A note on wild and Pagnatiello's experimental design strategy

OUR MAILBOX

A note on Wild and Pignatiello's experimental design strategy

Jack P. C. Kleijnen
Katholieke Universiteit Brabant
(Tilburg University)
School of Management and Economics
Department of Information Systems and Auditing
5000 LE Tilburg
Netherlands

In the December 1991 issue of this journal, Wild and Pignatiello proposed an experimental design strategy for designing robust systems using discrete-event simulation; their strategy used Taguchi's method for improving product and process quality in manufacturing. Now I propose two alternative techniques. One technique requires fewer observations than Taguchi's technique does. The second technique requires many more observations, but pays off since it quantifies the risk caused by uncontrollable environmental conditions.

Keywords: Taguchi method, risk analysis, fractional factorial design, simulation

Wild and Pignatiello [5, p.359] stated: "The goal is to find system designs that. . .are also robust or insensitive to uncountable system environmental conditions such as mean interarrival times and mean service times. . . . We distinguish the input factors as being either controllable (system design variables) or uncontrollable (noise variables). . . .the noise variables represent parameters that are unknown or uncertain and/or uncontrollable in the real world but controllable in the simulation experiments."

They further wrote, ". . .the strategy is applicable to any system with probabilistic components. . ."; [5, p. 360]. I would add that actually their strategy — and the two strategies I am going to propose — is applicable to systems with or without probabilistic components; that is deterministic simulation models also have controllable and uncontrollable inputs to which theses strategies can — and are — applied. Examples of deterministic simulation models are financial models (which have become popular since spreadsheet software has become widespread) and ecological models such as the one discussed in [4]. Other deterministic simulation models are discussed from the present perspective in [1].

The authors illustrated their strategy with a hypothetical jobshop example, assuming six controllable and six uncontrollable factors; for example, number of machines at station one and mean interval time respectively. The "feasible region" for the controllable factors was given in their table 3; for example, the number of machines at station one is 1, 2 or 3. In current practice, an exponential distribution is often assumed for the interarrival times (an environmental factor), and that distribution's parameter (say) λ is estimated from the arrival data collected for the system under consideration. The system is then simulated for this single value of λ. Also see [5, p.360].

That practice, however, is rejected by the authors. They propose to "investigate system performance using two setting for each of the noise variables"; [5, p.360]. So they investigate two values for the mean interrarrival time $1/\lambda$. Which two values to take, may be determined as follows: ". . .construct a confidence interval estimate of the estimated value of the input probability distribution parameters and use the end-points as the two settings"; [5, p.361]. I give the following example, to which I shall return. Suppose $\mu = 1/\lambda$ is estimated from a sample of n independent interrarrival times x_i:

$$\hat{\mu} = \sum_{i=1}^{n} \frac{x_i}{n} \tag{1}$$

Consequently the variance of this estimator is

$$var(\hat{\mu}) = \frac{var(x)}{n}. \tag{2}$$

Because of the central limit theorem I assume that the estimator in (1), which is an average, is normally distributed:

$$\hat{\mu} \; N(\mu, \frac{var(x)}{n}). \tag{3}$$

The mean of this normal distribution is estimated by (1), and its variance is estimated by combining

$$\hat{var}(x) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}. \tag{4}$$

with (2), A $1-\alpha$ confidence internal for μ is then obtained from the Student distribution with $n-1$ degrees of freedom:

$$(\hat{\mu}\; t_{n-1}^{a/2} \sqrt{v\hat{a}r(\hat{\mu})},\ \hat{\mu} + t_{n-1}^{a/2} \sqrt{v\hat{a}r(\hat{\mu})}. \tag{5}$$

If no sampling data are available, I propose to ask the users for their expert opinions and to formulate probability distributions reflecting that subjective knowledge. Such an approach is classical in the risk analysis of investment projects. Risk analysis of nuclear installations is discussed in [1]. Whether objective or subjective probabilities are sued, the Monte Carlo analysis of the resulting model is facilitated by software like "@Risk", a well-known add-on to Lotus 1-2-3.

The authors [5, p. 362] used a "resolution III 2^{6-3} design" for the six controllable factors, in order to "identify important variables quickly." They wanted to estimate the effects of changes in the uncontrollable factors, including "interactions between various input parameters". [5, p. 363]. Therefore they "cross" the design for the controllable factors with the design for the uncontrollable ones; [5, p. 364]. For the six environmental factors in their jobshop example they again take a resolution III 2^{6-3} design. I wonder whether this design does indeed permit the estimation of interactions between the uncontrollable factors; intuitively I think that interactions between controllable and uncontrollable factors can be estimated indeed! The authors did not present estimated main effects and interactions in their example. Instead they stated that from the data it is "apparent" that some jobshop environmental variables (which is an interaction between controllable and uncontrollable factors, I claim); [5, p.364]. They observed that the design for the environmental factors represents "a systematic sample from the entire noise space"; [5, p.365].

I propose two alternative techniques, one that requires fewer simulation runs, and one that takes many more runs but gives interesting information on the system's robustness. In the first technique I treat both controllable and uncontrollable factors the same, in the simulation experiment. So in the authors' example I propose a resolution III design for twelve factors (sox controllable and six uncontrollable factors); that is I suggest a 2^{12-8} design so that the twelve main effects and the overall mean are estimable from sixteen factor combinations. The authors use 64 combinations (crossing two 26-3 designs); that is, they use four times more simulation time! Later on they state "since several iterations of data collection. . .are likely to be needed,. . .initial designs matrices. . .be that environmental factors with nonsignificant main effects are eliminated. In a next iteration, interactions between controllable and uncontrollable factors can be estimated —similar to the authors' table 7 — several combinations of environmental factors are investigated, which gives an idea of the robustness of the system to be designed.

My second technique, risk analysis, takes a different route. The authors examine each environmental factor at only the $1 - \alpha$ confidence interval; in my example, these are the two values in (5). But obviously the value halfway this interval is much more likely! And values outside the $1 - \alpha$ interval are rare but do have a chance of α. The derivation of the confidence interval implies that the environmental factor m has the Gaussian probability function given in (3) with parameters estimated by (1), (2), and (4). Risk analysis takes 'many' samples from that distribution so that a good estimate of 'the' response variable is obtained. As the authors suggest, 'the' response may be "the average. . . [or]. . .the sample noise conditions": [5, p. 364]. That average and that variance can be estimated from the distribution provided by risk analysis, but we can also estimate other responses such as the probability of times larger than some given critical constant. The probability distribution can be estimated for several combinations of controllable factors; these factors can be investigated in a fractional factorial design such as the resolution III design. I give some more details on risk analysis and sensitivity analysis in [2 and 3, pp. 143-145], but the proper role of both techniques is still under debate. Note that the first technique (controllable and incontrollable factors combined into a single fractional design) may be applied to find out which environmental factors do not need to be sampled in the second technique (risk analysis).

I do not assert that the authors' design and analysis is wrong; all I want to claim is that there are alternative techniques to be considered. And of course I agree with the authors that "robustness" of a system design is important: in a case study concerning the optimization of keypunching at a computer center I noted that it "would have been wise to investigate the sensitivity of this solution to the arrival rate of jobs and their service rate, because changes in the future work load may require adaptation of the [estimated optimal setting of the controllable variables]"; [3, p.364]. But this message needs to be repeated again and again!

References

1. Iman R.L., and J.C., Helton (1988). "An Investigation of Uncertainty and Sensitivity Analysis Techniques for Computer Codes." *Risk Analysis 8*, 1: 71-90.
2. Kleijnen, J.P.C. (1983). "Risk Analysis and Sensitivity Analysis: Antithesis or Synthesis?" *Simuletter 14*, 1-4: 64-723.
3. Kleijnen, J.P.C. (1987). *Statistical Tools for Simulation Practitioners*. Marcel Dekker, New York.
4. Kleijnen, J.P.C., J. Rotmans and G. Van Ham (1992). " Techniques for Sensitivity Analysis of Si mulation Models: a Case Study of the CO2 Greenhouse Effect." *SIMULATION* (to be published).
5. Wild, R.H., and J.J. Pignatiello (1991). "An Experimental Design Strategy for Designing Robust Systems Using Discrete-event Simulation." *SIMULATION* 57, 6 ; 358-368.