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Published in	European Economic Review
Publication Date	2002
Link	https://research.tilburguniversity.edu/en/publications/539d10a7-be00-4a8e-9c9c-97cf0f810ece
Citation	Abbring, J H, van den Berg, G & van Ours, J C 2002, 'The anatomy of unemployment dynamics', European Economic Review, vol. 46, no. 10, pp. 1785-1824.
Download Date	2026-06-14 03:59:11
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The Anatomy of Unemployment Dynamics

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May 18, 2001

Abstract

This paper examines the relation between individual unemployment durations and incidence (inflow size) on the one hand, and the time-varying macroeconomic conditions in the economy on the other. We develop a model for the analysis of aggregate unemployment incidence and duration data, and estimate this model on quarterly French data over the period 1982–1994 stratified by sex. We find upward trends in both incidence and durations. The former is relatively important for females, the latter for males. Male incidence and durations are countercyclical, with only a minor role for cohort effects on durations. In contrast, female cohorts entering at the top of the cycle have relatively short unemployment spells, and the female incidence is, if anything, procyclical. There is strong seasonal variation, in particular in the incidence. Finally, we find relatively small non-monotonous individual duration dependence in the first 6 quarters of unemployment. Unobserved heterogeneity explains the observed negative duration dependence at these durations. We provide some evidence that negative individual duration dependence, and not heterogeneity, is important at higher durations.

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Keywords: unemployment, unemployment duration, business cycle, duration dependence, seasonality.

JEL codes: C41, E32, J64.

We would like to thank Tony Lancaster and François Laisney for some helpful suggestions. We also thank the editor and referees for comments. Gerard J. van den Berg acknowledges the Royal Netherlands Academy of Arts and Sciences for financial support. The Département du Marché du Travail of the Ministère du Travail, de l'Emploi et de la Formation Professionnelle of France kindly provided the data.

1 Introduction

Unemployment has been a top issue for economic research and policy for many decades. Traditionally, microeconomic research focuses on the incidence and duration of unemployment at the individual level, while macroeconomic research focuses on the macro unemployment rate and its behavior over the business cycle. In the micro approach, attention has recently concentrated on dynamic duration models for explaining individual variation in unemployment duration. These models typically assume the parameters to be independent of macroeconomic conditions, and these conditions are at most included as an additional regressor. At the same time, the recently expanding macro literature on aggregate flows between labor market states stresses that the distribution of unemployment durations and incidence changes markedly over the business cycle.

In the present paper, we aim to bridge the gap between these approaches by examining the relation between individual unemployment durations and incidence on the one hand, and the time-varying macroeconomic conditions in the economy on the other. We introduce a new model for the analysis of aggregate unemployment incidence and duration data, and estimate this model on French data for the period 1982–1994. We provide a complete decomposition of the dynamics of unemployment, and analyze the changes in the unemployment duration distribution over time. Our methodology offers a novel and integrated perspective on the interaction of macroeconomic conditions and individual unemployment durations and incidence. It is comparatively simple, and only uses readily available aggregate data.

The most basic decomposition of aggregate unemployment is in terms of the gross size of the incidence and the average duration. Throughout the paper, we pay most attention to macroeconomic effects that can be identified from the unemployment duration distribution and the way it changes over calendar time. The duration part of our model allows individual exit probabilities out of unemployment, as functions of the elapsed unemployment duration, to depend on calendar time. This contemporaneous calendar time dependence is modeled as the product of seasonal effects and flexibly specified (yearly) business cycle effects. In addition to this, we allow for cohort effects, *i.e.* variation with the moment of inflow into unemployment, in a similar way. The estimates of the calendar time effects that are supposed to capture the business cycle effects can then be compared to the behavior of traditional economic business cycle indicators. We estimate separate models for females and males, so that we can assess the differences in cyclical and seasonal patterns between the sexes.

As noted above, individual exit probabilities are allowed to depend on the elapsed duration of being unemployed. This represents genuine duration dependence due to *e.g.* stigma effects reducing the number of job opportunities of the long-term unemployed (*e.g.*, Vishwanath, 1989, and Van den Berg, 1990). Apart from this, we also allow for unobserved heterogeneity in the exit probabilities. In the case of (unobserved) heterogeneity,

individuals with the largest exit probabilities on average leave unemployment first. This dynamic sorting leads to a decline in the average quality of a cohort of unemployed in the course of time. Thus, negative duration dependence in observed aggregate exit probabilities may occur even in absence of genuine duration dependence at the micro level. This is important for policy analysis (*e.g.*, Layard, Nickell and Jackman, 1991, and Van den Berg and Van Ours, 1996).

The so called Mixed Proportional Hazard (MPH) assumption ensures identification of genuine and spurious duration dependence. The MPH model specifies the individual exit probability conditional on survival and the unobserved individual effect as the product of calendar time effects, the genuine duration dependence effect, and the unobserved individual effect (or ‘heterogeneity term’). As a result, the observed exit probabilities can be expressed as the product of calendar time effects, the genuine duration dependence effect, and the expected value of the heterogeneity term conditional on survival. It turns out that the latter is an interaction term of calendar time and elapsed duration. As such, the MPH specification provides a convenient way to structure time-duration interactions in the data. We will actually not restrict the interaction term to be consistent with the MPH interpretation when estimating the model, so that we can test for it. We extend earlier identification results by showing that the model can be non-parametrically identified from aggregate unemployment duration data.

In the MPH model, the cohort effects can be interpreted as effects of the composition of the inflow into unemployment. For example, a relatively high exit probability for individuals entering unemployment in a certain season can be viewed as evidence that the inflow in this season contains relatively many individuals with high unobserved quality. Similarly, it can be investigated whether the composition of the inflow during a recession differs from the composition of the inflow at the top of a cycle. This provides a test of the model of Darby, Haltiwanger and Plant (1985), who argue that in a recession the inflow into unemployment contains a relatively large amount of individuals with small exit probabilities, and that this is the major cause of the procyclicality of observed exit probabilities from unemployment.

The model and estimation method developed in this paper are designed to be applicable to discrete-time time-series data on aggregate numbers of individuals in different unemployment duration classes. Such data can be used to calculate the aggregate outflow from different duration classes for each calendar time point. In our context, the main advantage of aggregate data is that they cover a much longer time span than is usual in micro data. Clearly, for reliable estimation of business cycle effects, it is necessary to have data that include at least a complete cycle. Another major advantage of aggregate data is that usually they do not suffer from attrition. In the analysis of labor market transitions, attrition is a particularly serious problem, since attrition out of panel survey data may be induced by the occurrence of a transition (see Van den Berg, Lindeboom and Ridder, 1994). Finally, truly aggregate data in principle cover the whole population,

which makes such data better suited for the analysis of the overall impact of aggregate events like business cycles. The contribution of this paper is to show that much can be learned from aggregate duration and incidence data. Of course, complementary insights can be derived from analyses of micro data.

We estimate our model on quarterly French data for the period 1982.IV–1994.IV. We provide separate analyses for males and females, and find some remarkable differences between the sexes. We find upward trends in the incidence and downward trends in the exit probabilities. The incidence trend is relatively important for females and the duration trend for males. The male incidence is countercyclical and male exit probabilities are procyclical. The latter can be traced back to contemporaneous calendar time effects. Cohort effects on durations are relatively unimportant for males. In contrast, for females contemporaneous calendar time effects on exit probabilities are ambiguous, and exit probabilities depend procyclically on the moment of inflow. Furthermore, the female incidence is, if anything, procyclical. There are strong seasonal effects for both sexes. The season in the incidence is the most volatile, especially for females, but we also find cohort and contemporaneous seasonal effects in the exit probabilities. We find some evidence that size and quality of cohorts are positively related across seasons. Finally, we find non-monotonous individual duration dependence in the first 6 quarters of unemployment. Observed negative duration dependence is due to dynamic sorting because of unobserved heterogeneity. Extrapolation of the estimation results beyond the first 6 quarters suggests that negative individual duration dependence, and not dynamic sorting, is important at higher durations.

To date, a number of empirical studies have been published that focus on one or more of the issues we deal with in the present paper. Dynarski and Sheffrin (1990), Imbens and Lynch (1992) and Lollivier (1994) use micro data to estimate the effect of business cycle indicators like the unemployment rate on the unemployment duration distribution. Baker (1992a) uses aggregate data containing a large number of individual characteristics to investigate cyclical behavior of the determinants of unemployment. There have also been numerous studies on the relative importance of incidence and duration to explain variation in unemployment (see *e.g.* the survey in Layard, Nickell and Jackman, 1991). Concerning the distinction of genuine duration dependence and unobserved heterogeneity we generalize the existing literature, which typically assumes functional form restrictions. Van den Berg and Van Ours (1996) provide a non-parametric analysis of duration dependence and heterogeneity in data similar to ours, but they do not analyze business cycle effects. Also, their statistical model has no clear stochastic foundation, and they do not correct for cyclical changes in the composition of the inflow when evaluating the interaction of calendar time and duration. Below we will compare our results to those in the literature. It should be noted from the outset that the vast majority of this empirical literature is based on U.S. data.

The outline of the paper is as follows. In Section 2 we discuss the model and the empir-

ical implementation, and provide conditions under which the model is non-parametrically identified. Furthermore, we propose specification tests. In Section 3 we present the data. Section 4 discusses the estimation results and the results of the specification tests. Section 5 concludes.

2 The model and the empirical implementation

2.1 Observation of unemployment

This subsection sketches the type of data we use, and discusses the role of measurement errors. In the next subsection, we present the model for the exit probabilities out of unemployment.

Ideally, aggregate data give the total numbers of individuals in the labor market who are unemployed for t periods of time, $t = 0, 1, 2, \dots$, at calendar times $\tau = \tau_0, \tau_0 + 1, \tau_0 + 2$, etcetera. Here, both t and τ are discrete variables, measured on the same scale apart from the difference in origin. Denoting the number of individuals who are unemployed for t periods at calendar time τ by $U(t|\tau)$, we can calculate the fraction $\theta(t|\tau)$ of these unemployed who leave unemployment at τ as

$$\theta(t|\tau) = \frac{U(t|\tau) - U(t+1|\tau+1)}{U(t|\tau)}. \quad (1)$$

This fraction equals the aggregate exit probability out of unemployment at calendar time τ and duration t , conditional on survival up to t . We take $U(0|\tau)$ as the measure of the size of the inflow into unemployment at calendar time τ . We return to this in Section 3. Note that aggregate unemployment at time τ is given by $U(\tau) := \sum_{t=0}^{\infty} U(t|\tau)$.

In reality we do not exactly observe the numbers $U(t|\tau)$, and therefore neither $\theta(t|\tau)$. Sometimes the data are based on surveys of unemployed individuals. Respondents may have trouble recalling their elapsed unemployment duration. In that case they may be counted as being unemployed for t periods of time whereas in reality they are unemployed for $t-1$ or $t+1$ periods. Alternatively, they may tend to round off their duration to the nearest natural unit of time, like an integer number of months. If only a sample of the population is surveyed then the data contain sampling errors as well. If the data cover the whole population and are based on administrative records, then there may be misclassifications due to administrative errors.

Because of this, we allow for measurement errors in the model. From now on we place a \sim on top of observed values of variables, in contrast to true or unobserved values. We assume that

$$\tilde{U}(t|\tau) = U(t|\tau) \varepsilon_{t,\tau}, \quad (2)$$

with $\ln \varepsilon_{t,\tau}$ (jointly) normal with mean 0 and variance σ^2 .

In the empirical analysis we experiment with different types of correlation schemes for $\varepsilon_{t,\tau}$ over t and τ . If, at given calendar times τ , individuals are sometimes assigned to the wrong duration class adjacent to the right class, then we expect a negative correlation between $\varepsilon_{t,\tau}$ and $\varepsilon_{t+1,\tau}$ for every t . If the definition of unemployment used to count individuals at τ is less restrictive than the definition used elsewhere, then we expect a positive correlation between $\varepsilon_{t,\tau}$ and $\varepsilon_{t+1,\tau}$ for every t .

The observed exit probability out of unemployment $\tilde{\theta}(t|\tau)$ equals the right hand side of equation (1) with U replaced by \tilde{U} . By substituting equation (2) into this, we obtain

$$\ln\left(1 - \tilde{\theta}(t|\tau)\right) = \ln(1 - \theta(t|\tau)) + e_{t,\tau}, \quad (3)$$

with the error terms $e_{t,\tau} := \ln \varepsilon_{t+1,\tau+1} - \ln \varepsilon_{t,\tau}$ jointly normal. The mean of $e_{t,\tau}$ is 0 and the variance $2(1 - \text{corr}(\ln \varepsilon_{t,\tau}, \ln \varepsilon_{t+1,\tau+1}))\sigma^2$. Here, $\text{corr}(\cdot)$ denotes the correlation coefficient. Note that the errors in equation (3) are correlated even if the errors in equation (2) are mutually independent. In the latter case $\text{corr}(e_{t,\tau}, e_{t+1,\tau+1}) = -\frac{1}{2}$ for every t and τ , and all other types of correlations are zero.

Equation (3) links the data to the true exit probabilities. In the next subsection we present a model for these probabilities. Suppose we observe $\tilde{U}(t|\tau)$ for $K + 2$ duration classes $0, 1, \dots, K + 1$. Then (3) can be thought to represent $K + 1$ different equations, for $\tilde{\theta}(0|\tau)$ up to and including $\tilde{\theta}(K|\tau)$. The loss of information when going from $K + 2$ duration classes for U to $K + 1$ equations for θ , which is a first difference of U , concerns the level of unemployment. This is accounted for by the equation for the size of the total inflow into unemployment, *i.e.* $\tilde{U}(0|\tau) = U(0|\tau)\varepsilon_{0,\tau}$. In the next subsections we also present a simple model for $U(0|\tau)$.

2.2 The model

Usually, data sets on aggregate unemployment do not contain much information on individual characteristics that could be used as explanatory variables. At best the data are stratified into a small number of different types of individuals, in our case males and females. We estimate the model separately for each sex, and in the sequel we present the model for a given sex.

The aim is to provide a model for the true exit probabilities $\theta(t|\tau)$ appearing in the right hand side of equation (3). As stated in the introduction, we use a MPH model to describe these gross probabilities. The starting point for the MPH model is the specification of exit probabilities at the individual level. It is assumed that all variation in the individual exit probabilities out of unemployment can be explained by the prevailing unemployment duration t and calendar time τ and by unobserved heterogeneity across individuals. We denote the probability that an individual leaves unemployment right after t periods of unemployment, given that he is unemployed for t periods at calendar time τ ,

and conditional on his unobserved characteristics \mathbf{v} , by $\theta(t|\tau, \mathbf{v})$. We make the following assumptions on these individual (conditional) exit probabilities.

Assumption 1. (MPH) $\theta(t|\tau, \mathbf{v})$ has a mixed proportional hazard specification, *i.e.* there are positive functions ψ_1 and ψ_2 such that

$$\theta(t|\tau, \mathbf{v}) = \psi_1(t) \psi_2(\tau) \mathbf{v}. \quad (4)$$

For every t and τ , the distribution of \mathbf{v} conditional on calendar time τ and survival up to t is such that $\Pr(0 \leq \theta(t|\tau, \mathbf{v}) \leq 1) = 1$ and $\Pr(0 < \theta(t|\tau, \mathbf{v}) < 1) > 0$.

Assumption 2. Invariance of individual \mathbf{v} : \mathbf{v} does not change during unemployment.

The functions ψ_1 and ψ_2 represent the duration dependence and the calendar time dependence of the individual exit probabilities out of unemployment. Assumption 1 is reminiscent of the standard MPH assumption in reduced-form models for micro duration data (see Lancaster, 1990, and Van den Berg, 2001, for surveys). In models for micro duration data, dependence on calendar time is usually ignored, and the role of τ in the model above is replaced by the role of observed explanatory variables x . An important difference between the present model and MPH models for micro data is that here we have discrete time, whereas in micro studies time is usually treated as continuous. The present model should not be interpreted as an approximation to the continuous time MPH model. Rather, it should be regarded as a flexible accounting device for discrete aggregate duration data, with an appealing interpretation. Because of the discrete time framework, we have to introduce the last line of Assumption 1. Note that this implies that $0 < \theta(t|\tau) < 1$, and that the support of \mathbf{v} is bounded, so that all moments of \mathbf{v} exist (see Subsection 2.5).

We now turn to the cohort effects. We assume these to act by way of the composition of the inflow, *i.e.* by way of the shape of the distribution of \mathbf{v} in the inflow. In particular, we allow a scale parameter of the cumulative distribution function G_τ of \mathbf{v} in the inflow at calendar time τ to vary with τ , and we assume that this is the only way in which the distribution of \mathbf{v} varies between cohorts. Thus, we have

Assumption 3. The distribution G_τ of unobserved heterogeneity at moment of inflow τ satisfies, for every τ ,

$$G_\tau(v) = G\left(\frac{v}{\psi_3(\tau)}\right), \quad (5)$$

for some distribution G that does not depend on τ , and some positive function ψ_3 .

If $\psi_3(\tau) > \psi_3(\tau')$ then the individuals entering unemployment at τ on average have higher values of their unobserved characteristics \mathbf{v} (*i.e.*, higher exit probabilities) than individuals entering at τ' . It is intuitively clear that the class of functional forms for $\psi_3(\tau)$ has to

be restricted to obtain an identifiable model. We turn to identification issues in the next subsection.

Assumptions 1–3 define a model for the aggregate exit probabilities $\theta(t|\tau)$, which, from a formal point of view, generalizes the standard MPH setup by allowing the unobserved heterogeneity distribution to depend on calendar time, which is our ‘observed explanatory variable’. To express the exit probabilities $\theta(t|\tau)$, appearing in the right hand side of equation (3), in terms of the individual exit probabilities $\theta(t|\tau, \mathbf{v})$, we have to integrate \mathbf{v} out of the latter. Let \mathbf{t} denote the random unemployment duration, and t its realization. In obvious notation, there holds that

$$\theta(t|\tau) = \frac{\Pr(\mathbf{t} = t | \text{inflow at } \tau - t)}{\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t)} = \frac{\mathbb{E}_{\tau-t}(\Pr(\mathbf{t} = t | \text{inflow at } \tau - t; \mathbf{v}))}{\mathbb{E}_{\tau-t}(\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t; \mathbf{v}))}, \quad (6)$$

in which the expectations $\mathbb{E}_{\tau-t}$ are taken with respect to the distribution $G_{\tau-t}(v)$. Using standard relations between probability density functions, hazards and survival functions (see *e.g.* Lancaster, 1990), the probabilities $\Pr(\mathbf{t} = t | \text{inflow at } \tau - t; \mathbf{v})$ and $\Pr(\mathbf{t} \geq t | \text{inflow at } \tau - t; \mathbf{v})$ can be expressed in terms of $\theta(t|\tau, \mathbf{v})$. By doing this, and substituting equations (4) and (5), we get

$$\theta(t|\tau) = \psi_1(t) \psi_2(\tau) \psi_3(\tau - t) \frac{\mathbb{E}(\mathbf{v} \prod_{i=1}^t (1 - \psi_1(t - i) \psi_2(\tau - i) \psi_3(\tau - t) \mathbf{v}))}{\mathbb{E}(\prod_{i=1}^t (1 - \psi_1(t - i) \psi_2(\tau - i) \psi_3(\tau - t) \mathbf{v}))}, \quad (7)$$

in which we use the convention that $\prod_{i=1}^0 (1 - \psi_1(t - i) \psi_2(\tau - i) \psi_3(\tau - t) \mathbf{v}) = 1$. The expectations are now taken with respect to the distribution G (see Assumption 3). Substitution of equation (7) into equation (3) establishes the link between the observed exit probabilities and the model determinants.

The exit probability model developed so far is similar to the model analyzed in Van den Berg and Van Ours (1996). However, their analysis does take measurement errors into account, nor does it allow for cyclical cohort effects. Also, it does not address business cycle issues, and it does not analyze the incidence.

Our model is closed by the specification of an equation for the inflow size, or incidence. We simply take

$$U(0|\tau) = \psi_4(\tau), \quad (8)$$

for some positive function ψ_4 . Substitution of (8) into equation (2) for $t = 0$ establishes the link between the observed $\tilde{U}(0|\tau)$ and the unknown function $\psi_4(\tau)$.

2.3 Identification

The structural determinants in our model are the functions $\psi_1, \psi_2, \psi_3, \psi_4$ and G . As ψ_4 is trivially identified from the incidence data, we can restrict attention to identification of ψ_1, ψ_2, ψ_3 and G from the duration data.

Suppose we consider the duration model (7) for durations $0, 1, \dots, K$ and calendar times $\tau_0, \tau_0 + 1, \dots, \tau_0 + N$, with $K, N \geq 0$ and $K \leq N$, where possibly $N = \infty$ or $K = N = \infty$. It is convenient to exclude (τ, t) for which (7) involves $\psi_3(\tau - t)$ for $\tau - t < \tau_0$, *i.e.* to restrict attention to cohorts flowing into unemployment within the given calendar time frame. Formally, let $D_K = \{0, 1, \dots, K\}$ and $T_N = \{\tau_0, \tau_0 + 1, \dots, \tau_0 + N\}$ denote the sets of duration classes and calendar time periods considered. Furthermore, let $T_N(t) := \{\tau \in T_N | \tau - t \geq \tau_0\}$ contain all calendar time moments for which duration class t is considered, and let $\mathcal{T}_{N,K}(k) := \{(\tau, t) \in D_K \times T_N | t \leq k \wedge \tau \in T_N(t)\}$ combine all such pairs (τ, t) up to and including duration class k . We will discuss identification of our model for $(\tau, t) \in \mathcal{T}_{N,K} := \mathcal{T}_{N,K}(K)$.

Let $\mu_i := \mathbb{E}(\mathbf{v}^i)$ denote the moments associated with G . By expanding the product terms in equation (7) we find that $\theta(t|\tau)$ depends on $\{\psi_1(i), \psi_2(\tau - t + i), \psi_3(\tau - t), \mu_{i+1}, \text{ with } i = 0, 1, \dots, t\}$. Thus, the duration model (7) on $\mathcal{T}_{N,K}$ can be represented by a ‘parameter’ vector $\psi := (\psi_1, \psi_2, \psi_3, \mu_1, \gamma)$, where

$$\begin{aligned}\psi_1 &:= (\psi_1(0), \dots, \psi_1(K)), \\ \psi_2 &:= (\psi_2(\tau_0), \dots, \psi_2(\tau_0 + N)), \\ \psi_3 &:= (\psi_3(\tau_0), \dots, \psi_3(\tau_0 + N)), \\ \gamma &:= (\gamma_2, \dots, \gamma_{K+1}),\end{aligned}$$

and $\gamma_i := \mu_i / \mu_1^i$. Note that the number of ‘parameters’ grows with both N and K , and is infinitely large for $N = \infty$. So, although we use the word ‘parameter’, the analysis is not parametric. See also the discussion on estimation in the next subsection.

Now, let $\theta(t|\tau)$ and $\hat{\theta}(t|\tau)$ equal the r.h.s. of (7) evaluated at ψ and $\hat{\psi}$, respectively.

Definition 1. Two parameterizations ψ and $\hat{\psi}$ are *observationally equivalent* on $\mathcal{T}_{N,K}$ if $\theta(\tau, t) = \hat{\theta}(\tau, t)$ for all $(\tau, t) \in \mathcal{T}_{N,K}$.

We discuss identification of our model in terms of the set of observationally equivalent parameterizations. We restrict this set by two additional assumptions.

First, we assume that calendar time variation is separable in seasonal variation and variation over years covering full seasonal cycles. Let the number of seasons $S \geq 2$ be given, and label seasons by elements of $\mathcal{S} := \{1, 2, \dots, S\}$. Suppose that τ_0 corresponds to the first season. Then, the number of years equals $[N/S] + 1$, so that we can index the years by elements of $Y_N := \{0, 1, \dots, [N/S]\}$. Here, for any $x \in \mathbb{R}$, $[x]$ denotes the largest integer smaller than or equal to x . Let $y : T_N \rightarrow Y_N$ and $s : T_N \rightarrow \mathcal{S}$ map calendar time τ into years y and seasons s , respectively. So, $\tau = \tau_0 + Sy(\tau) + s(\tau) - 1$, with $y(\tau) = [(\tau - \tau_0)/S]$ and $s(\tau) = 1 + \tau - \tau_0 - Sy(\tau)$. Then, we have

Assumption 4. $\ln \psi_2$ and $\ln \psi_3$ are additively separable in seasonal and yearly terms:

$$\begin{aligned}\ln \psi_2(\tau) &= \omega_2(s(\tau)) + \alpha_2(y(\tau)) \quad \text{and} \\ \ln \psi_3(\tau) &= \omega_3(s(\tau)) + \alpha_3(y(\tau)),\end{aligned}$$

where ω_2 and ω_3 are functions of the prevailing season, and α_2 and α_3 functions of the prevailing year.

Thus, we assume that all calendar time variation at higher than yearly frequencies can be captured by seasonal effects that are repetitive between years. The lower frequency effects are modeled by ‘year dummies’, where the first season of each year is always an integer number of full seasonal cycles away from τ_0 .

Second, in order to identify the unobserved heterogeneity distribution, we need variation in the ‘regressor effects’, ψ_2 and ψ_3 :

Assumption 5. There exist $\tau, \tau' \in T_N(K)$ such that

$$\psi_2(\tau - t)\psi_3(\tau - K) > \psi_2(\tau' - t)\psi_3(\tau' - K) \text{ for all } t \in D_K.$$

Assumption 5 requires the existence of two cohorts of unemployed, flowing in at $\tau - K$ and $\tau' - K$ respectively, such that one cohort has higher mean exit probabilities in each duration class in D_K . The role of this assumption is standard. Unobserved heterogeneity induces negative duration dependence at the aggregate level: the fourth factor in the r.h.s. of (7), the conditional expectation of the unobserved heterogeneity term, decreases with t . If exit probabilities were constant over calendar time, this negative duration dependence would be the same for all τ , and indistinguishable from duration dependence at the individual level, ψ_1 . Assumption 5 ensures that observed duration dependence differs between the cohorts flowing in at $\tau - K$ and $\tau' - K$. Then, G , or, more precisely, γ can be identified from this difference, or from the interaction of τ and t in $\theta(t|\tau)$.

There is an analogy with MPH models for micro duration data, in which the role of τ is replaced by observed regressors. Elbers and Ridder (1982) provided the first identification proof using variation in observed regressors, and Melino and Sueyoshi (1990) and Van den Berg (2001) discuss the role of interaction of these regressors with duration dependence. The main difference with our model is that in these standard micro MPH models the regressors are assumed to be constant over the duration of the spells, so that Assumption 5 reduces to a simpler ‘static’ condition.

Assumption 5 is not very restrictive for small enough K , typically not larger than half the length of a business cycle. It should be noted that Assumption 5 is not necessary for identification. In particular, if K is large Assumption 5 will not hold, but weaker conditions can be applied. However, these conditions are not as transparent as Assumption 5. Therefore, and because our empirical analysis is based on a small number of duration classes only, we will not pursue this any further.

Let Ψ be the set of all parameter vectors ψ that satisfy Assumptions 1–5. In the appendix we prove the following proposition.

Proposition 1. *If two parameterizations $\psi, \hat{\psi} \in \Psi$ are observationally equivalent on $\mathcal{T}_{N,K}$, then $\hat{\psi}_1(t) = af^t\psi_1(t)$ for all $t \in D_K$, $\hat{\alpha}_2(y) = \alpha_2(y) + \ln b - (\tau_0 + Sy)\ln f$ and*

$\hat{\alpha}_3(y) = \alpha_3(y) + \ln c + (\tau_0 + Sy) \ln f$ for all $y \in Y_N$, $\hat{\omega}_2(s) = \omega_2(s) + \ln d - (s - 1) \ln f$ and $\hat{\omega}_3(s) = \omega_3(s) + \ln e + (s - 1) \ln f$ for all $s \in \mathcal{S}$, and $\hat{\mu}_1 = (abcde)^{-1} \mu_1$ and $\hat{\gamma} = \gamma$, for some positive constants a, b, c, d, e and f .

Proposition 1 implies that, under Assumptions 1–5, we can identify ψ up to 6 unknown parameters, a, b, c, d, e and f . The first 5 of those parameters redistribute the overall scale of the exit probabilities between the 6 factors in our multiplicative probability model, and can be pinned down by innocuous normalizations.

The sixth parameter, f , reassigns a linear trend between the duration (ψ_1), calendar time (ψ_2) and cohort (ψ_3) effects. Note that, in Proposition 1, $\hat{\psi}_2(\tau) = \exp(\hat{\alpha}_2(y(\tau)) + \hat{\omega}_2(s(\tau))) = bdf^{-\tau} \psi_2(\tau)$ and $\hat{\psi}_3(\tau - t) = \exp(\hat{\alpha}_3(y(\tau - t)) + \hat{\omega}_3(s(\tau - t))) = ce f^{\tau-t} \psi_3(\tau - t)$. So, the proposition claims that two parameterizations $(\psi_1, \psi_2, \psi_3, \mu_1, \gamma)$ and $(f^t \psi_1, f^{-\tau} \psi_2, f^{\tau-t} \psi_3, \mu_1, \gamma)$ are observationally equivalent for all positive constants f .

In our empirical analysis, we normalize this trend by imposing orthogonality of the log yearly cohort effect to a linear trend on Y_N :

$$\sum_{y \in Y_N} (2y - [N/S]) \alpha_3(y) = 0. \quad (9)$$

For practical purposes, we restrict the orthogonality condition to the year effects. This is sufficient for identification. Seasonal cycles are generally not orthogonal to a trend for finite N , but the ‘yearly trend’ embodied in a seasonal cycle is limited by its repetitive character and vanishes as $N \rightarrow \infty$.

The trend $2y - [N/S]$ in (9) is deliberately chosen to average 0 over Y_N . This ensures that the orthogonality condition does not interfere with the scaling of $\exp(\alpha_3(y))$: if (9) holds for some $\alpha_3(y)$, then it also holds for $\alpha_3(y) + \ln c$, for all positive constants c .

Clearly, if some $\alpha_3(y)$ is orthogonal to a linear trend in this sense, then, for any constant $g \neq 0$, $\alpha_3(y) + gy$ is not orthogonal to a trend. On the other hand, if $\alpha_3(y)$ is not orthogonal to a trend, we can find a constant g such that $\alpha_3(y) + gy$ is. Thus, the normalization in (9) effectively restricts the scope for reassigning an exponential trend from $\psi_3(\tau - t)$ on the one hand to $\psi_1(t)$ and $\psi_2(\tau)$ on the other, without changing the image of $\psi_1(t)\psi_2(\tau)\psi_3(\tau - t)$.

Note that we can only identify the moments of the unobserved heterogeneity distribution G . However, Assumption 1 implies that G has bounded support. Then, if we know $\{\mu_i\}_{i=1}^{\infty}$, i.e. if $K = \infty$, we can uniquely determine G . In practice $K < \infty$, and $\{\mu_i\}_{i=1}^{K+1}$ only provides bounds on G . We return to this in Subsection 2.5.

A final remark is that there will always be alternative models that are observationally equivalent to our model. A trivial example is the non-MPH model in which *individual* exit probabilities are given by equation (7) and in which there is no unobserved heterogeneity. Another example is a model in which the composition of the inflow is constant over time, and in which the individual exit probabilities $\theta(t|\tau, \mathbf{v})$ depend on the moment of inflow $\tau - t$ by way of a multiplicative factor $\psi_3(\tau - t)$. In that case the distribution of characteristics

in the inflow does not change, but becoming unemployed at certain dates gives the exit probabilities a boost. De Toldi, Gouriéroux and Monfort (1992) take this approach to model the effect of the season at the moment of inflow.

2.4 Empirical implementation and estimation

For expositional purposes, we assume that calendar time variation of the incidence can also be separated in seasonal and yearly effects, or

$$\ln \psi_4(\tau) = \ln(\nu_1) + \omega_4(s(\tau)) + \alpha_4(y(\tau)),$$

where ω_4 and α_4 are again functions of the prevailing season and year, respectively, and ν_1 is a positive constant. Note that we model $U(0|\tau)$ on T_{N+1} , and not just T_N , and that we have to extend the domains of s and y accordingly.

Without loss of generality, we can write α_2 , α_3 and α_4 as sums of cyclical components and linear trends in the sense of equation (9). Denote the cyclical part of α_j by α_j^c , $j = 2, 3, 4$. Then, the linear trend term in α_j is given by $\alpha_j - \alpha_j^c$, and can be represented by the yearly change of the trend, or $\Delta(\alpha_j(y) - \alpha_j^c(y))$, $j = 2, 3, 4$. Clearly, this yearly change is independent of y , so that we can simply write $\Delta(\alpha_j - \alpha_j^c)$. Obviously, $\Delta(\alpha_3 - \alpha_3^c) = 0$, and even $\alpha_3 - \alpha_3^c = 0$, by the normalization in (9).

Using this notation, we can fully characterize the duration model by

$$\begin{array}{ll} \mu_1, \gamma_2, \gamma_3, \dots, \gamma_{K+1}; & \ln \psi_1(0), \ln \psi_1(1), \dots, \ln \psi_1(K); \\ \omega_2(1), \omega_2(2), \dots, \omega_2(S); & \alpha_2^c(1), \alpha_2^c(2), \dots, \alpha_2^c([N/S]); \quad \Delta(\alpha_2 - \alpha_2^c) \\ \omega_3(1), \omega_3(2), \dots, \omega_3(S); & \alpha_3^c(1), \alpha_3^c(2), \dots, \alpha_3^c([N/S]), \end{array}$$

and the incidence model by

$$\begin{array}{ll} \nu_1; & \\ \omega_4(1), \omega_4(2), \dots, \omega_4(S); & \alpha_4^c(1), \alpha_4^c(2), \dots, \alpha_4^c([(N+1)/S]); \quad \Delta(\alpha_4 - \alpha_4^c), \end{array}$$

and, in each case, the disturbance parameters. In the remainder of the paper we will present results in terms of these quantities, which we will call the ‘parameters’ of the model.

Following the previous subsection, we ensure identification of the parameters by appropriate normalizations and orthogonality restrictions. We normalize $\ln \psi_1(0) = 0$, $\omega_2(1) = 0$, $\omega_3(1) = 0$, $\omega_4(1) = 0$, and the average values of $\alpha_2^c(y)$, $\alpha_3^c(y)$ and $\alpha_4^c(y)$ over Y_N , Y_N and Y_{N+1} , respectively, to 0. Furthermore, $\alpha_2^c(y)$, $\alpha_3^c(y)$ and $\alpha_4^c(y)$ are forced to be orthogonal to a linear trend over Y_N , Y_N and Y_{N+1} , respectively.

We do not impose any additional restrictions, and aim at a fully non-parametric analysis. In particular, we do not specify the α_j^c as functions of business cycle indicators, nor do we assume that the business cycles in the α_j are regular or periodical in any

sense. Instead, we assess the relation with the general business cycle by comparing the estimation results to the way in which traditional business cycle indicators behave over time. Drawback of this approach is that our model has incidental parameters in both the calendar time and the duration dimensions.

For convenience, and by lack of better alternatives, we estimate the model by maximum likelihood (ML). As we should expect standard asymptotic theory on the ML estimator to fail, we have performed some Monte Carlo simulations to assess its finite sample properties. Although we find some evidence for biases, deviations from the true parameter values are always small compared to both the Monte Carlo and the asymptotic ML estimates of the standard errors of the ML estimator. From this we conclude that ML provides acceptable estimates of the parameters of our model.

We also find that Monte Carlo estimates of the standard errors of the estimator are generally somewhat higher than the asymptotic ML estimates. It should be noted that any such deviations from predictions from standard asymptotic theory can both be due to differences between finite sample and asymptotic properties of the ML estimator and to the incidental parameter problem. In the sequel, we simply report the ML estimates, and bear the Monte Carlo results in mind. As we will see, test results are generally clear-cut, and do not seem critically dependent on using the ML standard errors.

In our analysis, we do not impose parametric structure on the exit probability model, but ML does exploit normality of the errors. Normality can be defended with standard arguments, but even without normality pseudo-ML arguments suggest our estimator is still appropriate. Alternatively, the model could be estimated by generalized nonlinear least squares in this case.

A related approach is the grouped-data method of Prentice and Gloeckler (1978). Meyer (1986) discusses a non-parametric extension that uses a discrete heterogeneity distribution with an unknown number of points of support (Heckman and Singer, 1984). The advantage of our method is that is relatively simple, and computationally convenient, as it directly expresses the observed exit probabilities in the moments of the heterogeneity distribution.

There are also some more recent advances in the estimation of the MPH model. Lenstra and Van Rooij (1998) develop a non-parametric estimator for a model with a binary regressor. Horowitz (1999) provides a semi-parametric estimator that allows for general duration dependence and heterogeneity. Both estimators exploit continuous-time features of the model and the data, and cannot be applied here.

2.5 Specification tests

As is clear from Subsection 2.3, the estimates of the duration dependence function ψ_1 and the unobserved heterogeneity distribution G crucially depend on the MPH specification. Thus, it is particularly important to test for this. In this subsection we develop tests that

are based on the estimates of μ_1 and γ .

The last line of Assumption 1 implies that $0 \leq \mathbf{v} \leq 1/(\psi_1(t)\psi_2(\tau))$ $G_{\tau-t}$ -almost surely for all t and τ . By using equation (5) it can be seen that this is equivalent to the requirement that

$$0 \leq \mathbf{v} \leq \bar{v} \quad G - \text{a.s.}, \quad \text{where } \bar{v} = \inf_{(\tau,t) \in \mathcal{T}_{N,K}} \left(\frac{1}{\psi_1(t)\psi_2(\tau)\psi_3(\tau-t)} \right). \quad (10)$$

Thus, the support of G is bounded from below by zero, and from above by $\bar{v} < \infty$. The sequence of moments $\{\mu_i\}$ corresponds to a distribution G satisfying this requirement if and only if $\{\bar{v}^{-i}\mu_i\}$ is *completely monotone*, i.e.

$$(-1)^j \Delta^j \bar{v}^{-i} \mu_i \geq 0 \text{ for } j = 0, 1, \dots, i \text{ and } i = 0, 1, \dots, K + 1, \quad (11)$$

where $\mu_0 = 1$. If $K = \infty$, G is uniquely determined. This follows from a simple change of variables in the Hausdorff moment problem, which is concerned with distributions concentrated on $[0, 1]$ (Shohat and Tamarkin, 1943). From a finite sequence of moment conditions bounds on G can be constructed. Note that the additional requirement that $\Pr(0 < \theta(t|\tau, \mathbf{v}) < 1) > 0$ puts mild further restrictions on the moment sequence, like $0 < \mu_1 < 1$.

Since complete monotonicity is not imposed on the moments while estimating the model, we can test for it. More precisely, if we do not require G to be a distribution function on $[0, \bar{v}]$, but allow G to be any function of bounded variation, the moments can take any value on the real line (Shohat and Tamarkin, 1943). Then, equation (7) specifies a more general class of models for the exit probabilities, which only requires a subset of the conditions introduced in this subsection to ensure that $0 < \theta(t|\tau) < 1$. Furthermore, the proof of Proposition 1 only relies on this subset of conditions. Thus, there exist moment sequences that correspond to $0 < \theta(t|\tau) < 1$, but that are not completely monotone. Furthermore, none of these moment sequences are observationally equivalent to a completely monotone moment sequence. This justifies testing for complete monotonicity.

In general, the test statistics implied by the inequalities are not very appealing. First, the upper bound equals the infimum of a number of different functions of parameters, so the distribution theory is non-standard. Moreover, they do not only depend on the μ_i estimates but also on the estimates of the other parameters of the model. For these reasons we do not construct tests from the complete monotonicity requirement directly.

Instead, we first focus on the lower bound of zero only. A necessary condition for the existence of a distribution with nonnegative support with moments $\{\mu_i\}_{i=1}^{K+1}$ is that the moment matrices $(\mu_{i+j})_{i,j=0\dots k_e}$ and $(\mu_{i+j+1})_{i,j=0\dots k_o}$ are positive semi-definite, for $k_e = 0, 1, \dots, [(K+1)/2]$ and $k_o = 0, 1, \dots, [K/2]$ (Shohat and Tamarkin, 1943). For $K = 4$,

this implies the following constraints on γ :

$$\begin{aligned}
q_2(\gamma) &= \gamma_2 - 1 \geq 0, \\
q_3(\gamma) &= \gamma_3 - \gamma_2^2 \geq 0, \\
q_4(\gamma) &= \gamma_2\gamma_4 - \gamma_3^2 - \gamma_4 - \gamma_2^3 + 2\gamma_2\gamma_3 \geq 0, \text{ and} \\
q_5(\gamma) &= \gamma_3\gamma_5 - \gamma_4^2 - \gamma_2^2\gamma_5 - \gamma_3^3 + 2\gamma_2\gamma_3\gamma_4 \geq 0.
\end{aligned} \tag{12}$$

If any of these inequalities is violated then no distribution with nonnegative support is able to generate these γ as normalized moments. For example, $q_2(\gamma) < 0$ would imply $\text{var}(\mathbf{v}) < 0$ and $q_3(\gamma) < 0$ would imply $\Pr(\mathbf{v} < 0) > 0$. So, a relatively simple procedure is to test null hypotheses $q_i(\gamma) \geq 0$ against alternatives $q_i(\gamma) < 0$ using one-sided pseudo- t -tests, separately for $i = 2, \dots, 5$. We do not perform joint tests. Joint tests are less straightforward due to the inequality constraints involved.

We also check a number of the remaining conditions, which are implied by the parameter restrictions that follow from the finite upper bound. For example, from equation (7) it is clear that $\bar{v} \leq \theta(0|\tau)/\mu_1$, so that (11) should at least hold for an upper bound of $\theta(0|\tau)/\mu_1$. Substituting in (11) and evaluating for $j = 1$ gives $\theta(0|\tau) \leq \gamma_2/\gamma_3$, $\theta(0|\tau) \leq \gamma_3/\gamma_4$, etcetera. By comparing the estimates of the right-hand sides of these inequalities to the observed $\tilde{\theta}(0|\tau)$, and taking account of standard errors, one can get a feeling on whether these inequalities hold.

The moment tests proposed above are informative on the validity of Assumption 1. Suppose that in reality $\theta(t|\tau, \mathbf{v})$ is not multiplicative in t , τ and \mathbf{v} , but instead contains interaction terms. Then, in particular cases, this shows up in the γ estimates being inconsistent with the moment restrictions above. For example, suppose that the duration dependence pattern for individuals with large v differs from that for individuals with small v in the following way: $\theta(t|\tau, \mathbf{v}) = \psi_1(t, \mathbf{v})\psi_2(\tau)$ with $\psi_1(0, \mathbf{v}) = \mathbf{v}$ and $\psi_1(1, \mathbf{v}) = 1/\mathbf{v}$. It can be shown that this generates an interaction effect similar to that generated by $\gamma_2 < 1$. Also, the tests may detect misspecification of the unit of time period. If in reality the model is correct for monthly periods but it is assumed to be correct for quarterly periods, then this may turn up in the γ estimates being inconsistent as moments of a distribution with bounded nonnegative support.

3 Data

We use quarterly unemployment data over the period 1982.IV–1994.IV, collected by the French public employment offices (A.N.P.E.), and subsequently collected on a nation-wide scale by the Department of Labor of the Ministry of Social Affairs and Employment (see ILO, 1989, for an extensive description). The data provide the number of unemployed at the *last* day of each quarter who have completed a given number of quarters of unemployment duration in their current spell. So, for example, the data for 1982.IV ($\tilde{U}(t|\tau_0)$) provide the number of individuals who have been unemployed for more than t and less

than $t + 1$ quarters at December 31, 1982, and the data for 1983.I ($\tilde{U}(t|\tau_0 + 1)$) similar information for March 31, 1983, etcetera. Unemployment here includes all individuals without employment who are immediately available for employment and actively searching for full-time permanent jobs. ‘Immediately’ here means within 15 days, and ‘full-time’ refers to more than 30 hours per week.

A simple regression of log unemployment on a linear trend reveals 4.0% (standard error 0.2%) and 2.8% (0.4%) yearly trends for females and males, respectively. Figure 1 shows female and male unemployment corrected for these trends and (multiplicative) seasonal effects. The overall pattern for males and females is similar. Clear peaks in unemployment can be identified at the end of 1984, early 1987, and late 1993/early 1994. Major troughs in unemployment are found early 1983, late 1985/early 1986, and late 1990. If anything, female unemployment lags one quarter after male unemployment. Fluctuations are stronger for males, in particular in the second half of the data period.

We relate cyclical patterns to two conventional business cycle indicators. The first is based on the capacity utilization ratio (CUR) provided by the OECD (Main Economic Indicators). We transform CUR into $\ln(\text{CUR}/(1 - \text{CUR}))$, and then deseasonalize the resulting series. The second is real GDP-growth, which is computed as the growth in real GDP between the current quarter and the same quarter one year earlier ($\times 10\%$; source: Comptes Nationaux, INSEE). Figure 2 graphs the resulting series, detrended and in deviation from their respective means. Note that, unlike the unemployment series, the CUR series are measured at the last day of the first month of each quarter. For convenience, we plot all series at the last day of each quarter, the measurement date of the quarterly unemployment series. We have also used a different time scale than in Figure 1, as we will experiment with lags and leads in our analysis. Both series roughly agree on the business cycle, but the subtle differences warrant using both in our analysis. A comparison with Figure 1 confirms that unemployment is countercyclical.

As explained in Subsection 2.1, our model and estimation method are designed to be applicable to quarterly exit probabilities computed from the discrete time unemployment duration data. Consistent with this design we use the number of unemployed in the first duration class ($U(0|\tau)$) as the measure of the size of the inflow into unemployment. The corresponding discrete time measure of the outflow from unemployment at time τ is the difference between the inflow into unemployment at time $\tau + 1$ and the growth of the stock of unemployed between τ and $\tau + 1$, or $U(0|\tau + 1) + U(\tau) - U(\tau + 1)$. The outflow rate is found by dividing by $U(\tau)$.

Figure 3 shows the development of the inflow into unemployment after adjusting for seasonal effects. Again, the general pattern in the fluctuations is the same for males and females. However, the trends are clearly different. In the beginning of the 1980s, the female inflow starts well below the male inflow, but, although the inflow of males into unemployment is slightly increasing over time, the inflow of females shows a much stronger increase. In the beginning of the 1990s, male and female inflow figures are roughly similar.

Then, the male inflow again rises above the female inflow.

Figure 4 shows the development of the outflow rate from unemployment. The cyclical patterns are similar for males and females. Overall, male exit probabilities are higher than female exit probabilities, but a steeper downward trend brings male exit probabilities down to the level of females in the early 1990s. Over the data period, the average quarterly outflow from male unemployment is 30.0% of total male unemployment, the average inflow is 30.7%. For females these figures are 26.5% for the outflow and 27.0% for the inflow.

The discrete time inflow measure that we use is smaller than the continuous time inflow because it excludes the persons who enter and leave unemployment between two measurement points. In the literature both measurement methods have been used. For example, Sider (1985) uses the latter whereas Layard, Nickell and Jackman (1991) use the same method as we do. According to Jackman and Layard (1991) the different measures exhibit similar behavior over the business cycle. In our case, from additional analysis it appears that many of the dynamic features of both series are similar, with seasonal fluctuations dominating cyclical and secular developments. It does seem that in the late 1980s the true inflow has increased more than the number of unemployed in the first duration class. This may however be caused by a change in the data collection procedure. We return to this issue below.

Figure 5 plots the exit probabilities out of unemployment as a function of unemployment duration, for different points in calendar time. At every point in time, the exit probability declines over the duration of unemployment. This decline can be due to unobserved heterogeneity, individual negative duration dependence, or a combination of both. Note, however, that the decline in the exit probability over the duration of unemployment is steeper in quarters in which the exit probability is larger to start with. This suggests that unobserved heterogeneity is an important factor in French unemployment dynamics. If the exit probability from the first duration class is high, then the dynamic sorting within a cohort of unemployed is faster, causing the average exit probability to decline more rapidly over the duration of unemployment (Abbring *et al.*, 2001, provide more discussion).

In 1986, some details of the procedure according to which the data are collected were changed. As a result, the time series on $\tilde{U}(t|\tau)$ exhibit ruptures at 1986.IV. This turns out to be particularly important for the series on $\tilde{U}(0|\tau)$. Further, the French policy towards youth unemployment changed substantially in the mid-1980s as well. The new policy basically entailed that young individuals were assigned to training jobs shortly after entering unemployment. This may be expected to affect the exit probability out of the first duration class $\theta(0|\tau)$. For these reasons, we add to the model a dummy variable $d(\tau)$ equaling one if τ is after 1986.IV and zero otherwise. Specifically, we multiply the expressions for $U(0|\tau)$ and $\theta(0|\tau)$ in the corresponding model equations by $\exp(d_{\geq 87} \cdot d(\tau))$, in which $d_{\geq 87}$ is a parameter to be estimated. Although this notation suggests otherwise, we do not impose $d_{\geq 87}$ to be the same in the equation for $U(0|\tau)$ and the equation

for $\theta(0|\tau)$. The results turn out to be insensitive with respect to small changes of the calendar time point defining the areas in which the dummy variable equals zero and one, respectively.

4 Estimation results

4.1 Some preliminary issues

We estimate our model using observations of unemployment in the first six duration quarters. This covers between 77.5% and 86.1% of the female unemployment stock, and between 78.9% and 87.9% of the male unemployment stock over the data period. From these observations $\tilde{U}(0|\tau), \dots, \tilde{U}(5|\tau)$ we can compute five quarterly exit probabilities from unemployment (*i.e.*, $K = 4$), and estimate a five equation duration model, as given by equation (7), for each sex separately. It turns out to be difficult to estimate models that include equations for exit probabilities from higher duration classes. Note that, as t increases, the degree of complexity and nonlinearity of $\theta(t|\tau)$ as a function of the parameters increases enormously. Below we show, however, that the estimation results can be used to make certain inferences on unemployment dynamics in higher duration classes. Estimation of incidence equation (8), using incidence observations $\tilde{U}(0|\tau)$, completes the estimation.

In order to detect the correlation structure of the measurement errors $\ln \varepsilon_{t,\tau}$ over t and τ we analyze the estimates of the errors $e_{t,\tau}$ of the equations (3), from an estimation with supposedly uncorrelated measurement errors. To make inferences, the correlation structures of the second type of errors has to be expressed in terms of those of the first type. If the $\varepsilon_{t,\tau}$ are i.i.d. then the only nonzero correlation in the $e_{t,\tau}$ concerns $\text{corr}(e_{t,\tau}, e_{t+1,\tau+1})$, which equals $-\frac{1}{2}$ for every t and τ . As argued in Section 2, there are various reasons for $\ln \varepsilon_{t,\tau}$ and $\ln \varepsilon_{t+1,\tau}$ to be correlated for every t ; some of these reasons implying a positive correlation and other a negative. In such cases the largest correlation between the errors of the equations (3), apart from the one noted above, is $\text{corr}(e_{t,\tau}, e_{t+1,\tau})$, which has the same value as $\text{corr}(\ln \varepsilon_{t,\tau}, \ln \varepsilon_{t+1,\tau})$. Similar results can be derived for other a priori plausible correlation schemes.

The residual analysis seems to suggest that there are nonzero (positive) correlations between measurement errors $\ln \varepsilon_{t,\tau}$ at one single calendar moment. We find no evidence for other correlation schemes like serial correlation over calendar time. Thus, we specify $\text{corr}(\ln \varepsilon_{t^*,\tau^*}, \ln \varepsilon_{t^{**},\tau^{**}}) = \rho^{|t^*-t^{**}|}$ if $\tau^* = \tau^{**}$, and 0 otherwise, with $-1 < \rho < 1$.

Table 1 gives estimates of the duration model with correlated measurement errors for both males and females. Residual analysis based on these estimates supports the specification of the measurement errors. The estimates of the correlation parameter indicate that the errors are positively correlated across duration classes at one calendar moment. We may conclude that misclassification of unemployed individuals into wrong duration classes

is not a major source of errors in the observed unemployment figures. Indeed, standard deviations of the measurement errors slightly below 0.02 show that measurement errors in unemployment numbers are generally small. Durbin-Watson statistics are satisfactory. The pseudo- R^2 statistics reveal a very good fit, especially for the lower duration classes and for males. Wald tests (not reported) show that each of the duration, cyclical and seasonal components in the exit probabilities is (jointly) significant at all conventional levels.

Recall that we let τ_0 coincide with the first season of a model year. Consistent with this setup, we have labeled the years by 1982.IV–1983.III, 1983.IV–1984.III, \dots , instead of the more generic $0, 1, \dots$ used before. Also, we denote the seasons by IV, I, II, III instead of the more generic $1, \dots, S$.

In the remainder of this section, we first discuss the estimates of the duration dependence and heterogeneity parameters, and the results of the model specification tests. Next, we include the estimation results of the incidence model in Table 2 in the discussion, and analyze the various calendar time effects.

4.2 Unobserved heterogeneity and duration dependence

The estimates of the individual duration dependence parameters indicate that there is significant non-monotonous duration dependence for both sexes. Individual female unemployed face a significant 17% rise in their exit probability after one quarter of unemployment, a small return to about 104% of the initial level in the next quarter, and further increases in the next two quarters, *ceteris paribus*. Male individual duration dependence follows a qualitatively similar pattern, except for some negative duration dependence between quarters 2 and 3. Overall, male duration dependence is slightly more negative. Clearly, these results are not consistent with stigma, loss of skills, or demotivation effects on exit probabilities in the first five quarters of unemployment. Furthermore, these non-parametric estimates are not compatible with frequently used monotonous parameterizations of genuine duration dependence like the Weibull function. Lollivier (1994) finds that there is genuine negative duration dependence from duration zero onwards, and that it is particularly strong when going from the second to the third quarter. This is in accordance to our results for quarters 1 and 2.

The results on genuine duration dependence imply that the decrease of the observed exit probabilities during the first five quarters of unemployment are mainly due to unobserved heterogeneity. Indeed, the estimates in the first five rows of Table 1 imply significant heterogeneity. For both males and females the second normalized moment γ_2 is significantly larger than 1, which implies a positive variance of the heterogeneity distribution G . This is consistent with the higher normalized moments being significantly larger than one.

As the resulting dynamic selection is stronger when exit probabilities are larger, the

aggregate exit probabilities decrease faster with duration in the top of the cycle than in the bottom. Figure 6 illustrates this effect. It plots the log of the expected value of the unobserved heterogeneity term conditional on survival at two different states of the business cycle. For expositional purposes, both series are normalized to 0 at $t = 0$. To understand how we constructed these series, first recall that the relevant conditional expectation is the fourth factor in the r.h.s. of equation (7). Note that it depends on all duration and time effects. To isolate the effect of dynamic sorting, we omit all duration dependence and seasonal effects, and take the business cycle effects to be constant across time and duration. In other words, we take $\psi_1(t)\psi_2(\tau)\psi_3(\tau - t)$ to be constant across τ and t . The two series in Figure 6 correspond to levels of $\psi_1(t)\psi_2(\tau)\psi_3(\tau - t)$ that reflect the top and the bottom of the cycle.¹ As expected, the decrease of the exit probabilities is stronger at the top of the cycle. It turns out that the selection due to heterogeneity is most severe in the early stages of unemployment, although there seems to be a further large selection effect in the last duration classes. The latter effect may not be significant, given the large standard errors of the estimates of the higher moments. Indeed, the moment-inequality statistics show that we can restrict the moments to belong to discrete distributions with two points of support, in which case the latter effect disappears (see Shohat and Tamarkin, 1943, and Lindsay, 1989).

We can also exploit this last result to informally extend our analysis to higher duration classes, without estimating a fully non-parametric model on data covering these duration classes. In Figure 5 we have seen that aggregate exit probabilities continue to decrease after the first 5 duration classes, to which we have restricted our analysis so far. To provide a first impression of the role of unobserved heterogeneity and genuine duration dependence in explaining this negative duration dependence beyond the first 1.5 years of unemployment, we extrapolate our model in the following way. First, we estimate the parameters of the discrete heterogeneity distributions. Fixing all but the heterogeneity parameters at their non-parametric estimates, and maximizing the restricted likelihood with respect to the heterogeneity parameters only, we find shares of 82.5% at 0.217 and 17.5% at 0.541 for females, and masses of 87.7% at 0.274 and 13.3% at 0.679 for males.² Next, if we assume that the higher moments, which we have not considered so far, also correspond to the same discrete distributions, we can extrapolate the effects of dynamic sorting beyond the fifth duration class. We find that most of the dynamic selection due to the two point heterogeneity is completed at the sixth quarter. As there is substantial observed negative duration dependence after the sixth quarter, we conclude that individual duration dependence turns negative after 1.5 years of unemployment.

Finally, we turn to the moment-inequality specification tests we proposed in Subsection 2.5. Five of the eight statistics based on equations (12) are actually positive, and therefore

¹We fix the log business cycle effect at one standard deviation of $\alpha_2(y(\tau)) + \alpha_3(y(\tau))$ below and above the mean value of these series. The standard deviations used are 0.091 for females and 0.106 for males.

²Similar results are found if we use a simple minimum distance procedure.

consistent (as point estimates) with the null. Of the remaining three statistics, even the lowest still has a p -value as high as 0.15. So, neither of the tests results in rejection at conventional levels, for males and females alike. This is an important result since, as we have seen, these moment-inequality tests have power against a wide range of model alternatives. As was noted in Subsection 2.5, the model also implies bounds for the exit probability out of the first duration class in terms of γ . The standard errors of these bounds turn out to be rather high, and observed exit probabilities out of the first duration class are well within the confidence intervals of these upper bounds. This again supports our MPH model.

4.3 Trends and cycles

There are significant trends in both the exit probabilities and the incidence. Table 1 shows that both female and male exit probabilities trend downward, at rates equal to -2.0% and -2.4% yearly, respectively. Table 2 lists the estimates of the incidence equation. There are significant positive trends in both female ($2.7\%/year$) and male ($1.8\%/year$) incidence. Note that the female trend in the incidence is larger, whereas the male trend in the outflow dominates.³ A possible explanation for the stronger trend in the female incidence is the development of labor market participation: whereas the female participation rate grew by a few percentage points, the male rate fell somewhat in the data period considered.

In Subsection 4.1, we have already seen that there is significant cyclical variation in the contemporaneous and cohort components of the exit probabilities. The Wald statistics for the incidence components in Table 2 show that there is also significant cyclical variation in the male incidence. The cycle in the female incidence, however, is not significant at a 5% level.

Figure 7 graphs the contemporaneous cycles in the exit probabilities. We can compare these to the business cycle indicators in Figure 2. Both cycles are roughly procyclical, decreasing in the early 1980s, increasing in the late 1980s, and again decreasing in the early 1990s. However, the initial fall in female exit probabilities is sharper and longer, and female probabilities seem to recover earlier in the 1990s. We will see later that the resulting pattern has an ambiguous relation to the business indicators.

The cohort cycles in Figure 8 are less pronounced, and, as we will confirm below, procyclical. The male cycle is particularly weak. Recall that these cycles are thought to capture cyclical variation in the unobserved composition of the inflow into unemployment.

Figure 9 shows the estimated cyclical variation of the incidence. Cyclical variation is fairly limited compared to the seasonal fluctuations and the trends. The female and male cycle are similar until 1987/1988, and agree on a (countercyclical) decrease in the incidence from 1993 onwards. However, only male incidence is clearly countercyclical in the period between 1988 and 1993. The female cycle is fairly flat, and slopes slightly

³See Cohen and Lefranc (1994) for an analysis of trends in French unemployment.

downward in the same period. Below, we present some evidence that this adds up to procyclical female incidence.

We further illustrate the cyclical patterns by regressions on the two business cycle indicators of Section 3, the inverse-logit transformed CUR and GDP growth ($\times 10\%$). As the cycles in our model only exhibit yearly variation, we create yearly indicator series by selecting data on one particular season in each of the years only, and discarding the data on the remaining 3 seasons. Different choices of the first, base season give different quarterly lags and leads. We define the yearly indicator series of which the first observation coincides with the first quarter in the unemployment data, 1982.IV, to be the “concurrent” series, and number lags and leads accordingly. In the regressions, we experiment with various lags and leads, and we include a trend because the indicators are not detrended.

We first discuss the regression results for males, which are clear-cut. The male contemporaneous cycle is well explained by the 5 quarter leading CUR, which gives an estimate on the CUR of 0.34 (standard error: 0.06) and an adjusted R -squared (R_a^2) of 0.71. Alternatively, a regression on concurrent real GDP growth gives an estimate of 0.38 (0.09) and $R_a^2 = 0.60$. These estimates are robust with respect to the choice of the lag or lead among the regressions that have some explanatory power. In general, this holds for the regressions discussed in the sequel. A regression of the male cohort cycle on 4 quarters lagged GDP growth gives an estimate of 0.08 (0.03), with $R_a^2 = 0.26$. For the CUR, we find estimates in the range 0.03–0.05 (0.02–0.03) for various leads and lags, with low R_a^2 's. Finally, we find a clear countercyclical pattern in the incidence. A CUR leading two quarters and concurrent GDP growth give parameter estimates of -0.20 (0.07) and -0.18 (0.08), and $R_a^2 = 0.34$ and 0.19, respectively.

For females, the contemporaneous cycle is not tracked well by the indicators, thus challenging our earlier conclusion from inspecting Figure 7. We only find relatively weak effects at 2 year lags (CUR) and leads (GDP), of ambiguous sign, and with large standard errors and low R_a^2 's. We find an unambiguous result for the cohort cycle. The cohort effects are clearly procyclical, with an estimate of 0.23 (0.06) on the 2 quarters leading CU ratio ($R_a^2 = 0.58$), and an estimate of 0.24 (0.07) with $R_a^2 = 0.45$ for concurrent GDP growth. A remarkable result is that the female incidence is (weakly) procyclical. A variety of lags and leads gives CUR effects in the range of 0.13–0.15 (0.05–0.07), with R_a^2 's around 0.1–0.3. Moreover, the estimate of the effect of four quarters lagged GDP growth is 0.21 (0.07), with $R_a^2 = 0.39$. This surprising result can possibly be traced back to a relatively high share of labor market entrants, as opposed to job losers, in female unemployment incidence. Also, the relatively high incidence in the top of the cycle may lead to congestion effects, which could explain the ambiguous results for the contemporaneous exit probability cycle.

In sum, the business cycle influences unemployment foremost by procyclical effects on individual outflow probabilities. For males, procyclical contemporaneous effects on the exit probability are dominant, but we also find a significant contribution of the incidence

to countercyclical unemployment fluctuations. There is only a very small procyclical cohort effect on the exit probabilities. For females, contemporaneous exit probability effects are ambiguous and the cyclicity of the incidence seems to be dampening the cyclicity of unemployment. We find stronger, procyclical cohort effects than for males. These effects, however, mainly follow the overall increase in the indicators until the start of the 1990s, and the overall decrease thereafter, and does not explain much of the more subtle fluctuations in female unemployment.

It may be interesting to compare these results to other studies, even though most of those are based on U.S. or U.K. data and some of them use data that cover only a small time span and/or only a specific subset of the population. There have been numerous studies examining the relative importance of incidence and duration to explain variation in unemployment. The evidence is mixed, and results differ between different countries and time periods. Layard, Nickell and Jackman (1991) present a survey based on aggregate data, from which it can be concluded that for most European countries, including France, the variation in unemployment duration is more important than the variation in incidence in explaining total variation in unemployment over the business cycle (see also Sider, 1985, and Pissarides, 1986).

There has also been some debate on whether the business cycle effect on durations works by way of an effect on the composition of the inflow or by way of a direct effect on the outflow probabilities. Darby, Haltiwanger and Plant (1985) argue that two groups of individuals can be distinguished, one group with high transition rates into and out of unemployment, and one with a high degree of specific human capital and with long-duration jobs and high unemployment durations. In a recession, firms in declining industries find it optimal to accelerate labor force reductions, and the inflow into unemployment will consist to a relatively large extent of individuals in the second group. In terms of our model this means that α_3^c should vary procyclically. However, we find that male cohort effects are very small, and that female cohort effects, although somewhat larger, only roughly track the business cycle. This is consistent with the empirical work of Dynarski and Sheffrin (1990), Imbens and Lynch (1992) and Baker (1992a). The first two studies use micro data while the third uses aggregate data containing a large number of observed explanatory variables. In these cases, certain attributes of the inflow are directly observed. All conclude that the business cycle affects individual outflow probabilities, and, in the case of the two last-mentioned papers, that the composition of the inflow is more or less constant, so that the business cycle effect on unemployment duration works primarily by affecting the individual outflow probabilities contemporaneously.

In the empirical literature there is a large agreement on the sign of the relation between observed outflow probabilities and the state of the business cycle. Butler and McDonald (1986), Dynarski and Sheffrin (1990), Imbens and Lynch (1992), Baker (1992a) and Lollivier (1994) all conclude that this sign is positive, and that therefore individual outflow probabilities are procyclical. This result is justified theoretically in Van den Berg (1994),

who generalizes previous theoretical papers by showing that in job search models the job offer arrival rate has a positive effect on the exit rate out of unemployment, under almost every possible configuration of model determinants. This is consistent with our findings for males. We have given some reasons for the ambiguous results for females above.

4.4 Seasonal effects

Finally, Tables 1 and 2 provide estimates of the seasonal effects in the components of unemployment. Note that the discrete incidence measure $U(0|\tau)$ can actually be thought to correspond to inflow between $\tau - 1$ and τ , whereas the outflow at τ , and the exit probabilities $\theta(t|\tau)$, refer to exits between τ and $\tau + 1$ instead. This has implications for the interpretation of seasonal patterns. For instance, the seasonal effects on the size ($\omega_4(\text{IV})$) and composition ($\omega_3(\text{IV})$) of the inflow at December 31 correspond to the inflow during the fourth quarter, whereas the contemporaneous seasonal effect on the exit probability ($\omega_2(\text{IV})$) affects the outflow during the first quarter of the next year. To facilitate reading of the tables, we have reordered the contemporaneous seasonal effects, and list quarter IV first. Furthermore, we have added the appropriate interpretation in parentheses.

There are strong and highly significant seasonal fluctuations in the incidence. For females (males) the incidence in the top season is 50% (39%) higher than the incidence in the bottom season. Incidence is relatively large in the second half of the year. One obvious reason for this is the fact that individuals usually leave school and enter the labor market in the third quarter.

We also find less pronounced but significant seasonal variation of the exit probabilities. Exit probabilities are 9–10% higher in the top contemporaneous and cohort seasons for females, and 9% higher in the top seasonal cohort and 15% higher in the top contemporaneous season for males. Relatively many exits occur during April-June (quarter I) and relatively few during January-March (quarter IV) of each year. The composition of the inflow is relatively favorable in the last two quarters of the year. This is not surprising since the inflow in those quarters consists to a large extent of individuals leaving school.

Note that the seasonal effects are rather similar between males and females. The main difference is that female incidence is relatively high in the third quarter. This may be due to layoffs of females from summertime service sector jobs. For females, seasonal fluctuations in the incidence are relatively large compared to the fluctuations in the exit probabilities.

There is a positive relation between the size and the quality of a cohort over seasons: if the inflow is large, the average exit probability of the inflowing cohort is high and *vice versa*. This is not an obvious relationship, as one might expect more competition for jobs between the individuals in larger cohorts, and therefore smaller individual exit probabilities for larger cohorts. However, if individuals also compete for jobs with individuals from

other cohorts, then one should expect the overall exit probabilities to decline in or just after large cohort seasons. The results provide some evidence for the latter: for instance, male incidence is much lower during the first half the year, and the exit probability is relatively high in April-June (quarter I). However, high exit probabilities in April-June can also be due to inflow into summer employment. Finally, the seasonal inflow in the third quarter is largely due to school leavers, which are more than averagely equipped to find a job, and thus have a higher than average exit probability.

5 Conclusion

In this paper we shed a light on unemployment dynamics by decomposing aggregate unemployment data. We develop and estimate a flexible model which allows the size of the inflow, the composition of the inflow, and the individual exit probabilities to depend on the state of the business cycle as well as the prevailing season. Moreover, we allow for unobserved heterogeneity and measurement errors in the data. We prove non-parametric identification of the model and develop and apply specification tests. We apply the method to quarterly French unemployment data on the period 1982.IV–1994.IV. The model specification is accepted and the results are robust with respect to various assumptions. The main empirical findings are the following.

First, upward trends in unemployment are both due to significant downward trends in exit probabilities and noticeable upward trends in the incidence. The incidence trend is particularly strong for females, possibly due to the relative increase in female labor force participation. The exit probability trend is somewhat stronger for males.

Business cycle effects are primarily found in the exit probabilities and to a lesser extent in the size of the inflow. As for the trends, we find notable differences between males and females. For males, we find strong procyclical exit probabilities within cohorts. Also, the incidence is clearly countercyclical. Cohorts entering at the top of the cycle have higher exit probabilities, but these effects are very weak. In contrast, contemporaneous cyclical effects on female exit probabilities are ambiguous. However, the female cohort effect is procyclical, although it only roughly tracks the business cycle indicators. This remarkable finding is coupled with, if anything, procyclical incidence. This may be due to the relatively large share of labor market entrants.

Across sexes, there are very large seasonal effects on the size of the inflow. In top seasons, the incidence can be as much as 50% higher than in bottom seasons. There are also smaller seasonal effects on the average quality of the inflow into unemployment and on the individual exit probabilities. Variation in the incidence is relatively important for females. Also, the size of the inflow in any season is positively related to the average quality of the inflow.

There is no negative duration dependence of individual exit probabilities during the

first 1.5 years of unemployment. Thus, stigma effects seem to be absent at those durations, and the decrease of the observed exit probabilities is almost completely due to the dynamic selection induced by unobserved heterogeneity. It turns out that this dynamic selection is mostly completed by the end of the 1.5 year period. This suggests that the negative duration dependence observed at higher durations can be traced back to negative individual duration dependence. Finally, we find that observed exit probabilities decrease less fast in a recession than in the top of the cycle. This is a robust feature of the data that is consistent with our model of dynamic sorting because of unobserved heterogeneity. It contradicts alternative models that predict an opposite interaction, like the ranking model (Blanchard and Diamond, 1994).

The relevance of the empirical results is threefold. First of all, the results on business cycle effects have implications for the plausibility of existing as well as future theoretical macroeconomic models of unemployment. Second, the magnitude of the business cycle and seasonal effects on the exit probabilities is such that they should be taken into account in standard micro-econometric unemployment duration analyses. Third, the results on the way in which business cycles affect unemployment, and the results on the role of unobserved heterogeneity versus genuine duration dependence, are of interest for unemployment policy.

Appendix: Proof of Proposition 1

We first introduce some notation. Let μ be the moment operator with respect to the moment sequence $\{\mu_i\}$, i.e. $\mu(\sum_i \beta_i v^i) = \sum_i \beta_i \mu_i$ for any sequence of constants $\{\beta_i\}$ (Widder, 1946). Similarly, let $\hat{\mu}$ be the moment operator with respect to the sequence $\{\hat{\mu}_i\}$. Define

$$\nu(\tau, t) = \frac{\mu\left(v \prod_{i=0}^{t-1} (1 - \psi_1(i)\psi_2(\tau - t + i)\psi_3(\tau - t)v)\right)}{\mu\left(\prod_{i=0}^{t-1} (1 - \psi_1(i)\psi_2(\tau - t + i)\psi_3(\tau - t)v)\right)},$$

and similarly define $\hat{\nu}$ in terms of $\hat{\mu}$, $\hat{\psi}_1$, $\hat{\psi}_2$, and $\hat{\psi}_3$. Finally, let $H(\tau, t) = \prod_{i=0}^t (1 - \theta(\tau - t + i, i))$ and $\hat{H}(\tau, t) = \prod_{i=0}^t (1 - \hat{\theta}(\tau - t + i, i))$.

Observational equivalence of ψ and $\hat{\psi}$ implies that

$$\forall(\tau, t) \in \mathcal{T}_{N,K} : \psi_1(t)\psi_2(\tau)\psi_3(\tau - t)\nu(\tau, t) = \hat{\psi}_1(t)\hat{\psi}_2(\tau)\hat{\psi}_3(\tau - t)\hat{\nu}(\tau, t), \quad (13)$$

and, equivalently, that $\forall(\tau, t) \in \mathcal{T}_{N,K} : H(\tau, t) = \hat{H}(\tau, t)$. Note that Assumptions 1–3 imply that $0 < \theta(\tau, t) < 1$ and, by implication, $0 < H(\tau, t) < 1$ for all $(\tau, t) \in \mathcal{T}_{N,K}$. Our proof proceeds in 3 steps.

- (i). Because of Assumption 5 it is possible to find a $\tau_1, \tau_1 + S \in T_N(1) : \theta(\tau_1 - 1, 0) \neq \hat{\theta}(\tau_1 - 1 + S, 0)$. Let $\tau^* = \tau_1$ if $s(\tau_1 - 1) = 1$, and $\tau^* = \tau_1 - 1$ otherwise. Because of Assumption 4, evaluating (13) at τ^* and $\tau^* + S$ and dividing gives

$$\frac{\alpha_2(\tau^*)\alpha_3(\tau^* - t)\nu(\tau^*, t)}{\alpha_2(\tau^* + S)\alpha_3(\tau^* + S - t)\nu(\tau^* + S, t)} = \frac{\hat{\alpha}_2(\tau^*)\hat{\alpha}_3(\tau^* - t)\hat{\nu}(\tau^*, t)}{\hat{\alpha}_2(\tau^* + S)\hat{\alpha}_3(\tau^* + S - t)\hat{\nu}(\tau^* + S, t)},$$

which, evaluated at $t = 0$ and $t = 1$, implies that

$$\frac{\nu(\tau^*, 1)}{\nu(\tau^* + S, 1)} = \frac{\hat{\nu}(\tau^*, 1)}{\hat{\nu}(\tau^* + S, 1)},$$

as $y(\tau^*) = y(\tau^* - 1)$ and $y(\tau^* + S) = y(\tau^* + S - 1)$. Multiplying by $H(\tau^*, 0)/H(\tau^* + S, 0) = \hat{H}(\tau^*, 0)/\hat{H}(\tau^* + S, 0)$, and substituting $\theta(\tau, 0)$ and $\hat{\theta}(\tau, 0)$ gives

$$\frac{1 - \theta(\tau^*, 0)\gamma_2}{1 - \theta(\tau^* + S, 0)\gamma_2} = \frac{1 - \hat{\theta}(\tau^*, 0)\hat{\gamma}_2}{1 - \hat{\theta}(\tau^* + S, 0)\hat{\gamma}_2}. \quad (14)$$

The left and right hand sides of (14) are the same function of γ_2 and $\hat{\gamma}_2$, respectively. Under Assumption 5 this function is strictly monotonous, implying that $\gamma_2 = \hat{\gamma}_2$.

- (ii). In turn, as we can write

$$\frac{\nu(\tau, t)}{\mu_1} = \frac{\mu\left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1}\right)\right)}{\mu\left(\prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1}\right)\right)}, \quad (15)$$

and similarly express $\hat{\nu}(\tau, t)/\hat{\mu}_1$ in terms of $\hat{\mu}_1$, $\hat{\nu}$ and $\hat{\mu}$, this implies that $\forall \tau \in T_N : \nu(\tau, 1)/\mu_1 = \hat{\nu}(\tau, 1)/\hat{\mu}_1$. Thus, for $t = 0$ and $t = 1$, (13) can be reduced to

$$\forall \tau \in T_N(t) : \psi_1(t)\psi_2(\tau)\psi_3(\tau - t)\mu_1 = \hat{\psi}_1(t)\hat{\psi}_2(\tau)\hat{\psi}_3(\tau - t)\hat{\mu}_1. \quad (16)$$

Evaluating (16) at any $\tau, \tau' \in T_N(t)$ and dividing gives

$$\forall \tau, \tau' \in T_N(0) : \frac{\psi_2(\tau)\psi_3(\tau)}{\psi_2(\tau')\psi_3(\tau')} = \frac{\hat{\psi}_2(\tau)\hat{\psi}_3(\tau)}{\hat{\psi}_2(\tau')\hat{\psi}_3(\tau')} \text{ for } t = 0, \text{ and} \quad (17)$$

$$\forall \tau, \tau' \in T_N(1) : \frac{\psi_2(\tau)\psi_3(\tau-1)}{\psi_2(\tau')\psi_3(\tau'-1)} = \frac{\hat{\psi}_2(\tau)\hat{\psi}_3(\tau-1)}{\hat{\psi}_2(\tau')\hat{\psi}_3(\tau'-1)} \text{ for } t = 1. \quad (18)$$

Substituting (18) in (17) and iterating gives

$$\forall \tau, \tau' \in T_N(t) : \frac{\psi_3(\tau)/\psi_3(\tau-t)}{\psi_3(\tau')/\psi_3(\tau'-t)} = \frac{\hat{\psi}_3(\tau)/\hat{\psi}_3(\tau-t)}{\hat{\psi}_3(\tau')/\hat{\psi}_3(\tau'-t)} \text{ for any } t \in D_K. \quad (19)$$

Evaluating (13) at any $(\tau, t), (\tau', t) \in \mathcal{T}_{N,K}$, dividing, and substituting (19) and (17) shows that

$$\forall (\tau, t), (\tau', t) \in \mathcal{T}_{N,K} : \frac{\nu(\tau', t)}{\nu(\tau, t)} = \frac{\hat{\nu}(\tau', t)}{\hat{\nu}(\tau, t)}.$$

Multiplying by $H(\tau', t)/H(\tau, t) = \hat{H}(\tau', t)/\hat{H}(\tau, t)$, and substituting $\theta(\tau, t)$ and $\hat{\theta}(\tau, t)$ gives

$$\begin{aligned} & \forall (\tau, t), (\tau', t) \in \mathcal{T}_{N,K} : \\ & \frac{\mu \left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau' - t + i, i)}{\nu(\tau' - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)}{\mu \left(\frac{v}{\mu_1} \prod_{i=0}^{t-1} \left(1 - \frac{\theta(\tau - t + i, i)}{\nu(\tau - t + i, i)} \mu_1 \frac{v}{\mu_1} \right) \right)} = \frac{\hat{\mu} \left(\frac{v}{\hat{\mu}_1} \prod_{i=0}^{t-1} \left(1 - \frac{\hat{\theta}(\tau' - t + i, i)}{\hat{\nu}(\tau' - t + i, i)} \hat{\mu}_1 \frac{v}{\hat{\mu}_1} \right) \right)}{\hat{\mu} \left(\frac{v}{\hat{\mu}_1} \prod_{i=0}^{t-1} \left(1 - \frac{\hat{\theta}(\tau - t + i, i)}{\hat{\nu}(\tau - t + i, i)} \hat{\mu}_1 \frac{v}{\hat{\mu}_1} \right) \right)}. \end{aligned} \quad (20)$$

We are now ready to prove by induction that $\nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ on $\mathcal{T}_{N,K}$ and $\gamma = \hat{\gamma}$:

- (a) Note that $\forall (\tau, t) \in \mathcal{T}_{N,K}(1) : \nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ and $\gamma_t = \hat{\gamma}_t$. Initialize $k = 2$.
 - (b) Suppose that $\forall (\tau, t) \in \mathcal{T}_{N,K}(k-1) : \nu(\tau, t)/\mu_1 = \hat{\nu}(\tau, t)/\hat{\mu}_1$ and $\gamma_t = \hat{\gamma}_t$ for $t = 1, \dots, k$. Then, the left and right hand sides of (20) evaluated at $t = k$ are the same function of γ_{k+1} and $\hat{\gamma}_{k+1}$, respectively. It is easy to check that, because of Assumption 5, there is a pair $\tau_k, \tau'_k \in T_N(k)$ for which this function is strictly monotonous.⁴ So, as (20) also holds for $\tau = \tau_k$ and $\tau' = \tau'_k$, it necessarily follows that $\gamma_{k+1} = \hat{\gamma}_{k+1}$. Then, (15) implies that $\forall \tau \in T_N(k) : \nu(\tau, k)/\mu_1 = \hat{\nu}(\tau, k)/\hat{\mu}_1$.
 - (c) If $k < K$, let $k = k + 1$ and repeat (b).
- (iii). To complete the proof, note that, as a result, (16) holds for all $t \in D_K$:

$$\forall (\tau, t) \in \mathcal{T}_{N,K}(\tau, t) : \psi_1(t)\psi_2(\tau)\psi_3(\tau-t)\mu_1 = \hat{\psi}_1(t)\hat{\psi}_2(\tau)\hat{\psi}_3(\tau-t)\hat{\mu}_1. \quad (21)$$

Furthermore, we know from (19) that $\psi_3(\tau)/\psi_3(\tau-1)$ and $\hat{\psi}_3(\tau)/\hat{\psi}_3(\tau-1)$ are proportional, which implies that $\hat{\psi}_3(\tau) = \hat{a}f^\tau\psi_3(\tau)$ for some constants $\hat{a}, f > 0$. Also, we know from (17) that $\psi_2(\tau)$ and $\hat{\psi}_2(\tau)f^\tau$ are proportional given t , implying that $\hat{\psi}_2 = \hat{b}f^{-\tau}\psi_2$ for some constant $\hat{b} > 0$. Finally, evaluating (21) at arbitrary $(\tau, t), (\tau, t') \in \mathcal{T}_{N,K}$ we find that

$$\forall (\tau, t), (\tau, t') \in \mathcal{T}_{N,K} : \frac{\hat{\psi}_1(t')\hat{\psi}_3(\tau-t')}{\psi_1(t')\psi_3(\tau-t')} = \frac{\hat{\psi}_1(t)\hat{\psi}_3(\tau-t)}{\psi_1(t)\psi_3(\tau-t)}.$$

Therefore, $\psi_1(t)$ and $\hat{\psi}_1(t)f^{\tau-t}$ are proportional given τ , and necessarily $\hat{\psi}_1(t) = \hat{c}f^t\psi_1(t)$ for some constant $\hat{c} > 0$. \square

⁴Assumption 5 implies that, for $t = 1, 2, \dots, K$, there exist $\tau_t, \tau'_t \in T_N(t)$ such that the numerator and the denominator in (20) are different. Clearly, Assumption 5 is not a necessary condition for this to be true. However, milder conditions are not very transparent as these typically depend on all parameters of the model.

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Tables and figures

Table 1: Estimation results duration model

	females		males	
unobserved heterogeneity (G)				
μ_1	0.277	(0.005)	0.329	(0.004)
γ_2	1.295	(0.079)	1.192	(0.046)
γ_3	2.517	(0.429)	1.796	(0.231)
γ_4	7.221	(2.236)	3.287	(0.983)
γ_5	24.906	(11.533)	6.055	(4.128)
duration dependence ($\ln \psi_1(t)$)				
quarter 0	0		0	
quarter 1	0.160	(0.040)	0.077	(0.030)
quarter 2	0.043	(0.075)	-0.020	(0.045)
quarter 3	0.071	(0.079)	-0.066	(0.051)
quarter 4	0.132	(0.091)	-0.001	(0.056)
contemporaneous trend and cycle ($\alpha_2(y)$)				
trend ($\Delta(\alpha_2 - \alpha_2^c)$)	-0.020	(0.002)	-0.024	(0.001)
cycle ($\alpha_2^c(y)$):				
1982.IV–1983.III	0.138	(0.019)	0.007	(0.014)
1983.IV–1984.III	-0.026	(0.014)	-0.105	(0.011)
1984.IV–1985.III	-0.035	(0.013)	-0.010	(0.009)
1985.IV–1986.III	-0.106	(0.015)	-0.027	(0.011)
1986.IV–1987.III	-0.044	(0.019)	0.035	(0.013)
1987.IV–1988.III	-0.011	(0.019)	0.072	(0.013)
1988.IV–1989.III	0.031	(0.017)	0.093	(0.013)
1989.IV–1990.III	0.045	(0.016)	0.085	(0.013)
1990.IV–1991.III	-0.037	(0.015)	-0.017	(0.012)
1991.IV–1992.III	0.021	(0.014)	-0.025	(0.011)
1992.IV–1993.III	0.006	(0.015)	-0.093	(0.013)
1993.IV–1994.III	0.017	(0.020)	-0.014	(0.017)

(table continued on next page)

Table 1: (Continued)

	females		males	
contemporaneous season ($\omega_2(s)$)				
quarter IV (January-March)	0		0	
quarter I (April-June)	0.083	(0.017)	0.140	(0.012)
quarter II (July-September)	0.026	(0.015)	0.046	(0.012)
quarter III (October-December)	0.054	(0.017)	0.021	(0.013)
cycle composition inflow ($\alpha_3(y) = \alpha_3^c(y)$)				
1982.IV–1983.III	-0.072	(0.018)	0.016	(0.014)
1983.IV–1984.III	-0.003	(0.013)	0.011	(0.009)
1984.IV–1985.III	-0.005	(0.011)	-0.017	(0.008)
1985.IV–1986.III	-0.010	(0.014)	-0.028	(0.009)
1986.IV–1987.III	0.040	(0.016)	-0.001	(0.011)
1987.IV–1988.III	0.030	(0.016)	-0.003	(0.011)
1988.IV–1989.III	0.037	(0.016)	0.010	(0.012)
1989.IV–1990.III	0.052	(0.015)	0.012	(0.012)
1990.IV–1991.III	0.050	(0.013)	0.021	(0.011)
1991.IV–1992.III	0.000	(0.012)	-0.019	(0.010)
1992.IV–1993.III	-0.057	(0.013)	-0.006	(0.011)
1993.IV–1994.III	-0.063	(0.019)	0.003	(0.015)
season composition inflow ($\omega_3(s)$)				
quarter I (January-March)	-0.072	(0.006)	-0.076	(0.004)
quarter II (April-June)	-0.060	(0.006)	-0.084	(0.006)
quarter III (July-September)	0.021	(0.005)	0.001	(0.004)
quarter IV (October-December)	0		0	
measurement error				
σ	0.017	(0.001)	0.015	(0.001)
ρ	0.323	(0.086)	0.326	(0.086)
$d_{\geq '87}$	0.159	(0.019)	0.133	(0.013)
$N + 1(K + 1)$	48(5)		48(5)	

(table continued on next page)

Table 1: (Continued)

	females	males
Pseudo-R^2		
R_0^2	0.80	0.92
R_1^2	0.84	0.93
R_2^2	0.74	0.91
R_3^2	0.39	0.79
R_4^2	0.46	0.47
Durbin-Watson statistics		
DW_0	1.70	1.57
DW_1	2.66	2.34
DW_2	2.00	1.83
DW_3	1.97	2.09
DW_4	1.27	1.51
Moment-inequality pseudo-t-statistics (p-values)		
$\gamma_2 - 1$	3.74	4.16
$\gamma_3 - \gamma_2^2$	3.03	2.78
$\gamma_2\gamma_4 - \gamma_3^2 - \gamma_4 - \gamma_2^3 + 2\gamma_2\gamma_3$	1.72	-0.24 (0.41)
$\gamma_3\gamma_5 - \gamma_4^2 - \gamma_2^2\gamma_5 - \gamma_3^3 + 2\gamma_2\gamma_3\gamma_4$	-0.14 (0.44)	-1.05 (0.15)

Explanatory note: Standard errors are in parentheses. R_i^2 , $i = 0, \dots, 4$, are pseudo- R^2 statistics for the $i + 1$ -th equation ($\tilde{\theta}(i|\tau)$). DW_i , $i = 0, \dots, 4$, are Durbin-Watson statistics for the $i + 1$ -th equation ($\tilde{\theta}(i|\tau)$). The moment-inequality tests are explained in Subsection 2.5; pseudo- t -values are reported here (normal p -values in parentheses). The verbal description of the seasons in parentheses is justified in Subsection 4.4, and is added to support the discussion there. We have also computed Wald statistics for the null hypotheses $\ln \psi_1(1) = \ln \psi_1(2) = \ln \psi_1(3) = \ln \psi_1(4) = 0$, and $\alpha_k^c(1982.IV-1983.III) = \dots = \alpha_k^c(1993.IV-1994.III) = 0$ and $\omega_k(I) = \omega_k(II) = \omega_k(III) = 0$ for $k = 1, 2$. We have omitted these statistics as all tests lead to rejection of the null at all conventional significance levels.

Table 2: Estimation results incidence model

	females		males	
constant (ν_1)	359.437	(9.493)	455.327	(7.328)
trend and cycle incidence ($\alpha_4(y)$)				
trend ($\Delta(\alpha_4 - \alpha_4^c)$)	0.027	(0.004)	0.018	(0.003)
cycle ($\alpha_4^c(y)$):				
1982.IV–1983.III	-0.017	(0.015)	-0.020	(0.009)
1983.IV–1984.III	0.003	(0.016)	0.051	(0.010)
1984.IV–1985.III	-0.044	(0.019)	-0.012	(0.011)
1985.IV–1986.III	-0.055	(0.021)	-0.042	(0.013)
1986.IV–1987.III	0.017	(0.019)	0.047	(0.012)
1987.IV–1988.III	0.062	(0.023)	0.041	(0.014)
1988.IV–1989.III	0.038	(0.021)	-0.025	(0.013)
1989.IV–1990.III	0.022	(0.019)	-0.082	(0.011)
1990.IV–1991.III	0.038	(0.017)	-0.028	(0.010)
1991.IV–1992.III	0.012	(0.016)	-0.001	(0.010)
1992.IV–1993.III	0.020	(0.017)	0.078	(0.010)
1993.IV–1994.III	-0.028	(0.018)	0.019	(0.011)
1994.IV(–1995.III)	-0.067	(0.028)	-0.028	(0.017)
season incidence ($\omega_4(s)$)				
quarter I (January-March)	-0.172	(0.014)	-0.202	(0.009)
quarter II (April-June)	-0.200	(0.014)	-0.331	(0.009)
quarter III (July-September)	0.207	(0.014)	-0.020	(0.009)
quarter IV (October-December)	0		0	
measurement error				
σ	0.033	(0.003)	0.020	(0.002)
$d_{\geq '87}$	0.026	(0.040)	-0.039	(0.024)
$N + 2$	49		49	
Wald statistics (p-values)				
cycle incidence	18.6	(0.068)	226.1	(0.000)
season incidence	1137.1	(0.000)	2095.3	(0.000)
Pseudo-R^2				
R^2	0.97		0.98	
Durbin-Watson statistic				
DW	2.09		2.27	

Explanatory note: Standard errors are in parentheses. The Wald test statistics concern the null hypotheses $\alpha_4^c(1982.IV-1983.III) = \dots = \alpha_4^c(1994.IV-1995.III) = 0$ and $\omega_4(I) = \omega_4(II) = \omega_4(III) = 0$. p -values are given in parentheses. DW is the Durbin-Watson statistic and R^2 the pseudo- R^2 for the incidence equation. The verbal description of the seasons in parentheses is justified in Subsection 4.4, and is added to support the discussion there.

Figure 1: Unemployment ($\tilde{U}(\tau)$; deseasonalized and detrended)

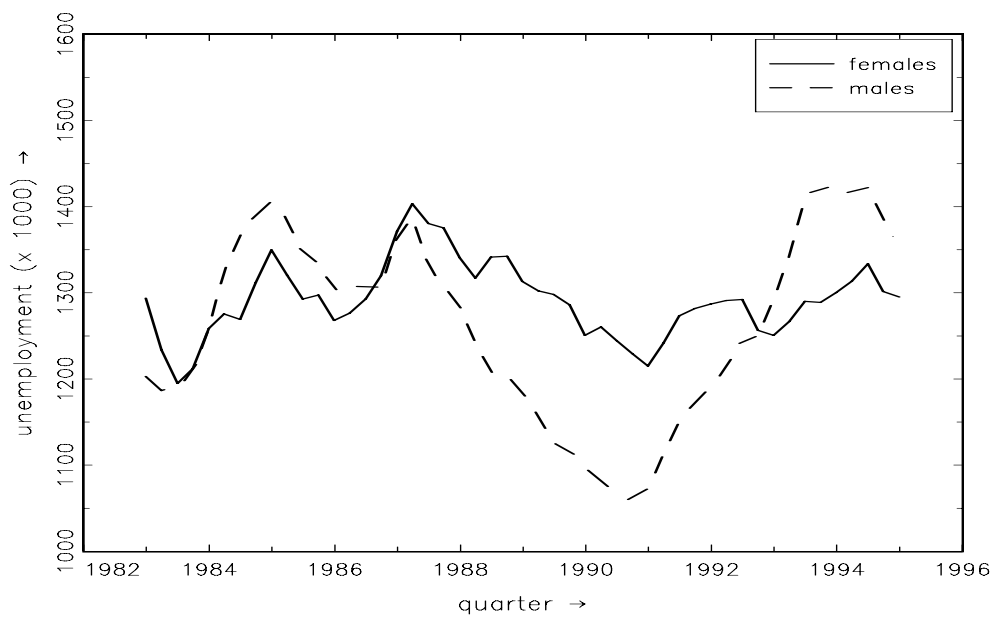


Figure 2: Business cycle indicators (detrended and deseasonalized; normalized means)

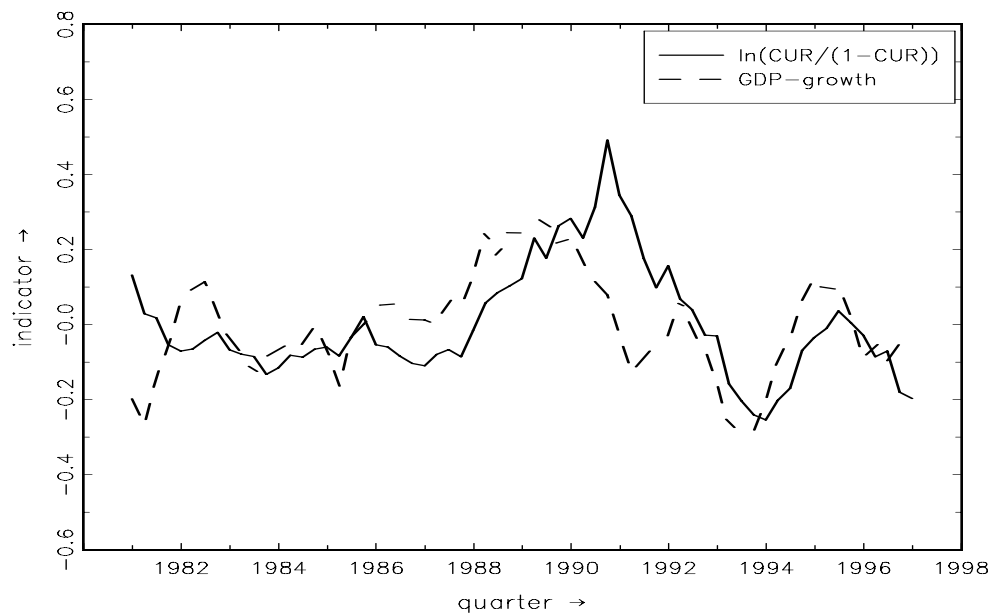


Figure 3: Inflow into unemployment ($\tilde{U}(0|\tau)$; deseasonalized)

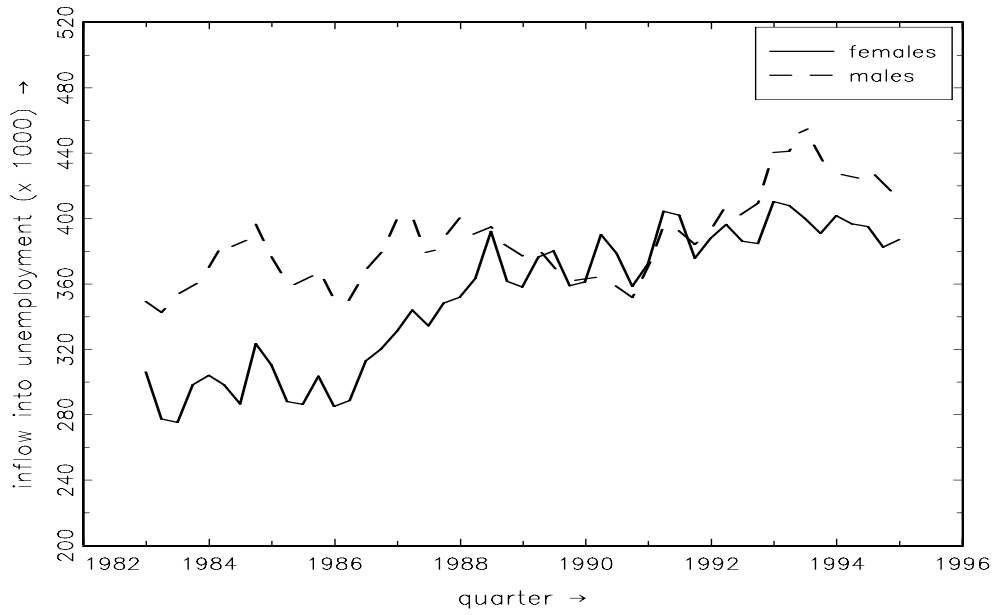


Figure 4: Outflow rate from unemployment ($\frac{\tilde{U}(0|\tau+1)+\tilde{U}(\tau)-\tilde{U}(\tau+1)}{\tilde{U}(\tau)}$; deseasonalized)

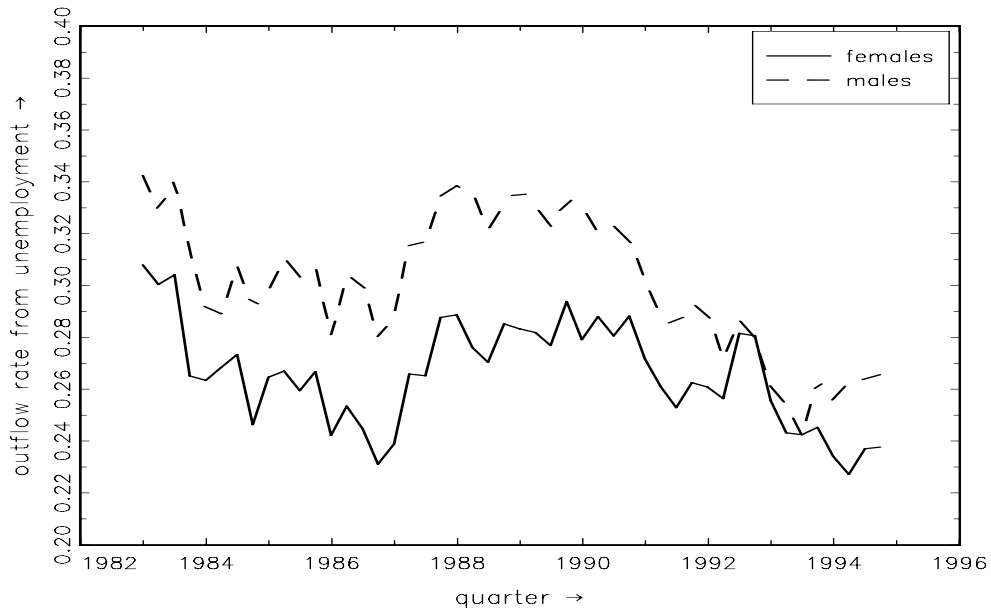


Figure 5: Quarterly exit probabilities by duration class ($\tilde{\theta}(t|\tau)$); females (top) and males (bottom)

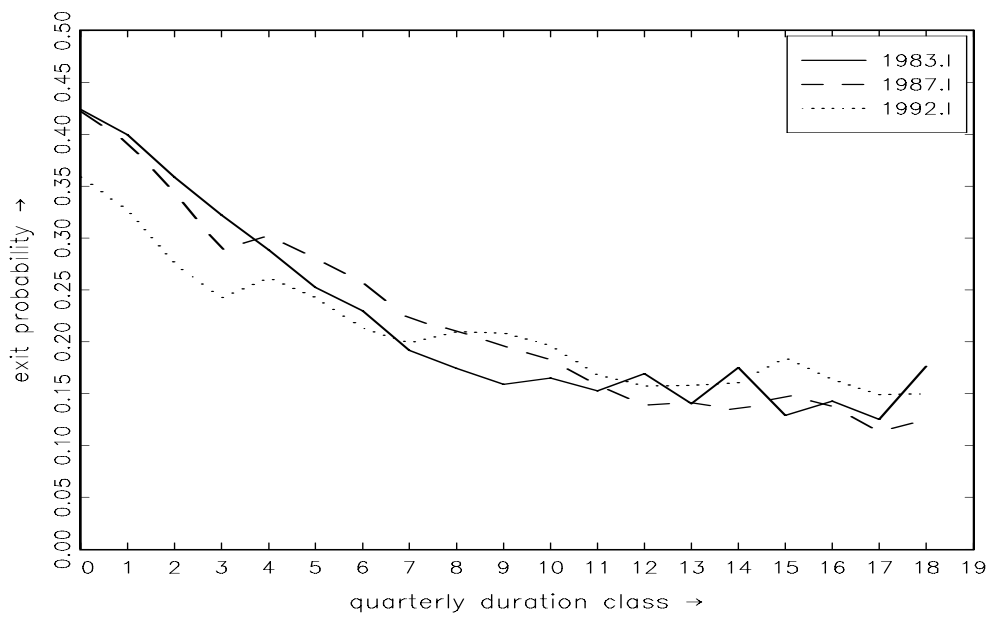
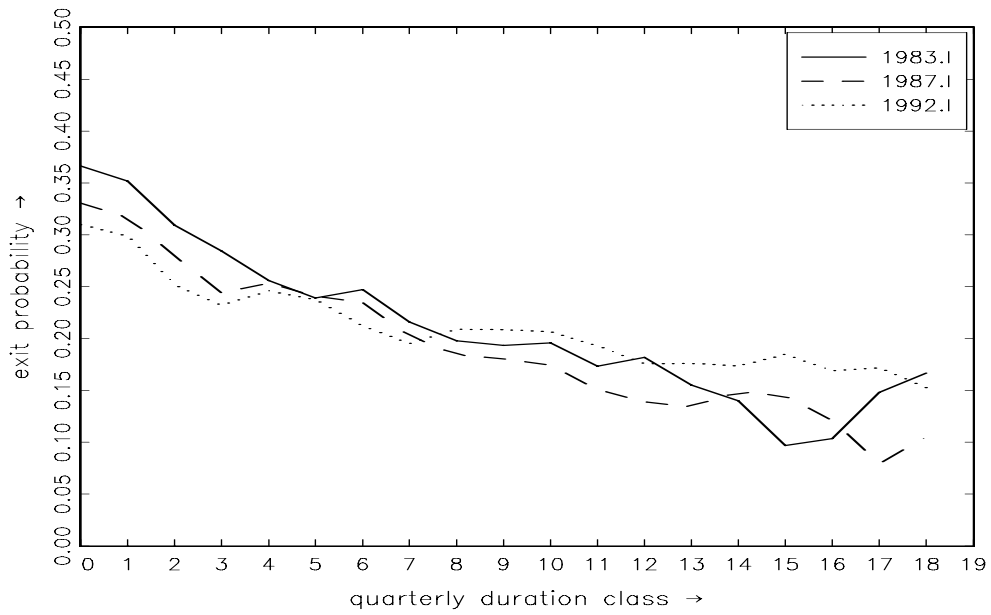


Figure 6: Duration dependence effect unobserved heterogeneity (log interaction term)

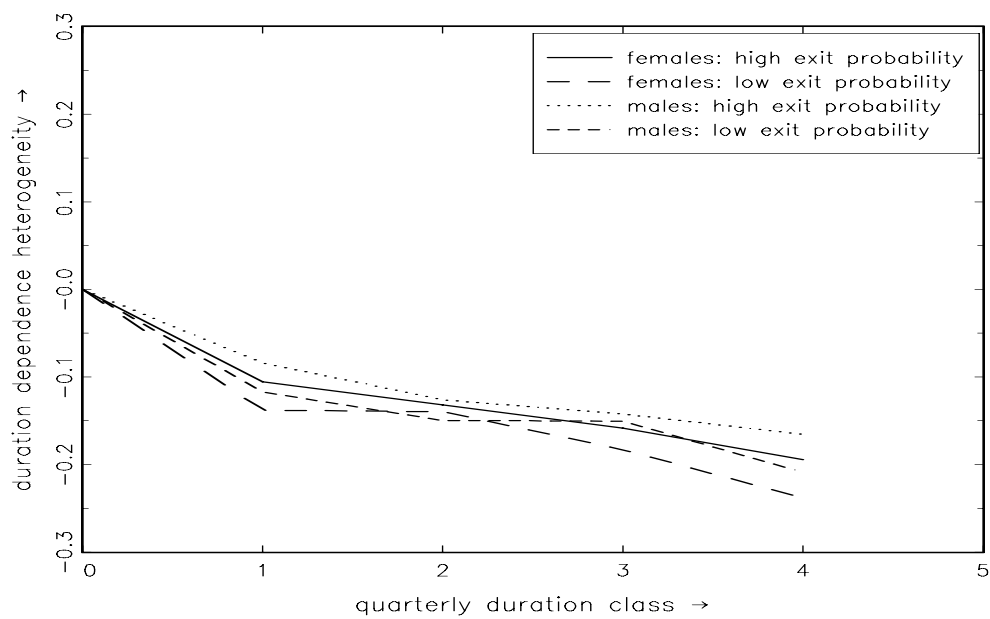


Figure 7: Contemporaneous cycle exit probabilities ($\alpha_2^c(y(\tau))$)

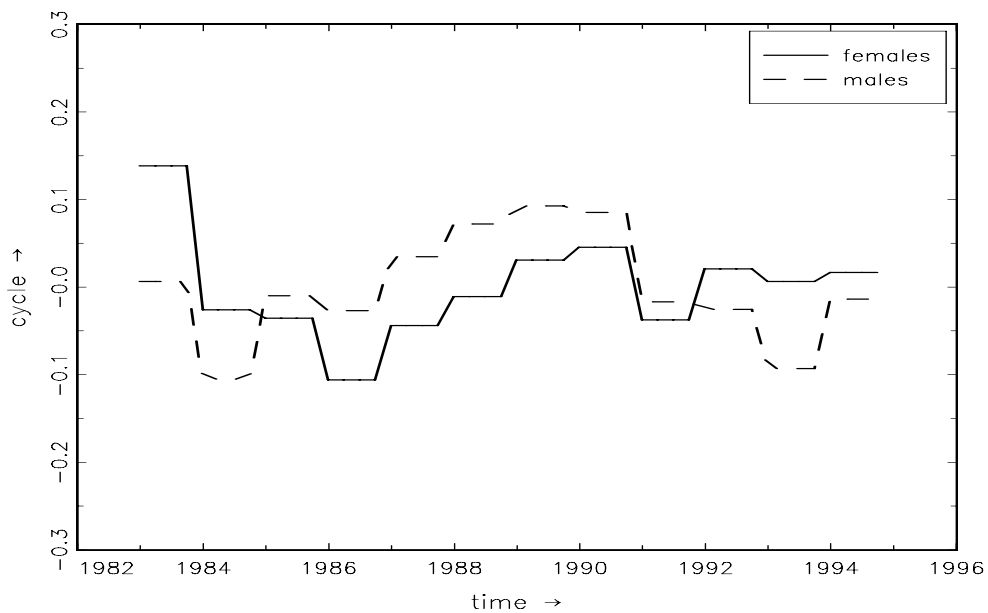


Figure 8: Cycle composition inflow ($\alpha_3^c(y(\tau)) = \alpha_3(y(\tau))$)

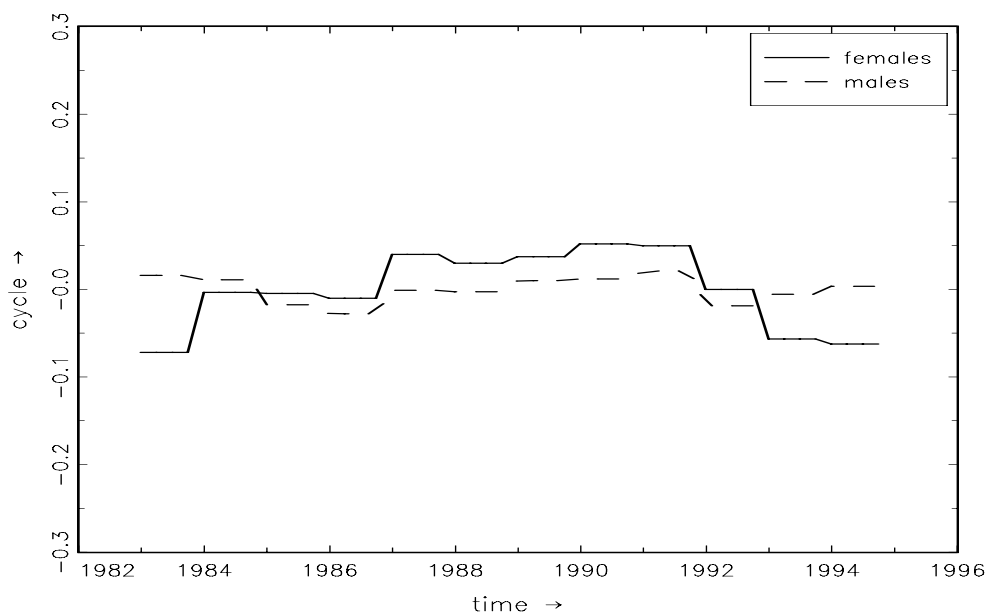


Figure 9: Cycle incidence ($\alpha_4^c(y(\tau))$)

