to minimize the distance between the two functions $h(\omega; \lambda_1)$ and $h(\omega/s; \lambda_s)$ according to that chosen metric (and possibly restricting ω to be in some relevant range). However, it seems hard to argue in favor of any particular metric especially a priori. Instead, we simply check whether some simple rule for adjusting λ works well, and provide a visual as well as point-by-point analytic comparison. A simple criterion is to multiply λ with some power of the frequency adjustment, i.e. to choose

$$\lambda_s = s^n \lambda_1 \tag{3}$$

The literature has suggested to choose $n = 2$, see e.g. Backus and Kehoe (1992), or even $n = 1$, see e.g. Cooley and Ohanian (1991) or Correia, Neves and Rebelo (1992). We will show that $n = 4$ (or at least something very close to it) is the most sensible choice.

[1] This is the Fourier transform of the HP-filter assuming that the number of observations tends to ∞. This will be very close to the finite sample filter except for very short samples and near to the initial and final observations in the sample.

3.1 A visual comparison ...

To provide a visual comparison, we show the transfer functions for the quarterly frequency vis-a-vis transfer functions for annual data, plotted over quarterly frequencies. For the annual data we illustrate the transfer function for $n = 1; 2; 3; 4; 5$. With $\lambda_1 = 1600$, one gets

$$\lambda_{0.25} \in \{400; 100; 25; 6.25; 1.56\}$$

where 400 corresponds to linear adjustment, where 100 relates to the "square" rule and where 6.25 is our suggestion from adjusting with the fourth power of the frequency ratios.

The results can be seen in figure 2. In that figure hn, $n = 1, \ldots, 5$ corresponds to using $\lambda_{0.25} = 0.25^n \lambda_1$ and hence, the filter transfer function for annual data, plotted over quarterly frequencies, whereas h_quarterly corresponds to the transfer function of the usual HP-filter in quarterly data. Note, how h4 matches h_quarterly most closely: their two curves are extremely close. To show their difference, figure 3 is provided. For a more fine-tuned comparison, we have also included non-integer powers between $n = 3.8$ and $n = 4.05$. The suggested value of $n = 4$ works very well, as one can see: the difference between the transfer functions is nowhere larger than 0.025.

Even more striking results are obtained for a comparison between quarterly and monthly frequencies. We just show the differences between the transfer functions in figure 4. There, the differences between these functions is virtually negligible for $n = 4$ and nowhere larger than 0.002. Of course, the switch to monthly rather than quarterly frequencies is less common in the literature than the switch to annual rather than quarterly frequencies discussed above.

n	3.75	3.80	3.85	3.90	3.95	4.00
$\lambda_{0.25} = 1600 * 0.25^n =$	8.84	8.25	7.69	7.18	6.70	6.25

Table 1: *Values for the HP filter parameter $\lambda_{0.25}$ for annual data, when adjusting with noninteger powers close to 4. Depending on the context, adjustments with these values might give slightly better results, but the resulting differences are unlikely to matter much in practice.*

Figures 3 and 4 thus indicate, that the adjustment with $n = 4$ is practically perfect when moving from quarterly to monthly data, whereas it is doing somewhat worse when moving from quarterly to annual data, with values such as $n = 3.9$ or $n = 3.95$ perhaps slightly preferable. We continue to find this effect also in our other comparisons below. Thus, table 1 lists the resulting values for the HP filter parameter for annual data, when using noninteger powers for n close to $n = 4$. Depending on the context, adjustments with these values might give slightly better results, but the resulting differences are unlikely to matter much in practice. We thus stick to our recommendation to pick $n = 4$ and thus $\lambda_{0.25} = 6.25$ as a useful and simple rule and to establish a literature benchmark. Obviously, an analysis using any value for $\lambda_{0.25}$ between 6.25 and, say, 8.25, is perfectly reasonable too. Most importantly, whatever one picks within this range will be quite different from the values $\lambda_{0.25} = 100$ or even $\lambda_{0.25} = 400$ used in the literature so far, and we strongly recommend to discontinue their use.

To summarize, the results so far suggest that λ in the HP filter should move with the fourth power of the frequency of observations. In particular, given the standard choice of

$$\lambda_1 = 1600$$

for quarterly data, one should use

$$\lambda_3 = 129600$$

for monthly data, and

$$\lambda_{0.25} = 6.25$$

for annual data. For annual data, any value in the range $6.25 \leq \lambda_{0.25} \leq 8.25$ is a reasonable choice too. We will proceed to show further arguments in favour of these recommendations.

3.2 ...and an analytic argument.

One can look at the comparison with analytic methods as well. To this end, we consider a marginal change in the observation frequency ratio s around $s = 1$, and look at its differential impact on the HP-filter. We assume λ_s to be the function (3) of s, taking the power n as parameter. For the correct adjustment, is should be the case that

$$\frac{d}{ds} h(\omega/s; \lambda_s) \approx 0 \tag{4}$$

where $\frac{d}{ds}$ denotes taking the total derivative with respect to s. For each ω and s, this equation can be solved for the parameter $n = n(s, \omega)$: one finds

$$n(s, \omega) = 2 \frac{\omega/s \sin(\omega/s)}{1 - \cos(\omega/s)} \tag{5}$$

If the power specification is appropriate, then this expression should be approximately constant over the range of relevant frequencies ω. Here, "relevant" should mean the range of frequencies over which the HP-filter for quarterly frequencies is not close to a constant anyhow as a function of ω, since the derivative of equation (4) will be close to zero there anyhow as well. Inspecting figure 1, it certainly suffices to restrict attention to values $0 \leq \omega \leq \pi/5$. Table 2 lists values of $n = n(s, \omega)$ at $s = 1$. More generally, note that $n(s, \omega) = n(\omega/s)$, which means that these values are also valid for quite different values of s, provided ω is adjusted suitably as well.

ω	0	$\pi/20$	$\pi/10$	$\pi/5$
$n(1, \omega)$	4	3.992	3.967	3.868

Table 2: *The optimal power adjustment at frequency ω for an adjustment locally around a quarterly sampling rate. As one can see, the optimal adjustment is generally between* 3.8 *and* 4.0 *at the relevant frequencies.*

What this suggests is that $n = 4$ or something close to it is an excellent choice if one wishes to make the transfer function invariant to the sampling frequency. The analysis furthermore shows that $n = 4$ is the exact outcome only at $\omega = 0$: otherwise, a slightly lower number between, say, $n = 3.8$ and $n = 4$ might be more appropriate.

4 A Time-Domain Perspective.

To clinch our case, we will complement the frequency-domain perspective with some results in the time domain by filtering some artificially generated data, taking a detailed look at one sample and a more general Monte-Carlo study.

4.1 One sample...

More precisely, we generate a "quarterly" series from an AR(1) process,

$$y_t = \theta y_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2) \tag{6}$$

and then draw "annual" observations from it by taking every fourth observation. We then apply the HP-filter to the quarterly data as well as to the annual data. For the annual data, we again use the five λ values as stated above, and then compare the resulting time trends, which are then usually subtracted from the original series in order to get the residual "business cycle component".

The results from comparing time trends can be seen in figure 5. To generate it, we have set $\theta = 0.95$. In this figure $tn, n = 1, \ldots, 5$ corresponds to using $\lambda_{0.25} = 0.25^n \lambda_1$, whereas t_quarterly corresponds to the trend generated by the usual HP-filter in quarterly data. Again, note, how t4 matches t_quarterly most closely! As with the frequency-domain analysis, this suggests that λ in the HP-filter should move with the fourth power of the frequency of observations.

We also generated the annual observations by averaging each four quarterly observations to see whether there is a difference between sampling or averaging: apart from affecting the persistence of the series we don't expect this to make much of a difference when calculating trends, since the HP-filter pertains to lower-frequencies[2]. A visual comparison of the trends yielded a picture which is extremely similar to figure 5 and we have therefore chosen to exclude it, although it would have helped our case: if anything, t4 matched t_quarterly even more closely than in figure 5.

When visually checking the business cycle components, differences remain, regardless of which $\lambda_{0.25}$ is chosen, see figure 6. This shouldn't surprise: higher-frequency data will always contain additional sources of noise at these higher frequencies. It is thus more appropriate to compare what has been *taken out*, i.e. to compare the trend components as in figure 5 rather than to compare what has been left in as here.

4.2 ... and a Monte Carlo Analysis.

We also report Monte Carlo results for the standard deviations and first-order autocorrelations calculated from the business cycle component, since comparing moments such as these are typically at the heart of applications of the HP-filter. These Monte Carlo experiments were also performed for the comparison of quarterly data and annual data. We study how different values of the smoothing parameter affects two standard moments studied in the business cycle literature: the percentage standard deviation and the first-order autocorrelation of the business cycle components. In all experiments we used the standard value of $\lambda_q = 1600$ for the quarterly data as the reference point.

We generated artificial data from the process stated in equation (6), using values for θ between 0.9 and 1.0. We also looked at a case where the growth rate (rather than the level) of the data is an autoregressive series:

$$\Delta y_t = \theta \Delta y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, \sigma^2) \tag{7}$$

This latter specification is relevant for two reasons. First, Deaton (1992) argues that this specification seems to be a proper description of US output. Secondly, processes such as this has been studied in some business cycle studies, see e.g. Rotemberg and Woodford's (1996) study of whether the growth model can account for the forecastable movements in the US data.

We studied two frequency changes: (a) where the higher frequency data relate to quarterly observations and the lower frequency data to annual observations, and

[2]One way to think about this issue is in terms of stock and flow variables. For stock variables, the sampling technique would be the appropriate way of going from e.g. monthly data to quarterly data. For flow variables, averaging (or simple summation) would be the appropriate technique.

(b) where the higher frequency data relate to monthly observations and the lower frequency data to quarterly observations.

In going from the higher frequency data to the lower frequency data, we use both the techniques studied above, i.e. we sample the lower frequency data from a particular observation in the higher frequency data and we average the higher frequency data to generate the lower frequency data. For example, from going to quarterly data to annual data, we use either the fourth quarter observation as the annual data or the average over the four "quarter" of a "year" as the annual data. We refer to these two techniques as "sampling" and "averaging". Finally, we choose the sample periods such that there are 200 quarterly observations in all experiments.

Before examining this comparison it also important to remember the lesson learned from the previous subsection: if the cyclical adjustment is done appropriately, then it is the trend components which should match, not the business cycle components. One should not expect the moments of the business cycle components to coincide exactly, since the higher frequency data contains additional high-frequency noise. For this reason, the numbers reported in empirical studies need to always report, whether, say, annual or quarterly data was used, since different business cycle volatilities will be obtained for the different types of data. Desiring to find the same volatilities means that one forces the filter to use random movements of longer frequencies to "make up" for the random movements at high frequencies, which get lost if one moves from quarterly to annually sampled data: one should instead desire slightly higher volatilities at quarterly sampling rates. A good choice for the frequency adjustment is therefore one, under which the volatility of the business cycle component ends up slightly higher for the data sampled at higher frequency.

It turns out again, that $n = 4$ is an excellent choice. Tables 3 and 4 show that one obtains roughly the same moments for some adjustment between $n = 3$ and $n = 4$: with $n = 4$, one gets the desired slight overstatement of the volatility on the quarterly sampling rate compared the annual sampling rate[3].

Tables 5 and 6 report on the same statistics for monthly versus quarterly data: the adjustment $n = 4$ wins the competitions by a mile for sampled data, and looks extremely good for averaged data as well. In particular, regardless of whether the level of the series or the growth rates of the series are assumed to be stationary we find that: (1) the maximum difference between the standard deviation of the monthly series and the quarterly series is less than 0.5 percent, and (2) the first-order autocorrelations of the quarterly sampled data and the third-order autocorrelations of the monthly data are practically identical. Thus, our suggested correction of the

[3]From the Monte-Carlo analysis we also found that when adjusting to the annual frequency, an adjustment of the smooting parameter with n=3.75 produces business cycle statistics almost identical to the quarterly data for $\lambda = 1600$.

smoothing parameter appears to produce business cycle moments that are very similar whereas the alternative adjustment procedures suggested in the literature perform relatively poorly.

5 Recomputing the Facts.

Based on the above analysis it seems natural to ask whether the modification of the rule for adjusting the smoothing parameter matters for reported business cycle "facts". At the outset, it should be clear that 'wild' values of λ of course can lead to radical adjustments of the business cycle moments. Here, however, we will see whether the adjustment advocated for in the previous two sections leads to changes in the conclusions drawn in the literature on the moments of business cycles. To investigate this issue we recompute of some of the results reported by Backus and Kehoe (1992) for a cross-section of OECD countries using historical annual data.

Backus and Kehoe (1992) used a value for λ_{annual} of 100 corresponding to the "square" rule ($n = 2$). Our analysis above suggests using $\lambda_{annual} = 6.25$. One of their most interesting findings were that output volatility was higher in the interwar period than during the postwar period but that there is no general rule as far as a comparison of the postwar period with the prewar (pre WWI) period is concerned. This result is in contrast to the conventional wisdom of e.g. Burns (1960), Lucas (1977), and Tobin (1980) that output volatility declined after WWII (hence that output volatility has been lower in the postwar period than in either earlier period). Another interesting result was that prices changed from generally being procyclical before World War II to being countercyclical thereafter.

Table 7 lists the results for output volatility when using the alternative value for the smoothing parameter. The results of this analysis are quite interesting. We find that (i) generally, the difference in volatility between the prewar and the postwar period narrows; (ii) for most countries, there was has been a decline in volatility in the postwar period relative either the interwar period or the prewar period. These results differ from Backus and Kehoe's (1992) results but are in line with the traditional wisdom quoted above. This is an important result. The reason is that the traditional wisdom of a decline in output volatility after WWII was interpreted by Baily (1978) and Tobin (1980) in terms of the successfulness of stabilization policy. Tobin (1980, p.48) states that

"*.. Martin Baily has proved once more that a picture is worth more than a thousand words. His picture ... shows how much more stable real output has been in the United States under conscious policies of built-in and discretionary stabilization adopted since 1946 ..*"

Clearly, Backus and Kehoe's (1992) result puts serious doubt on this interpretation, but our analysis shows that the traditional wisdom may indeed be correct (although we are not willing to hypothesize about the underlying reasons for the decline in volatility). These results are, in our view, reassuring and indicate that the concern of this paper is important. Finally, it should also be noted that our estimates of the volatilities are in many cases considerably smaller than those reported in Backus and Kehoe (1992). One obtains a decline of 50 percent in the standard deviation in many cases.

Table 4 reports the results for Backus and Kehoe's (1992) result on the cyclical behavior of the price level. They found that prices have become countercyclical in the postwar period and that the interwar period historically was the period where procyclicality was most pronounced. These are in line with other studies, see e.g. Cooley and Ohanian (1991) and Ravn and Sola (1995). Hence, a fundamental change in these results would lead one to be somewhat sceptical about our arguments.

This table indicates that Backus and Kehoe's (1992) results on the change in the behavior of prices is robust to the change in the smoothing parameter that we have advocated for. The only major exception is Norway for which we have a big change in the correlation relative to Backus and Kehoe for the postwar period (they estimate this correlation to be -0.63). Thus, for these results, the indication is that the adjustment proposed in the present paper matters less for the qualitative results.

6 Conclusions

The major conclusion of this paper is that the parameter λ, which governs the behavior of the HP-filter, should be adjusted according to the fourth power of a change in the frequency of observations. This adjustment strategy makes sure that the same low-frequency movements are excluded from the data, regardless of the frequency of the observations. However, the cyclical variability of some series, calculated by first removing the HP-trend and then calculating the standard deviation of the residual, will depend on high-frequency events as well. They will thus depend on the frequency of the observations as well as on whether the data is time averaged or time-sampled. This dependence should be acknowledged by stating the nature of the underlying data as precisely as possible in any empirical work. Some well-known comparisons of business cycles moments across countries and time periods have been recomputed, using the recommended fourth-power adjustment. In particular, we overturn a finding by Backus and Kehoe (1992) and return instead to older conventional wisdom (Baily, 1978, Lucas, 1977, Tobin, 1980): based on the new HP-Filter adjustment rule, output volatility turns out to be lower in the postwar period compared to the prewar period.

References

[1] Backus, David K. and Patrick J. Kehoe (1992), "International Evidence on the Historical Properties of Business Cycles", *American Economic Review* vol. 82, no.4, 864-888.

[2] Backus, David K., Patrick J. Kehoe and Finn E. Kydland (1992), "International Real Business Cycles", *Journal of Political Economy* vol.101, 745-775.

[3] Baily, Martin N. (1978), "Stabilization Policy and Private Economic Behavior", *Brookings Papers on Economic Activity* 1, 11-50.

[4] Blackburn, Keith and Morten O. Ravn (1992), "Business Cycles in the U.K.: Facts and Fictions", *Economica* vol.59, 383-401.

[5] Brandner, Peter and Klaus Neusser (1992), "Business Cycles in Open Economies: Stylized Facts for Austria and Germany", *Weltwirtschaftliches Archiv* vol.128, 67-87.

[6] Burns, Arthur (1960), "Progress Towards Economic Stability", *American Economic Review* vol. 50, 1-19.

[7] Canova, Fabio (1997), "Detrending and Business Cycle Facts", forthcoming, *Journal of Monetary Economics.*

[8] Canova, Fabio (1994), "Detrending and Turning-Points", *European Economic Review* vol.38, nos.3/4, 614-623.

[9] Cogley, Timothy and James M. Nason (1995), "Effects of the Hodrick-Prescott Filter on Trend and Difference Stationary Time Series: Implications for Business Cycle Research", *Journal of Economic Dynamics and Control* vol.19, 253-278.

[10] Cooley, Thomas F. and Gary D. Hansen (1989), "The Inflation Tax in a Real Business Cycle Model", *American Economic Review* vol.79, 733-48.

[11] Cooley, Thomas J. and Lee E. Ohanian (1991), "The Cyclical Behaviour of Prices", *Journal of Monetary Economics* vol.28, 25-60.

[12] Correia, Isabel H., Joao L. Neves, and Sergio T. Rebelo (1992), "Business Cycles from 1850-1950 - New Facts about Old Data", *European Economic Review* vol.36, nos.2/3, 459-467.

[13] Danthine, Jean-Pierre and John B. Donaldson (1993), "Methodological and Empirical Issues in Real Business cycle Theory", *European Economic Review* vol.37, 1-35.

[14] Danthine, Jean-Pierre and Michel Girardin (1989), "Business cycles in Switzerland. A Comparative Study", *European Economic Review* vol.33, 31-50.

[15] Deaton, Angus (1992), **Understanding Consumption**, Oxford, UK: Clarendon Press.

[16] Fiorito, Ricardo and Tryphon Kollintzas (1994), "Stylized Facts of Business Cycles in the G7 from a Real Business Cycles Perspective", *European Economic Review* vol.38, no.2, 235-269.

[17] Hansen, Gary D. (1985), "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics* vol.16, 309-27.

[18] Harvey, Andrew C. and A. Jaeger (1993), "Detrending, Stylized Facts and the Business Cycle", *Journal of Applied Econometrics* vol.8, 231-247.

[19] Hodrick, Robert J. and Edward C. Prescott (1980), "Postwar U.S. Business Cycles: an Empirical Investigation", Discussion Paper no.451, Carnegie Mellon University.

[20] King, Robert G. and Sergio T. Rebelo (1993), "Low Frequency Filtering and Real Business Cycles," *Journal of Economic Dynamics and Control* vol. 17, no. 1-2, 207-231.

[21] Kydland, Finn E. and Edward C. Prescott (1982), "Time to Build and Aggregate Fluctuations", *Econometrica* vol.50, 1345-1370.

[22] Kydland, Finn E. and Edward C. Prescott (1990), "Business Cycles: Real Facts and a Monetary Myth", *Federal Reserve Bank of Minneapolis Quarterly Review* vol.14, 3-18.

[23] Lucas, Robert E. Jr. (1977), "Understanding Business Cycles", in K.Brunner and Allan H. Meltzer (eds.), **Stabilization of the Domestic and International Economy**, Carnegie-Rochester Conference Series 5, Amsterdam: North-Holland, 7-29.

[24] Ravn, Morten O. and Martin Sola (1995), "Stylized Facts and Regime Changes: Are Prices Procyclical?", *Journal of Monetary Economics* vol.36, no.3, 497-526.

[25] Rotemberg, Julio J. and Michael Woodford (1996), "Real-Business-Cycle Models and the Forecastable Movements in Output, Hours, and Consumption", *American Economic Review* vol.86(1), 71-89.

[26] Sichel, Daniel E. (1993), "Business-Cycle Asymmetry - a Deeper Look", *Economic Record* vol.31(2), 224-36.

[27] Söderlind, Paul (1994), "Cyclical Properties of a Real Business Cycle Model", *Journal of Applied Econometrics* vol.9, S113-S122.

[28] Tobin, James (1980), **Asset Accumulation and Economic Activity**, Chicago: University of Chicago Press.

7 Tables

θ	Quar.	Annual: Sampled				Annual: Averaged			
		$n=1$	$n=2$	$n=3$	$n=4$	$n=1$	$n=2$	$n=3$	$n=4$
Specification (6): $y_t = \theta y_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2)$									
1	2.536	4.172	3.531	2.953	2.430	3.891	3.212	2.595	2.038
0.99	2.547	4.170	3.539	2.963	2.440	3.886	3.218	2.603	2.046
0.97	2.559	4.069	3.504	2.961	2.451	3.774	3.176	2.594	2.052
0.95	2.558	3.907	3.427	2.934	2.450	3.597	3.086	2.558	2.044
0.90	2.521	3.486	3.174	2.811	2.409	3.133	2.798	2.407	1.982
Specification (7): $\Delta y_t = \theta \Delta y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, \sigma^2)$									
0.4	3.666	6.601	5.489	4.481	3.574	6.323	5.172	4.120	3.173
0.2	2.980	5.101	4.285	3.549	2.884	4.823	3.970	3.192	2.490
0.1	2.735	4.586	3.868	3.220	2.635	4.308	3.552	2.864	2.243

Table 3: **Annual Data: Standard Deviations.** *Moments were computed from 1000 replications each of a length of 200 quarters using 20 observations for initializations,* $\sigma = 2.$*. The 'sampled' ('averaged') data refers to the 4'th 'quarter's' observation (average over the 4 quarters of the year).*

θ	Quar.		Annual: Sampled				Annual: Averaged			
	corr	corr_4	$n=1$	$n=2$	$n=3$	$n=4$	$n=1$	$n=2$	$n=3$	$n=4$
Specification (6): $y_t = \theta y_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2)$										
1	0.704	0.094	0.557	0.425	0.253	0.042	0.665	0.554	0.401	0.198
0.99	0.703	0.093	0.553	0.423	0.252	0.041	0.662	0.553	0.400	0.198
0.97	0.701	0.088	0.531	0.408	0.242	0.036	0.644	0.540	0.391	0.192
0.95	0.695	0.079	0.497	0.382	0.224	0.025	0.617	0.518	0.375	0.181
0.90	0.674	0.044	0.396	0.298	0.162	-0.016	0.535	0.447	0.318	0.140
Specification (7): $\Delta y_t = \theta \Delta y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, \sigma^2)$										
0.4	0.846	0.165	0.633	0.514	0.352	0.143	0.701	0.599	0.456	0.263
0.2	0.784	0.118	0.589	0.463	0.294	0.082	0.678	0.571	0.421	0.222
0.1	0.747	0.104	0.572	0.443	0.272	0.061	0.671	0.561	0.410	0.209

Table 4: **Annual Data: Autocorrelations.** *See notes to table 3.* corr_4 *refers to the fourth autocorrelation, which is the appropriate number to compare to the first autocorrelation of annual data.*

θ	Quarterly		Monthly			
	Sampl.	Aver.	$n=1$	$n=2$	$n=3$	$n=4$
Specification (6): $y_t = \theta y_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2)$						
1	4.432	4.239	2.948	3.383	3.878	4.357
0.99	4.427	4.233	2.960	3.393	3.883	4.431
0.97	4.298	4.095	2.962	3.576	3.826	4.303
0.95	4.099	3.884	2.943	3.322	3.714	4.104
0.90	3.609	3.358	2.839	3.120	3.383	3.616
Specification (7): $\Delta y_t = \theta \Delta y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, \sigma^2)$						
0.4	7.047	6.878	4.416	5.195	6.069	7.044
0.2	5.429	5.250	3.519	4.081	4.717	5.429
0.1	4.877	4.692	3.204	3.695	4.252	4.878

Table 5: **Monthly Data: Standard Deviations.** *See notes to table 3*

θ	Quarterly		Monthly							
	Sampled	Averaged	$n=1$		$n=2$		$n=3$		$n=4$	
			corr	corr_3	corr	corr_3	corr	corr_3	corr	corr_3
Specification (6): $y_t = \theta y_{t-1} + \epsilon_t, \epsilon_t \sim \mathcal{N}(0, \sigma^2)$										
1	0.708	0.782	0.780	0.415	0.830	0.534	0.868	0.631	0.898	0.709
0.99	0.705	0.780	0.779	0.414	0.829	0.533	0.868	0.629	0.897	0.706
0.97	0.685	0.764	0.776	0.406	0.824	0.522	0.861	0.614	0.890	0.685
0.95	0.654	0.740	0.769	0.392	0.816	0.502	0.851	0.589	0.877	0.656
0.90	0.562	0.667	0.742	0.341	0.784	0.436	0.815	0.509	0.837	0.564
Specification (7): $\Delta y_t = \theta \Delta y_{t-1} + \epsilon_t,\ \epsilon_t \sim \mathcal{N}(0, \sigma^2)$										
0.4	0.781	0.820	0.891	0.528	0.919	0.635	0.939	0.719	0.953	0.781
0.2	0.741	0.797	0.843	0.462	0.881	0.577	0.909	0.669	0.930	0.741
0.1	0.724	0.789	0.814	0.437	0.857	0.554	0.890	0.649	0.915	0.725

Table 6: **Monthly Data: Autocorrelations.** *See notes to table 3. $corr_3$ refers to the third autocorrelation, which is the appropriate number to compare to the first autocorrelation of quarterly data.*

	Standard Deviations (%)			$n = 4$		$n = 2^*$	
	I.Prewar	II.Interwar	III.Postwar	I/III	II/III	I/III	II/III
Australia	3.77(0.37)	2.47(0.35)	1.40(0.14)	2.69	1.77	3.3	2.5
Canada	3.13(0.27)	5.06(0.77)	1.50(0.21)	2.09	3.38	2.0	4.4
Denmark	2.20(0.17)	2.45(0.37)	1.35(0.15)	1.63	1.82	1.6	1.8
Germany	2.32(0.21)	5.26(0.88)	1.80(0.24)	1.29	2.92	1.5	4.4
Italy	2.13(0.20)	2.60(0.30)	1.51(0.14)	1.41	1.72	1.2	1.8
Japan	2.10(0.27)	2.47(0.38)	1.45(0.18)	1.45	1.70	0.8	1.0
Norway	1.07(0.09)	2.89(0.56)	1.06(0.12)	1.01	2.72	1.1	2.0
Sweden	1.73(0.22)	2.41(0.47)	1.03(0.09)	1.68	2.34	1.7	2.6
United Kingdom	1.54(0.16)	2.50(0.30)	1.27(0.17)	1.21	1.97	1.3	2.1
United States	3.30(0.35)	4.91(0.70)	1.58(0.17)	2.09	3.11	1.9	4.1

Table 7: **Output Volatility.** **Numbers from Backus and Kehoe (1992). Numbers in parentheses are standard errors computed from GMM estimations of the unconditional moments.*

	$n = 4$			$n = 2^*$		
	I.Prewar	II.Interwar	III.Postwar	I.Prewar	II.Interwar	III.Postwar
Australia	0.29	0.30	-0.26	0.60	0.59	-0.47
	(0.14)	(0.18)	(0.18)	(0.10)	(0.12)	(0.11)
Canada	0.11	0.69	-0.01	0.41	0.77	0.12
	(0.15)	(0.12)	(0.15)	(0.13)	(0.08)	(0.16)
Denmark	0.18	0.02	-0.60	0.18	-0.26	-0.48
	(0.12)	(0.26)	(0.09)	(0.12)	(0.25)	(0.11)
Germany	0.04	0.86	-0.17	-0.01	0.71	0.01
	(0.13)	(0.06)	(0.14)	(0.15)	(0.09)	(0.16)
Italy	0.01	0.14	-0.33	-0.02	0.58	-0.24
	(0.10)	(0.15)	(0.14)	(0.11)	(0.09)	(0.14)
Japan	-0.49	-0.18	-0.37	-0.45	0.03	-0.60
	(0.11)	(0.25)	(0.18)	(0.11)	(0.22)	(0.10)
Norway	0.47	0.16	0.57	0.65	0.16	-0.63
	(0.11)	(0.16)	(0.10)	(0.08)	(0.19)	(0.08)
Sweden	-0.08	0.23	-0.38	0.15	0.30	-0.53
	(0.17)	(0.09)	(0.09)	(0.13)	(0.10)	(0.07)
U.K.	0.16	0.14	-0.72	0.26	0.20	-0.50
	(0.14)	(0.24)	(0.08)	(0.12)	(0.21)	(0.14)
U.S.	0.05	0.75	-0.25	0.22	0.72	-0.30
	(0.11)	(0.09)	(0.21)	(0.11)	(0.13)	(0.16)

Table 8: **The Correlation of Prices and Output.** **Numbers taken from Backus and Kehoe (1992). Numbers in parentheses are standard errors.*

8 Figures

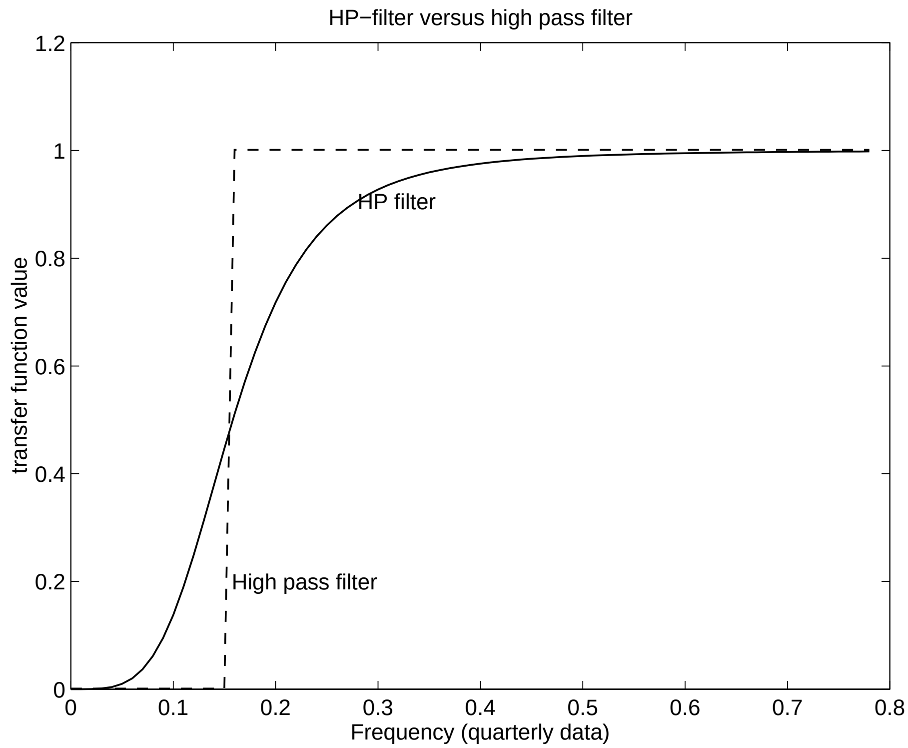

Figure 1: *This figure compares the HP-filter for quarterly data and* $\lambda = 1600$ *with a high pass filter, cutting off frequencies below* $\pi/20$, *in frequency domain. The two filters are rather similar: both attribute peak-to-peak cyclical movements of less than ten years of duration to be part of the business cycle component.*

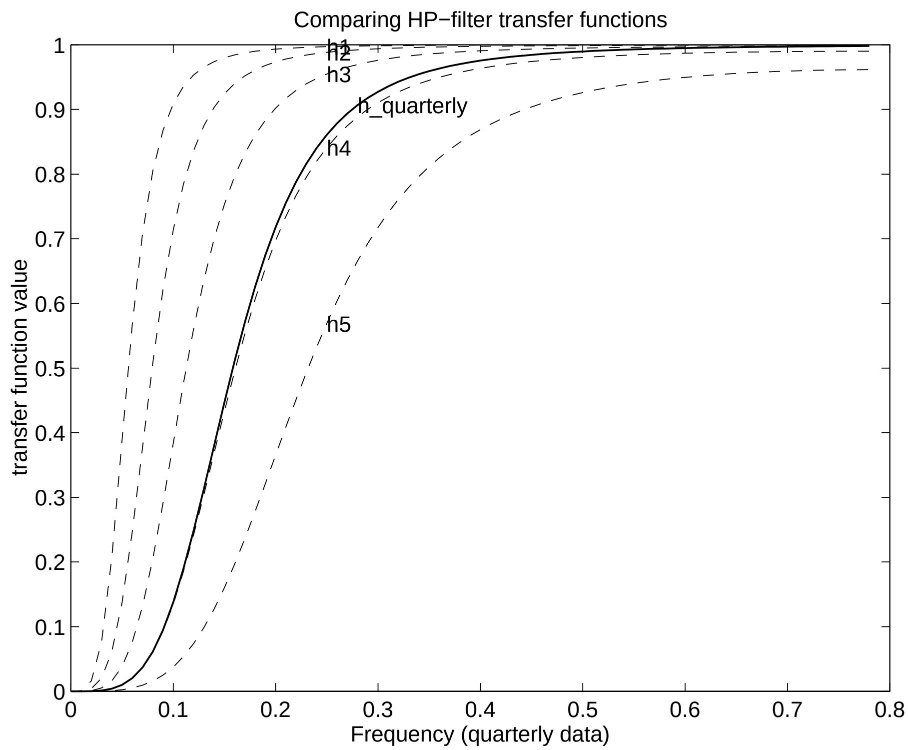

Figure 2: *This figure compares different ways of adapting the HP-Filter by comparing their transfer functions in frequency domain. Note, how h4 matches h_quarterly most closely: their two lines are extremely close. That suggests that lambda in the HP filter should move with the fourth power of the frequency of observations.*

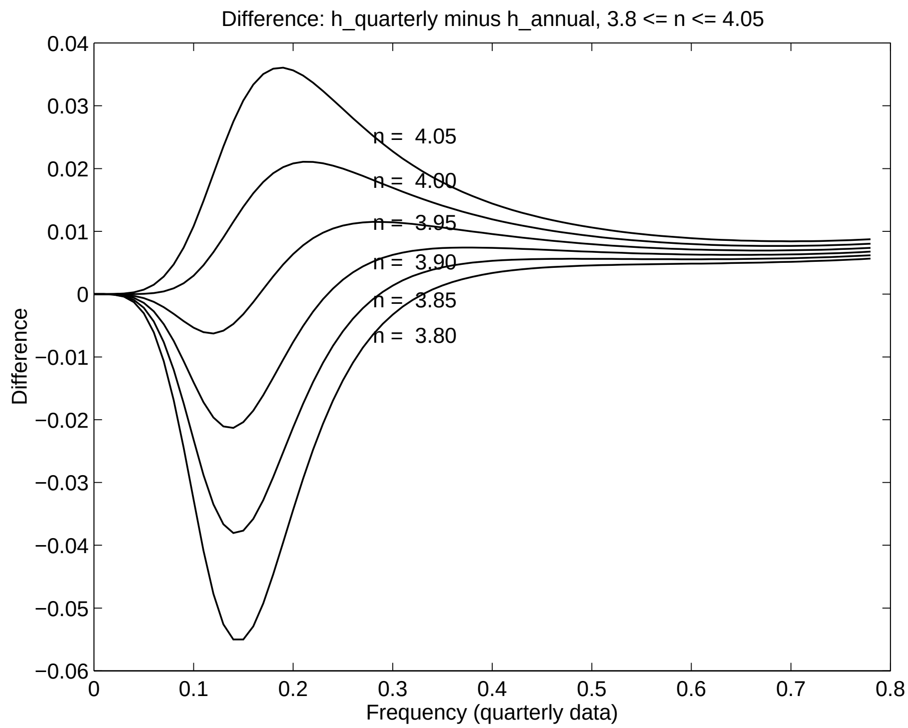

Figure 3: *This figure plots the difference between the transfer function for quarterly data and* $\lambda_1 = 1600$ *minus the transfer function for annual data with* $\lambda_{0.25}$ *adjusted with various powers* n *of the frequency ratio* $s = 0.25$*. For* $n = 4$ *and hence* $\lambda_{0.25} = 6.25$*, the difference is strictly smaller than* 0.025 *in absolute value everywhere. We also showed the difference for some noninteger values of* n*: apparently,* $n = 3.95$ *or even* $n = 3.9$ *might be even slightly preferable to* $n = 4$*. In applications, the differences between* $n = 3.9$ *and* $n = 4$*, say, are unlikely to matter much, though.*

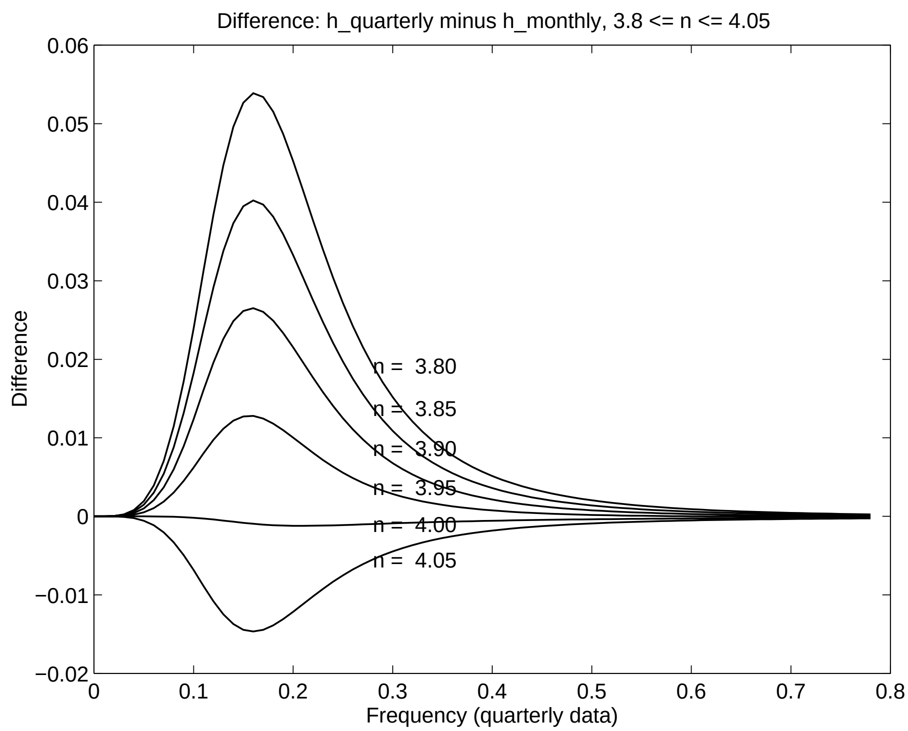

Figure 4: *This figure plots the difference between the transfer function for quarterly data and* $\lambda_1 = 1600$ *minus the transfer function for monthly data with* λ_3 *adjusted with various powers* n *of the frequency ratio* $s = 3$*. For* $n = 4$ *and hence* $\lambda_3 = 129600$*, the difference is strictly smaller than* 0.002 *in absolute value everywhere, clearly dominating any of the other noninteger values close to* $n = 4$ *shown in this figure.*

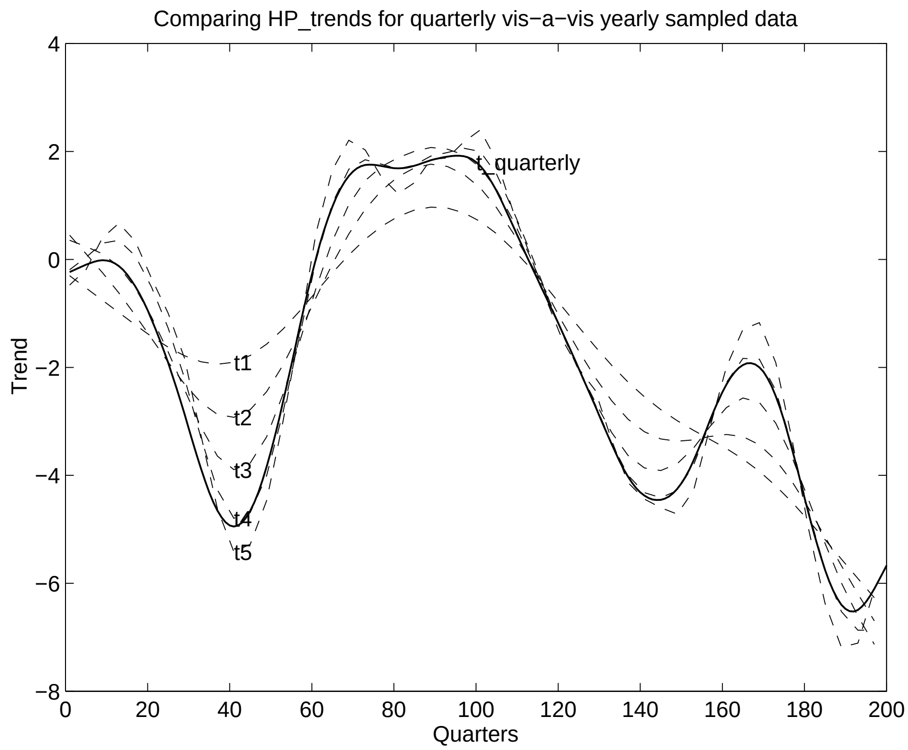

Figure 5: *This figure compares different ways of adapting the HP-Filter by comparing the resulting time trends when filtering some artificially generated data. In this figure, the data is* sampled*, i.e the annual data corresponds to taking every fourth observation of the quarterly series. Note, how t4 matches t_quarterly most closely. That suggests that lambda in the HP filter should move with the fourth power of the frequency of observations.*

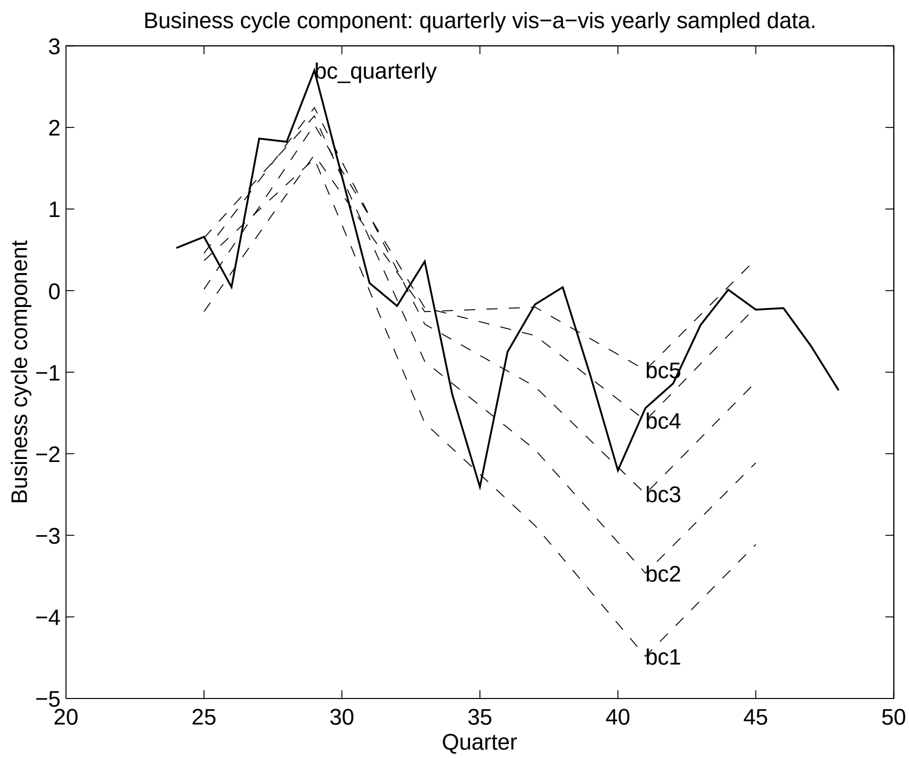

Figure 6: *This figure compares different ways of adapting the HP-Filter by comparing the resulting business cycle components after removing the HP-trend when filtering some artificially generated data. In this figure, only the sampled annual data in its five filtered versions (and not the time-averaged data) is compared to the quarterly business cycle component. One can clearly see, that a difference remains, regardless of which* λ_m *is chosen. This shouldn't surprise: higher-frequency data will always contain additional sources of noise at these higher frequencies. It is thus more appropriate to compare what has been* taken out, *i.e. to compare the trend components as in figure* **??** *rather than to compare what has been left in as here.*