On Superimposed Recurrent Cycles

By

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1. Introduction

In the course of time the study of business cycles developed from the theories of self-sustaining cycles into the theories of cyclical response to exogenous shocks, mainly of monetary origin. The older theories from the thirties and forties had the common feature of being based on an endogenous dynamism, resulting from the interaction between the multiplier and some form of acceleration principle. Especially according to the range of values in which the parameters of the investment function were chosen, these models produced explosive, damped or constant cycles.

After World War II there was a rise of interest in the case of explosive fluctuations, in the meantime neglecting expectational and other important aspects of economic reality. Afterwards it is quite understandable that these theories of endogenous instability induced strong reactions, by monetarism but also by macroeconometric modelbuilding.[1] But after the disappointing experience of the early seventies, there arose a new chain of theories, now based on the hope to reconcile business cycles with the postulates of microeconomic (competitive) theory. Initially this approach relied upon random monetary and related shocks that were held to be responsible for cyclical fluctuations that after the stable sixties proved to be well alive. Later on rational expectationists tried to introduce more identifiable exogenous factors into this view, that actually was as one-sided as the older theories of predominantly endogenous cycles.

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[1] About forty relevant articles from the period 1933-1963 have been republished in Gordon and Klein (1966).

In an extensive review of the literature on business cycles, Zarnowitz (1985) confronts these theories with what he calls the "stylized facts". These indeed tell us that the observed fluctuations do not resemble the deterministic wave motions which sometimes arise in the natural sciences. But on the other hand it appears that cycles are persistent, lasting long enough to permit the development of cumulative movements, both in the downward and in the upward direction. The observed movements moreover have in common that they show up in many ways, not only in macroeconomic variables, but also in spatial and sectoral magnitudes. For the United States in particular, it has been well established that the mean duration of business cycles remained approximately stable at four years, in which during the last four decades expansions covered about three years and contractions about one year.

At the end of his survey Zarnowitz pleads for a synthesis between the theories of self-sustaining cycles and the new mainstream literature on rational expectations models. We agree with this, but we think it is a pity that he does not incorporate the lessons which can be learned from macroeconometric modelbuilding into his proposal. Some of these lessons are told in Klein (1983). And starting from these we will present here a methodological device to give some ground to the desired synthesis.

Central to this is the hypothesis that fluctuations in economic time series are essentially recurrent. First we show how it is possible to combine enough cosine functions to approximate any given time series. This is Klein's method, which will be described in section 2. In section 3 we present a numerical example, which is meant to illustrate that the framework proposed can be given empirical content, namely by combining the theory of self-sustaining cycles with the hypothesis of identifiable exogenous shocks. In section 4 the method is applied to some time series of real life: the rate of capacity utilization in U. S. manufacturing, a time series for price indices of Southern England and a series of fluctuations around trend in U.S. real income. The epilogue gives some further linkages with existing literature.

2. Klein's Method

Some years ago Klein (1983, pp. 124-128) presented a method (derived from the work of Otsuki) for measuring periodicities, which was basically meant to see whether a (macroeconometric) model is capable of reproducing cyclical history. In this section we will give a description of this method.

As a starting point, consider the second order difference equation

$$y_t + \varrho\, y_{t-1} + y_{t-2} = 0.$$

Notice that the coefficient of y_{t-2} has been put at unity, so that the constant term of the corresponding characteristic equation also assumes the value one:

$$\lambda^2 + \varrho\,\lambda + 1 = 0.$$

The roots of this equation are conjugate complex if $\varrho^2 < 4$. For, if this inequality holds, the sign of the discriminant in

$$\frac{-\varrho \pm \sqrt{\rho^2 - 4}}{2}$$

is negative, so that the roots have the form

$$a \pm ib.$$

The equivalent trigonometric form for complex numbers is:

$$a = r\cos\vartheta,\ b = r\sin\vartheta, \text{ with modulus } r = \sqrt{a^2 + b^2}.$$

For our equation the modulus of the roots

$$\sqrt{\frac{\varrho^2 + 4 - \varrho^2}{4}} = 1,$$

so that the solution will show a recurrent cycle. Then, the angle of oscillation can be calculated at:

$$\vartheta = \cos^{-1} a = \cos^{-1}\left(-\frac{1}{2}\varrho\right).$$

Next, consider the time series, y_t. For this, Klein recommends to fit the equation

$$y_t + y_{t-2} = -\varrho\, y_{t-1} = u_t,$$

by minimizing the residual error in the square. This procedure restricts the coefficient of y_{t-2} to be one, thus insuring no dampening. The estimated parameter then can be used to estimate ϑ in

$$\hat{\vartheta} = \cos^{-1}\left(-\frac{1}{2}\hat{\varrho}\right).$$

In this way the angle of oscillation of the best-fitting undamped sinusoid to the data provides an estimate of the periodicity of the series. If for instance, the estimated value of $\hat{\varrho} = 1$, then $\hat{\vartheta} = 120°$, or if $\hat{\varrho} = 0$, then $\hat{\vartheta} = 90°$, or if $\hat{\varrho} = -1$, then $\hat{\vartheta} = 60°$. From this it follows that we have found cycles of respectively 3, 4 and 6 periods.

As Klein shows, this method can easily be extended to the more general case of superimposed cycles. In the case of the fourth order equation

$$y_t + \varrho_1\, y_{t\text{-}1} + \varrho_2\, y_{t\text{-}2} + \varrho_3\, y_{t\text{-}3} + y_{t\text{-}4} = 0,$$

the characteristic equation is

$$\lambda^4 + \varrho_1\, \lambda^3 + \varrho_2\, \lambda^2 + \varrho_3\, \lambda + 1 = 0.$$

The roots of this equation will appear as two pairs of conjugate complex roots, if the following factoring is possible:

$$(\lambda - a \pm ib)(\lambda - c \pm id) = 0\,.$$

By multiplying out, we find

$$\lambda^4 - (2a + 2c)\lambda^3 + (a^2 + b^2 + c^2 + d^2 + 4ac)\lambda^2 - \{2c(a^2 + b^2) + 2a(c^2 + d^2)\}\lambda + (a^2 + b^2)(c^2 + d^2) = 0.$$

For one pair of conjugate complex roots we already know that no dampening is guaranteed if

$$a^2 + b^2 = 1 \text{ and/or } c^2 + d^2 = 1.$$

Using these conditions, the characteristic equation can be simplified to

$$\lambda^4 - (2a + 2c)\lambda^3 + (2 + 4ac)\lambda^2 - (2a + 2c)\lambda + 1 = 0.$$

Notice that $\varrho_1 = \varrho_3$, so that it is clear that in the underlying case it is recommended to regress

$$y_t + y_{t-4} = -\rho_1(y_{t-1} + y_{t-3}) - \rho_2 y_{t-2} + u_t\,.$$

Then, analogous to the second order case, the estimated parameters can be used to estimate the angles of oscillation in

$$\hat{\vartheta}_1 = \cos^{-1}\hat{a} \text{ and } \vartheta_2 = \cos^{-1}\hat{c}.$$

If for instance it has been found that $\hat{\varrho}_1 = 1$ and $\hat{\varrho}_2 = 2$, then $\hat{a} = -0.5$ and $\hat{c} = 0$, so that $\hat{\vartheta}_1 = 120^o$ and $\hat{\vartheta}_2 = 90^o$.

It is worthwhile to note that the time series belonging to this example shows a cycle which can be calculated at 12 periods. We shall call this the dominant cycle and the other ones the underlying cycles. It is easy to see that the periodicity of the dominant cycle equals the least common multiple of the periodicities of the underlying cycles. (This will be illustrated in the next section.)

As Klein indicates, the procedure outlined above can be extended as far as one wants. In the case of a sixth order equation, we regress

$$y_t + y_{t-6} = -\varrho_1(y_{t-1} + y_{t-5}) - \varrho_2(y_{t-2} + y_{t-4}) - \varrho_3 y_{t-3} + u_t\,.$$

And analogously to before the estimated ϱ's can be used to estimate the ϑ's in

$$\hat{\vartheta}_1 = \cos^{-1}\hat{a}, \hat{\vartheta}_2 = \cos^{-1}\hat{c} \text{ and } \hat{\vartheta}_3 = \cos^{-1}\hat{e},$$

Where *a*, *c*, and *e* are calculated from

$$\varrho_1 = -2a - 2c - 2e,$$

$$\varrho_2 = 3 + 4\,ac + 4\,ae + 4ce,$$

$$\varrho_3 = -4a - 4c - 4e - 8\,ace.$$ [2]

If for instance, the estimated parameters are

$$\hat{\varrho}_1 = -2.146, \hat{\varrho}_2 = 2.303 \text{ and } \hat{\varrho}_3 = 1.843,$$

then

$$\hat{a} = -0.5, \hat{c} = 0.707 \text{ and } \hat{e} = 0.866,$$

so that

$$\hat{\vartheta}_1 = 120^o, \hat{\vartheta}_2 = 45^o \text{ and } \hat{\vartheta}_3 = 30^o.$$

This is an example with a dominant cycle of 24 periods, being the least common multiple of the three underlying cycles, the first of 3, the second of 8 and the third of 12 periods.

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[2] More in general the coefficient ϱ_k^{2n} in the 2n-th order difference equation

$$\sum_{j=0}^{2n}\left(\rho_j^{2n} y_{t-j}\right) = \rho_0^{2n} y_t + \rho_1^{2n} y_{t-1} + \rho 2^{2n} y_{t-2} + ... + \rho 2_n^{2n} y_{t-n} = 0$$

with undamped complex roots, can be calculated by:

$$\varrho_k^{2n} = 1, \text{ if } k = 2n \text{ or if } k = 0;$$

$$\varrho_k^{2n} = (-1)^k \sum_i^k \left[1/2 \binom{n-i}{k-i}\right] S_i^n,$$

for $i = 1, 3, 5, \ldots, k$ if k is an odd number and $1 \le k \le n$ and
for $i = 0, 2, 4, \ldots, k$ if k is an even number and $0 < k \le n$;

$$\varrho_k^{2n} = \varrho\, 2_{n-k}^{\;2n}, \text{ if } n < k < 2n.$$

Here $\binom{a}{b} = \frac{a!}{b!(a-b)!}$ and S_i^n represents the sum of all possible combinations of i elements from the subset of the first n elements of $S = (2\,a, 2\,c, 2\,e, 2\,g, \ldots)$, with $S_0^n = 1$.

Some other examples are:

$\hat{\varrho}_1$	$\hat{\varrho}_2$	$\hat{\varrho}_3$	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$	$\hat{\vartheta}_3$	Dominant cycle
-0.3473	2.0000	-0.3473	120^{o}	80^{o}	60^{o}	18 periods
-3.5202	6.0778	-7.0404	90^{o}	36^{o}	18^{o}	20 periods
-2.4142	4.4142	-4.8284	90^{o}	60^{o}	45^{o}	24 periods

Notice that the angle of oscillation of the dominant cycle always equals the smallest difference between the ϑ's of the underlying cycles.

3. A Theoretical Example

We now will demonstrate how Klein's method can be used to combine the theory of self-sustaining cycles with the hypothesis of identifiable exogenous shocks. To this end consider the (fictive) time series that is reproduced by Fig. 1a. (The corresponding figures are given in the Appendix.)

Fig. 1 a. Time series with irregular fluctuations

In order to discover the autoregressive equation that describes the series best, several specifications have been tested. These (OLS) regressions are set out in Table 1. The Eqs. (1)-(8) are unrestricted relationships. From the given statistics (sum of squared residuals in proportion to the total sum of squares) it is easy to sec that the lower order equations are not very adequate. Moving to a higher order provides us with a better result.

Fig. 1b. Time series with dominant cycle of 12 periods

However, this process is clearly not infinite, for the seventh and eighth order equations do not add much to the satisfying result that is obtained when using a sixth order equation. Eq. (6) also reveals that some of the estimated coefficients do not deviate from zero significantly, whereas at the same time it appears that $\varrho_2 \approx \varrho_4$ and $\varrho_6 \approx 1$, thus giving ground to the hypothesis that the sequence can be considered as a recurrent cycle. This is tested by regressing $(y_t + y_{t\text{-}6})$ on $(y_{t\text{-}2} + y_{t\text{-}4})$. The result is Eq. (9).

Comparing Eq. (9) with Eq. (6) it is seen that the residual variance has increased. (This is always the case if the unrestricted regression does not yield exactly the same estimated coefficients as does the restricted regression.) However, the hypothesized restriction is not rejected by the F-test, so that it can be stated that Eq. (9) indeed detects the (superimposed) recurrent cycle, which is inherent to the time series under consideration.[3]

The residuals of Eq. (9), defined as the difference between the actual value of $(y_t + y_{t\text{-}6})$ in a certain period and its predicted value, are reproduced by Fig. 2. From this it is clear that some relatively large residuals can be observed, namely in the periods 14, 24, 32, 38 and 44. This strongly suggests that certain shocks, which are both scarce and apparent, have been active.

[3] The following F-statistic is computed:

$$F = \frac{(SSE_r - SSE)/r}{SSE/(n-k-1)},$$

where SSEr and SSE are the residual variances of respectively the restricted and the unrestricted regression equation, r is the number of restrictions, n is the number of observations and k is the number of regressors from the unrestricted equation.

*Table 1. Regressions on the Fictive Time Series**

1. $y_t = 0.049 - 0.060\, y_{t-1}$
   (0.67) (0.149)

$SSE = 1028$, $SST = 1032$, $n = 49$.

2. $y_t = 0.208 - 0.113\, y_{t-1} - 0.664\, y_{t-2}$
   (0.53) (0.117) (0.117)

$SSE = 596$, $SST = 1031$, $n = 48$.

3. $y_t = 0.216 - 0.064\, y_{t-1} - 0.658\, y_{t-2} + 0.086\, y_{t-3}$
   (0.54) (0.152) (0.120) (0.164)

$SSE = 592$, $SST = 1030$, $n = 47$.

4. $y_t = 0.264 + 0.003\, y_{t-1} - 1.154\, y_{t-2} + 0.066\, y_{t-3} - 0.873\, y_{t-4}$
   (0.33) (0.092) (0.092) (0.099) (0.099)

$SSE = 203$, $SST = 1026$, $n = 46$.

5. $y_t = 0.213 + 0.006\, y_{t-1} - 1.152\, y_{t-2} + 0.077\, y_{t-3} - 0.866\, y_{t-4} + 0.011\, y_{t-5}$
   (0.34) (0.159) (0.093) (0.211) (0.101) (0.172)

$SSE = 198$, $SST = 1017$, $n = 45$.

6. $y_t = 0.317 - 0.029\, y_{t-1} - 1.891\, y_{t-2} + 0.038\, y_{t-3} - 1.925\, y_{t-4} + 0.026\, y_{t-5} - 0.985\, y_{t-6}$
   (0.14) (0.066) (0.065) (0.087) (0.087) (0.070) (0.071)

$SSE = 31$, $SST = 1016$, $n = 44$.

7. $y_t = 0.405 - 0.209\, y_{t-1} - 1.885\, y_{t-2} - 0.331\, y_{t-3} - 1.904\, y_{t-4} - 0.372\, y_{t-5} - 0.968\, y_{t-6} - 0.226\, y_{t-7}$
   (0.15) (0.170) (0.066) (0.328) (0.088) (0.344) (0.072) (0.187)

$SSE = 29$, $SST = 980$, n, $= 43$.

8. $y_t = 0.537 - 0.287\, y_{t-1} - 2.079\, y_{t-2} - 0.468\, y_{t-3} - 2.297\, y_{t-4} - 0.499\, y_{t-5} - 1.396\, y_{t-6} - 0.285\, y_{t-7}$
   (0.17) (0.173) (0.171) (0.332) (0.327) (0.346) (0.344) (0.187)

$- 0.247\, y_{t-8}$
(0.188)

$SSE = 27$, $SST = 954$, $n = 42$.

9. $y_t = 0.323 - 1.922\, y_{t-2} - 1.922\, y_{t-4} - 1.0\, y_{t-6}$
   (0.14) (0.035) (0.035)

$SSE = 36$, $SST - 2584$, $n = 44$.

10. $y_t = -2.0\, y_{t-2} - 2.0\, y_{t-4} - 1.0\, y_{t-6} + 3.0\, D_{14} + 3.0\, D_{24} + 3.0\, D_{32} + 3.0\, D_{38} + 3.0\, D_{44}$

$SSE = 0$, $SST = 2584$, $n = 44$.

$\hat{\vartheta}_1 = 120^o$, $\hat{\vartheta}_2 = 90^o$, $\hat{\vartheta}_3 = 60^o$

* Numbers between brackets are standard deviations. The sum of squared residuals is SSE and the total sum of squares is SST. The number of observations is *n*.

Of course this can be investigated with the aid of dummy variables. Adding these, the result is the last equation of Table 1, of which can be said that it describes the time series perfectly. (This is so by construction, and hence not really surprising.)

Fig. 2. Residuals

According to Eq. (10) we now are able to conclude that we have identified five exogenous shocks, each equal to 3. (Notice that by adding 5 dummies *all* the residuals of Eq. (9) have disappeared, which is due to the "recurrence" feature of the time series under consideration.) At the same time we have discovered that the "real" autoregressive part of the sequence is given by

$$\text{I} \quad y_t = -2.0\, y_{t-2} - 2.0\, y_{t-4} - 1.0\, y_{t-6}\ .$$

It should be remarked that this equation is also found if the number of observations is reduced substantially. Actually we only need a time series embodying a representative part of the dominant cycle.

Equation I enables us to construct a new time series, taking the first 6 observations of the original series as starting values. The result, which shall be referred to as the "clear" series, is depicted in Fig. 1b. From this it appears that equation I describes a cycle with a periodicity of 12 periods. This is the dominant cycle, for it can be decomposed into three underlying cycles, of 3, 4 and 6 periods respectively. This is illustrated by Fig. 3.

Looking at Fig. 3 it is important to realize that a difference equation of recurrent cycles does not imply any specific cyclical pattern. Only the periodicity of the cycle is determined by it. Other features of the path, such as the amplitude, also depend on the starting values. This is not hard to see, for in the absence of shocks any recurrent cycle has to reproduce its initial conditions at its end. The latter also holds for the underlying cycles. Take as an example the case that the initial conditions of the dominant cycle amount to the value 1 (one) in every period. These can be decomposed as follows:

Period	Dominant cycle	Cycle of 3 periods	Cycle of 4 periods	Cycle of 6 periods
-5	1			-3
-4	1			0
-3	1		-3	3
-2	1	1	-3	3
-1	1	-2	3	0
0	1	1	3	-3

Fig. 3. Decomposition of dominant cycle

(see numerical example, above)

The initial conditions of the underlying cycles uniquely determine the starting values of the 12-period cycle. Over this dominant cycle, the initial conditions of the underlying cycles will be repeated respectively 4, 3 and 2 times.

Of course it is always possible to describe the inherent cyclical pattern of a recurrent cycle by the (isolated) effects of shocks.

*Table 2. Effects of Shocks**

Shock in period 1			Shocks in periods 1-6			Shocks in periods 1-12		
Period	Effect	Cumu-lated	Period	Effect	Cumu-lated	Period	Effect	Cumu-lated
1	3	3	1	3	3	1	3	3
2	0	3	2	3	6	2	3	6
3	-6	-3	3	-3	3	3	-3	3
4	0	-3	4	-3	0	4	-3	0
5	6	3	5	3	3	5	3	3
6	0	3	6	3	6	6	3	6
7	-3	0	7	-3	3	7	0	6
8	0	0	8	-3	0	8	0	6
9	0	0	9	3	3	9	0	6
10	0	0	10	3	6	10	0	6
11	0	0	11	-3	3	11	0	6
12	0	0	12	-3	0	12	0	6
13	3	3	13	3	3	13	0	6
14	0	3	14	3	6	14	0	6
15	-6	-3	15	-3	3	15	0	6
16	0	-3	16	-3	0	16	0	6
17	6	3	17	3	3	17	0	6
18	0	3	18	3	6	18	0	6
19	-3	0	19	-3	3	19	0	6
20	0	0	20	-3	0	20	0	6
21	0	0	21	3	3	21	0	6
22	0	0	22	3	6	22	0	6
23	0	0	23	-3	3	23	0	6
24	0	0	24	-3	0	24	0	6
* Calculated with equation I; the value of a shock is 3 per period.								

Table 2 contains some examples, constructed with the aid of equation I. Using this information it is easy to (re)construct the time series of Fig. 1a from the "clear" series of Fig. 1b. It is also interesting to note that only if the shock is sustained for exactly the length of the dominant cycle (here 12 periods), this cycle will not be repeated. Else, if a shock of shorter duration is going on, the effects are lasting, consequently changing the shape of the series to which they are added.

Another important point to realize is that a difference equation of superimposed recurrent cycles may exhibit a very long dominant cycle. The following example illustrates this:

$$\text{II} \quad y_t = 0.382\, y_{t-1} - 2.382\, y_{t-2} - 0.764\, y_{t-3} - 2.382\, y_{t-3} + 0.382\, y_{t-3} - y_{t-6}.$$

At first sight this relationship differs much from equation I. However actually they have two ϑ's in common, whereas the third ϑ is 108^o instead of 120^o. Notwithstanding this small difference, equation II shows a dominant cycle, which is 5 times as long as that of example I: 60 instead of 12 periods. This is due to the fact that the periodicity of the cycle only can be observed as an integer, being the least common multiple of the periodicities of the underlying cycles. This has been illustrated by Fig. 4. Fig. 4a is a reproduction of Fig. 1b. It shows a recurrent cycle of 12 periods, which is clearly observable from period 10 onwards. Fig. 4b is based on equation II, starting from the same initial conditions as for equation I, namely $y_{-5} = -6$, $y_{-4} = -5$, $y_{3-} = 5$, $y_{-2} = 6$, $y_{-1} = -1$ and $y_0 = -3$.

Fig. 4 a. Time series according to equation I

Figure 4 demonstrates that the two time series coincide in the periods 55-60. Evidently this is the time interval that both equations reproduce the same initial conditions. (Notice that the cumulated value of both series is zero at t = 60.) However of more importance is the coincidence of most peaks and throughs, thus showing the "near" dominancy of the 12-period cycle in the case of equation II.

Regressing actual time series, it will be clear that the estimated ϑ's hardly ever shall appear as integers. Therefore in empirical investigations, it is recommended to search for the periodicity of (what could be called) the "nearly" dominant cycle. In most cases this will be possible because the periodicities of the underlying cycles are estimated parameters, for which it always can be tested whether they fit to an integer or not.

Fig. 4 b. Time series according to equation II

4. Some Empirical Examples

The theory of (superimposed) recurrent cycles has always been a fascinating subject. In this respect, efforts to test the existence of a Kondratieff-cycle strike the imagination most. Sometimes these attempts incorporate additional (or preceding) findings on other cycles, such as the Juglar and the Kuznets, but on the whole they lack an integrating framework.

The method described above may provide us with a good starting point to develop such a framework. For, using difference equations conditioned for no dampening, it is not hard to detect long waves. This is illustrated by the following regression, which is based on a very long time series (1264-1954) for price indices of Southern England:

$$y_t = 10.45 + 1.982\, y_{t-1} - y_{t-2}.$$

The time series used has been taken from Ramsey (1971). The equation shows that a recurrent cycle of approximately 47 years ($\hat{\vartheta}=7.7^{o}$) is inherent to this series.

Imitating a single economic time series by the simple tool of an autoregressive relationship is also associated with the Lucas critique that macroeconometric models ought to include the implications of the rational expectations hypothesis. In Lucas's own words (1977, p. 3):

"Let me begin to sharpen the discussion by reviewing the main qualitative features of economic time series which we call ‘the business cycle’. Technically, movements about trend in gross national product in any country can be well described by a stochastically disturbed difference equation of very low order."

It must be said that "technically spoken" Lucas is right, thus making the facts of life much more simple than they look. Consequently it is not surprising that in the meantime this view has penetrated into introductory courses on economics.

An example of the latter is the textbook of Parkin (1984). There it is stated that the fluctuations around trend in U.S. real income are well described by a second order difference equation, whereas the residuals are interpreted as purely random disturbances.

From a statistical point of view Parkin is right in stipulating that an (unconditioned) second order equation fits best to his detrended growth rates. Using Parkin's data (n = 80) the following results show this:

$y_t = -0.065 + 1.174\, y_{t-1} - 0.392\, y_{t-2}$ SSE = 2284 and SST = 9297

(0.609) (0.104) (0.104)

$y_t = -0.067 + 1.684\, y_{t-1} - y_{t-2}$

(0.728) (0.068) SSE = 304 and SST = 29515.

By conditioning the residual variance has increased to such an extent that the restriction should be rejected by the F-test. However, it remains to be seen whether this statistical result tells us enough to accept (Parkin’s) empirical validation of the theory of dying cycles, which must be kept alive with random shocks. Adding dummy variables to capture the influence of exogenous shocks shows that the conditioneel equation fits equally well or even better.

We conclude this section by employing the method presented above more extensively to a variable which is known to be an important indicator of the state of the U.S. economy, namely the rare of capacity utilization in U.S. manufacturing.

The time series used here is drawn from OECD (1984, p. 59) and involves quarterly material over the period 1964-1983. Figure 5 displays this series. Given the well-known qualifications on aggregate capital utilization measures, this series seems to exhibit a recurrent cycle of medium-term length. This will be tested now.

Table 3 shows the difference equations that have been investigated. From this it may be concluded that the conditioned equations perform better than the corresponding unconditioned specifications, at least when comparing the coefficients of determination.

Fig. 5. Rate of capacity utilization in U. S. manufacturing

(see original article)

This evidently is explained by the fact that additional information is introduced by imposing restrictions upon the estimated parameters, including the (forced) enlargement of the total sum of squares.

This raises the question whether the right information is used by pinning the results into the preconceived direction. In the case of the fictive time series of Table 1 this was clearly justified. But now, from the unconditioned equations in Table 3, it appears that the expected restrictions are not in evidence. This suggests that it is not allowed to conclude that the rate of capacity utilization in U. S. manufacturing exhibits a (dominant) selfsustaining cycle.

However, on closer investigation it is seen that we are faced here with the problem of having variables with a high degree of multicollinearity. (For the fictive time series of Table 1 this is not the case.) Consequently the estimates of the separate effects of the regressors are not reliable, thus preventing us from the conclusion that the time series under consideration exhibits a dying cycle, which has been kept alive with external shocks.

At this stage the problem is undecided. But there are additional criteria which give more support to the ultimate conclusion that the cycle in the rate of capacity utilization is self-sustaining instead of dying. In the first place it is seen that the estimated parameters of the unconditioned equations all show the same sign as their conditioned counterparts. Further, moving up from the second to a higher order specification, the coefficient of y_{t-2} approaches (minus) one, this being the restriction imposed upon Eq. (7).

*Table 3. Regressions for the Rate of Capacity Utilization in U. S. Manufacturing**

1. $y_t = 5.249 + 0.936\, y_{t-1}$
   (3.38) (0.041)
   $SSE = 312,\ SST = 2441,\ n = 79.$

2. $y_t = 8.545 + 1.442\, y_{t-1} - 0.546\, y_{t-2}$
   (2.93) (0.097) (0.097)
   $SSE = 219,\ SST = 2432,\ n = 78.$

3. $y_t = 6.748 + 1.543\, y_{t-1} - 0.820\, y_{t-2} + 0.195\, y_{t-3}$
   (3.12) (0.115) (0.194) (0.119)
   $SSE = 211,\ SST = 2418,\ n = 77.$

4. $y_t = 8.892 + 1.584\, y_{t-1} - 1.007\, y_{t-2} + 0.561\, y_{t-3} - 0.247\, y_{t-4}$
   (3.24) (0.115) (0.211) (0.213) (0.121)
   $SSE = 199,\ SST = 2401,\ n = 76.$

5. $y_t = 8.187 + 1.598\, y_{t-1} - 1.048\, y_{t-2} + 0.663\, y_{t-3} - 0.370\, y_{t-4} + 0.087\, y_{t-5}$
   (3.50) (0.118) (0.221) (0.242) (0.222) (0.125)
   $SSE = 191,\ SST = 2357,\ n = 75.$

6. $y_t = 8.442 + 1.606\, y_{t-1} - 1.069\, y_{t-2} + 0.660\, y_{t-3} - 0.407\, y_{t-4} + 0.142\, y_{t-5} - 0.036\, y_{t-6}$
   (3.76) (0.122) (0.231) (0.259) (0.258) (0.233) (0.127)
   $SSE = 191,\ SST = 2305,\ n = 74.$

7. $y_t = 11.233 + 1.864\, y_{t-1} - 1.0\, y_{t-2}$
   (3.24) (0.039)
   $\vartheta = 21.3^{o}$ $SSE = 283,\ SST = 8718,\ n = 78.$

8. $y_t = 15.238 + 1.696\, y_{t-1} - 1.575\, y_{t-2} + 1.696\, y_{t-3} - 1.0\, y_{t-4}$
   (3.78) (0.124) (0.235) (0.124)
   $\vartheta_1 = 16.5^{o},\ \vartheta_2 = 96.4^{o}$ $SSE = 309,\ SST = 7036,\ n = 76.$

9. $y_t = 17.441 + 1.657\, y_{t-1} - 1.398\, y_{t-2} + 1.270\, y_{t-3} - 1.398\, y_{t-4} + 1.657\, y_{t-5} - 1.0\, y_{t-6}$
   (4.72) (0.130) (0.258) (0.328) (0.258) (0.130)
   $\vartheta_1 = 14.1^{o}\ \vartheta_2 = 126.1^{o},\ \vartheta_3 = 63.4^{o}$ $SSE = 355,\ SST = 5366,\ n = 74.$

* Quarterly figures from OECD (1984).

However, the most important indication arises after the addition of dummy variables to the restricted equations, meant to catch the exogenous pressure on the rate of capacity utilization of the worldwide supply shocks of 1973-1974 and 1979-1980. From this it appears that the values of the estimated parameters remain almost unchanged, whereas at the same time the residual variances decrease to a considerable extent. This altogether leads to the conclusion that the rate of capacity utilization in U.S. manufacturing is adequately described by a self-sustaining cycle of approximately 4 years, which is inherent to Eq. (7).

It is an interesting experiment to use an autoregressive relationship for prediction purposes, certainly in the case that self-sustaining cycles are involved. Using Eq. (7), Table 4 gives the results of such an experiment. In this case the initial conditions originate from the upswing of 1983.

*Table 4. Rate of Capacity Utilization in U. S. Manufacturing (1984-1989)**

1984-1	80.9	1985-1	88.3	1986-1	85.3	1987-1	77.4	1988-1	79.0	1989-1	87.2
1984-2	83.1	1985-2	88.7	1986-2	83.1	1987-2	76.6	1988-2	81.0	1989-2	88.3
1984-3	85.3	1985-3	88.3	1986-3	80.9	1987-3	76.6	1988-3	83.3	1989-3	88.7
1984-4	87.1	1985-4	87.1	1986-4	78.9	1987-4	77.5	1988-4	85.4	1989-4	88.2
* Forward calculation with Eq. (7) from Table 3.											

Therefore, it is not surprising to see that the rate of capacity utilization moves through an expansion in 1984 to a peak in 1985 and then turns into a further contraction phase. The next lower turning point of the cycle is found in 1987, so that from thereupon the following expansion can come through. It goes without saying that this forward calculation is mechanistic, neglecting the influence of the recent supply shock.

5. Epilogue

In the beginning of the last century, an engineer named Joseph Fourier announced to the French Academy of Sciences that an arbitrary function could be expanded in an infinite series of sines and cosines. According to Braun (1984, p. 485) Fourier’s announcement caused a loud furor in the Academy. Many members, including Lagrange, thought this to be pure nonsense. However, just recently, mathematicians have succeeded in establishing exceedingly sharp conditions for the so called Fourier series to converge, which ranks as one of the great mathematical theorems of the twentieth century.

As known, the Fourier analysis is also applied in economics. An example is in Hillinger (1982). Hillinger estimates recurrent cycles by cosine functions applied to residuals from polynomial trends. However, as Zarnowitz (1985, p. 542) points out, detrending time series may not be a good practice. His main argument is that detrended cycles tend to be relatively symmetrical, thus contradicting the evidence of business cycles which show a strong asymmetry in expansions lasting longer than contractions.

Another application of the Fourier analysis, which is closely related to the method described above, is found in Koshimura (1986). Koshimura (fully) decomposes a time series of investment in fixed capital into 5 recurrent cycles, respectively of 10, 5, 31/a, 2.5 and 2 years. The 10-year cycle is called the "fundamental wave" and the other ones the "harmonic waves".

It is clear that Koshimura's approach shows much resemblance with our distinction between the dominant cycle at the one side and the underlying cycles at the other side. Actually the periodicity of what Koshimura calls the "fundamental" wave shows up as the least common multiple of the periodicities of the "harmonic waves". However, contrary to the method described above, Koshimura's approach is completely deterministic. It does not leave any room for a synthesis between the theories of self-sustaining cycles and the hypothesis of identifiable exogenous shocks, which was the main intention to elaborate Klein's method here.

Appendix

Series of the Figures 1, 2 and 3*

	(1)	(2)	(3)	(4)	(5)	(6)		(1)	(2)	(3)	(4)	(5)	(6)
1	-2	*	-2	0	2	-4	26	-4	-0.79	-1	-1	2	-2
2	-1	*	-1	-1	2	-2	27	1	-0.09	1	1	-2	2
3	1	*	1	1	-2	2	28	2	-0.01	2	0	-2	4
4	2	*	2	0	-2	4	29	3	-0.25	3	-1	2	2
5	3	*	3	-1	2	2	30	4	-0.17	1	1	2	-2
6	1	*	1	1	2	-2	31	-6	-0.63	-6	0	-2	-4
7	-6	-0.63	-6	0	-2	-4	32	-5	2.21	-5	-1	-2	-2
8	-5	-0.56	-5	-1	-2	-2	33	5	-0.09	5	1	2	2
9	5	-0.09	5	1	2	2	34	0	-0.25	6	0	2	4
10	6	-0.01	6	0	2	4	35	-1	-0.25	-1	-1	-2	2
11	-1	-0.25	-1	-1	-2	2	36	6	0.07	-3	1	-2	-2
12	-3	-0.40	-3	1	-2	-2	37	-2	-0.63	-2	0	2	-4
13	-2	-0.63	-2	0	2	-4	38	-4	2.21	-1	-1	2	-2
14	2	2.44	-1	-1	2	-2	39	1	-0.09	1	1	-2	2
15	1	-0.09	1	1	-2	2	40	-4	-0.48	2	0	-2	4
16	-4	-0.25	2	0	-2	4	41	3	-0.25	3	-1	2	2
17	3	-0.25	3	-1	2	2	42	10	0.30	1	1	2	-2
18	7	-0.17	1	1	2	-2	43	-6	-0.63	-6	0	-2	-4
19	-6	-0.63	-6	0	-2	-4	44	-5	2.21	-5	-1	-2	-2
20	-8	-0.56	-5	-1	-2	-2	45	5	-0.09	5	1	2	2
21	5	-0.09	5	1	2	2	46	-6	-0.71	6	0	2	4
22	6	-0.25	6	0	2	4	47	-1	-0.25	-1	-1	-2	2
23	-1	-0.25	-1	-1	-2	2	48	12	0.53	-3	1	-2	-2
24	0	2.83	-3	1	-2	-2	49	-2	-0.63	-2	0	2	-4
25	-2	-0.63	-2	0	2	-4	50	-7	-0.79	-1	-1	2	-2

(1) Time series with irregular fluctuations (Fig. 1 a).
(2) Residuals (Fig. 2).
(3) Series with the dominant cycle of 12 periods (Fig. 1 b).
(4) Series with the underlying cycle of 3 periods (Fig. 3).
(5) Series with the underlying cycle of 4 periods (Fig. 3).
(6) Series with the underlying cycle of 6 periods (Fig. 3).

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