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# Optimal Competition: a benchmark for competition policy

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## Abstract

This paper introduces optimal competition: the best form of competition in an industry that a competition authority can achieve under the information constraint that it cannot observe firms' efficiency levels. We show that the optimal competition outcome in an industry becomes more competitive as more money is spent in the industry, as the competition authority puts less weight on producer surplus and more weight on employment. The relation between competition and entry costs is U-shaped. Finally conditions are derived under which Cournot competition is too competitive compared to the optimal competition outcome.

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## 1 Introduction

Competition authorities around the world face a daunting task: with limited means they have to monitor a huge number of sectors to see where intervention could be welfare enhancing. A natural strategy in such a situation is to target industries where there is (or seems to be) a lack of competition and where intervention by the competition authority could make a big difference. The problem is that economic theory does not give any guidance on this issue. Generally speaking, economic models show how market imperfections create distortions

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relative to the first best outcome. However, no intervention effort by a competition authority will create the first best outcome. Hence a competition authority which bases its selection process of industries to scrutinize on the comparison of the current market outcome with the first best outcome is likely to waste its scarce resources on industries where its intervention will hardly make a difference.

This paper introduces a benchmark industry outcome, called 'optimal competition', which is better achievable for a competition authority than the first best outcome. This benchmark is called optimal competition to signal that in contrast to 'perfect competition' competition is optimized under (information) constraints. The policy recommendation is that competition authorities should target their resources on industries where there is a big gap between the current industry outcome and the optimal competition outcome. Before introducing the details of this approach, consider the following motivating example.

Consider two industries, denoted  $I$  and  $II$ , which have the same structural characteristics except for their entry costs. In particular, assume that in both industries consumers are willing to spend 100 (dollar, say) in total. Consumers' utility for the products in each industry is given by a CES function  $\left(\int_0^{N_j} q_{ji}^{0.5} di\right)^2$  where  $q_{ji}$  denotes product  $i$  in industry  $j$  and  $N_j$  denotes the number of products in industry  $j$  ( $= I, II$ ). Further, assume that in both industries output is produced with a constant returns to scale technology using only labor, where the wage is normalized to 1 (one). Efficiency (or productivity)  $n$  is defined as the amount of output produced by one unit of labor. Assume that  $n$  in both industries is distributed uniformly on  $[0, 10]$ . Finally, the entry cost in industry  $I$  equals  $\gamma_I = 31$  and in industry  $II$  it equals  $\gamma_{II} = 150$ . Now you are told that industry profits (sum of profits of all firms in the industry minus the entry costs paid) in industry  $I$  equal 18.21 and in industry  $II$  they equal 0.72. Further, in industry  $I$  firms with productivity above 3.52 are active and in industry  $II$  firms with productivity above 3.55 are active. Which of these two industries should a competition authority target for further examination?<sup>1</sup> Guided by the first best benchmark, a number of people may choose to target industry  $I$ : entry costs are lower than in  $II$  while industry profits are a lot higher. Defining an entry barrier as the amount of industry profits that can be sustained in excess of the entry cost, clearly entry barriers are higher in industry  $I$  than they are in  $II$ . Moreover, in industry  $I$  almost 20% of total expenditure goes into firms' pockets as net profits. However, using this intuition to decide on which sector the competition authority should focus its attention is incorrect! As we will show below, the industry outcome and profits in industry  $I$  are identical to the optimal competition outcome. Hence no intervention by the competition authority will reduce these industry profits.<sup>2</sup> In industry  $II$ , however, compe-

<sup>1</sup> Clearly one would need more information than just industry profits to make this decision. And below we will introduce the example in more detail. However, for illustrative purposes the information given is sufficient to make the point.

<sup>2</sup> That is to say, no intervention which does not reduce welfare. Clearly, if the competition authority would introduce a maximum price just above  $\frac{1}{10}$  (marginal cost level of the most

tion is inefficiently weak. That also explains why industry *II* attracts almost the same number of firms as industry *I* while its entry costs are considerably higher. The optimal competition outcome in industry *II* differs from the outcome described above. Therefore, it is better to target industry *II* because there the competition authority can make a difference and raise welfare.

The optimal competition benchmark introduced here is the solution to a mechanism design problem, where the designer does not observe firms' efficiency levels but does know the distribution of these costs. In particular, the mechanism offers a menu of contracts with different output-revenue options. Firms then self-select their optimal output-revenue combination based on their efficiency level. In other words, the difference with the first best outcome is that the optimal competition benchmark does not assume that a competition authority will be able to observe firms' efficiency levels. This assumption seems natural and is, in fact, the standard assumption in mechanism design problems.

An obvious criticism is that even the solution to this mechanism design problem may be more than a competition authority may hope to achieve. Our response to this criticism are the following three arguments. First, in an industry context it is not unreasonable to assume that information is available on firms' efficiency distribution and consumers' utility function. In fact, an important part of the empirical literature (see Reiss and Wolak (2002) for a survey) is devoted to estimating industries' cost and demand structures. In other words, the informational problems may not be worse here than in other mechanism design problems, like regulating a monopolist or designing optimal auctions. Second, it may seem far fetched to have a competition authority offering a menu of choices from which firms must choose, however, this is beside the point. To illustrate, consider the optimal tax literature. Although, to the best of our knowledge, no government has ever proposed a tax code given by the differential equation that follows from an optimal tax problem, a lot has been learned from the optimal tax literature on how taxes should be designed. Below we will derive a number of properties of optimal competition, which will be surprising to people using the first best benchmark. For instance, we will derive conditions under which Cournot competition is, in fact, optimal competition. Since we allow for a set of mechanisms which may seem to be stretching what is realistically feasible, this is a very strong result indeed. Third, worrying about the feasibility of the optimal competition benchmark goes to the core of this paper: our point is exactly that using the first best outcome as a benchmark is not helpful. It is not realistic to assume that a competition authority can intervene in a way that will create the first best outcome in an industry. Hence optimal competition is a first step toward a better benchmark for competition authorities. In the light of the mechanism design literature it is a natural step, but it need not be the last step.

The main motivation of the paper is the optimal competition benchmark as a tool for competition authorities to help them select which industries require efficient firm), industry profits would be lower. But, as shown below, welfare would be lower as well.

further investigation. However, there are broader lessons that can be learned from the optimal competition outcome. This paper makes the following five additional contributions to the Industrial Organization literature.

First, the economic analysis of the effects of intensity of competition on welfare is rather complex. We will first explain where this complexity comes from and then how this paper contributes to this issue. When modelling the effect of competition on welfare there are, broadly speaking, two ways in which competition can be intensified. First, one can increase the number of firms in the industry. This is the route taken in papers, like Dixit and Stiglitz (1977) and Mankiw and Whinston (1986). These papers derive the optimal number of firms (or optimal product variety) in an industry with given competitive behavior (say, Cournot competition). Another way in which competition can be intensified is more aggressive interaction between existing firms. The way in which this is often formalized is by increasing the elasticity of substitution between goods (see, for instance, Aghion et. al. (2002) and Blanchard and Giavazzi (2001)). This, however, implies changing the consumers' utility function. Thus welfare comparisons become hazardous. Another option to model more aggressive interaction (without affecting agents' utility function) is a switch from Cournot to Bertrand competition. This approach has two disadvantages. It is rather messy in terms of the mathematics and it only considers two possibilities instead of working with a continuum.<sup>3</sup>

A major contribution of this paper is that by viewing the problem as a mechanism design problem we both generalize the modelling of competition outcomes<sup>4</sup> and we manage to make the mathematics simpler. The innovation is the identification of intensity of competition in a mechanism design problem. In this way, we determine the optimal number of firms and the optimal intensity of competition using a simple two dimensional graph with two curves: a downwardsloping budget constraint and an upwardsloping entry condition.

The second contribution of the paper is the following. People guided by the first best benchmark tend to believe that prices should be equal (or at least close) to marginal cost. This leads them to believe that Bertrand competition is always preferable to Cournot competition from a welfare perspective. Although Bertrand competition is welfare maximizing in the case where firms produce perfect substitutes, this is not the case when consumers value variety. We will show that in the case where consumers' utility function is of the CES form (with a finite elasticity of substitution) and the efficiency distribution in an industry follows the Pareto distribution, Cournot competition can be optimal. Moreover, we will show that if the least efficient firms cannot enter the market, Cournot competition is, in fact, too fierce. Welfare would be increased if competition

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<sup>3</sup>One way in which one can introduce a continuous variable here is to model the, so called, conjectural variations. However, this approach has its own limitations (no clear foundation for where the firms' conjectures come from) and it is rather messy as well.

<sup>4</sup>More formally, let  $q(\cdot)$  denote a firm's output level as a function of its efficiency level,  $n$ . Then comparing the Cournot and Bertrand competition outcomes boils down to calculating welfare under two specific functional forms,  $q^B(\cdot)$  and  $q^C(\cdot)$ , determined by the Bertrand Nash and Cournot Nash equilibrium resp. In contrast, below we consider *any* functional form  $q(\cdot)$  that satisfies incentive compatibility.

would become less intense, because then more firms could enter the industry with differentiated products.

Third, this idea that competition in the market can be too fierce from a social point of view is an important conceptual contribution. To illustrate, Bolton et al. (2000) state that 'Predatory pricing poses a dilemma that has perplexed and intrigued the antitrust community for many years. On the one hand, ... predatory pricing can be an instrument of abuse; on the other hand, price reductions are the hallmark of competition and the tangible benefit that consumers perhaps most desire from the economic system'. The idea implicit in this dilemma is that more competition is always better from a welfare point of view. Bolton et al. (2000) survey how predatory pricing can indeed be optimal for a firm and welfare reducing in a dynamic world with incomplete information. However, we will show below that competition can be too fierce also in a static context with symmetric information among firms. Thus one can formalize that firms may have to be punished for competing too fiercely as well as for lack of competition. Comparing the optimal competition outcome with the market outcome yields that the monopoly power effect causes too little output and the appropriability effect too little entry in the market outcome. On the other hand, the rent creation effect leads to excessive production and entry levels in the market outcome as compared to the optimal competition outcome.

One can also relate this issue to a competition authority's task of approving mergers. Consider a certain industry where the market outcome is more competitive than the optimal competition outcome. Then a merger that reduces the intensity of competition can be welfare enhancing. Hence a competition authority should take a more favourable stance towards such a merger in this case than in an industry where the market outcome is less competitive than the optimal outcome.

Fourth, we derive the following comparative static results. As the amount of money spent in the industry goes up, the optimal competition outcome becomes more competitive. This suggests that, *ceteris paribus*, a competition authority should spend more resources monitoring mature industries than starting industries. Further, as competition authorities put more weight in their objective function on consumer surplus (as compared to producer surplus) and more weight on employment, the outcome should become more competitive. The employment effect may be surprising as competition authorities often claim that they allow soft competition in an industry to protect employment. This argument overlooks that low competitive pressure creates rents thereby reducing output and therefore employment. Finally, the relation between the level of the sunk entry cost and the intensity of competition in the optimal outcome is U-shaped.

Finally, the optimal competition benchmark allows us to derive sufficient conditions under which regulating a monopolist is preferable to liberalizing the industry and inviting entry. Without the optimal competition concept, it is hard to derive such sufficient conditions because there is no upperbound on welfare achievable in the market outcome. To illustrate, one can show that regulating the monopolist leads to higher welfare than a market outcome with

Cournot competition. But that does not prove much as it leaves open the question whether there are other market outcomes (like Bertrand competition) that yield higher welfare than the regulation outcome. Optimal competition yields (by definition) the highest welfare that any market outcome can achieve. Hence, showing that the regulated outcome leads to higher welfare than optimal competition makes a strong case for regulation.

This paper is related to several strands of the literature. First, the optimal competition benchmark is derived as the solution to a mechanism design problem. In particular, it is reminiscent of the literature on optimal auctions, see for instance Bulow and Roberts (1989) and Bulow and Klemperer (1996), and the optimal allocation of prizes in a contest as in Moldovanu and Sela (2001). This will become even more clear later on when we introduce the notion of industry marginal costs (as compared to an individual firm's marginal costs) which is closely related to marginal revenue in auctions. The difference with optimal auctions is that we maximize welfare, not revenue. Second, the efficiency distribution is the realized distribution of the firms in the industry. That is, it is not a probability distribution from which agents are drawn. Finally, firms pay a sunk entry cost to enter the industry and hence the number of firms is determined endogenously. In Bulow and Roberts (1989) the number of participants in the auction is exogenously given.

Second, the result that an increase in sunk entry costs can reduce the intensity of competition in the optimal outcome is similar to a result by Gilbert and Klemperer (2000). They show that because of a sunk entry cost for buyers it may be optimal ex ante for a seller to commit to rationing. In particular, in this case competition is reduced (rationing instead of market clearing prices) by the seller to encourage entry by weak (low valuation) buyers. We generalize this idea by considering the optimal competition outcome as a function of the entry cost and find a U-shaped relationship.

The rest of this paper is organized as follows. The next section introduces the model, defines the optimal competition outcome and represents the solution in a simple diagram. Section 3 derives the implications for competition policy of the optimal competition benchmark. Section 4 compares the optimal competition outcome with a market outcome and derives conditions under which they coincide. Section 5 discusses three extensions of the basic model. What happens to the optimal competition benchmark if the competition authority attaches value to producer surplus and employment? When should deregulation in one market spill over into more intense competition in another market? And, finally, when is regulating a monopolist better from a social point of view than breaking up the monopoly and inviting entry into the industry? Mathematical proofs are given in the appendix.

## 2 Model

This section formalizes the concept optimal competition. It is the solution to a mechanism design problem where firms' efficiency levels cannot be observed.

A competition authority offers a menu of output and revenue combinations and firms select the most profitable combination. The difference between the optimal competition benchmark and the first best is precisely the assumption that firms' efficiency levels cannot be observed. Hence under optimal competition, the competition authority cannot force firms to price at marginal costs. Since it seems indeed unrealistic to assume that a competition authority can observe firms' marginal cost levels, the optimal competition outcome gives a better benchmark for authorities to decide which sectors should be scrutinized. We first introduce the demand side of the economy and then the supply side.

Consider an economy with sectors  $j \in [0, 1]$  where each sector consists of a number of firms producing goods. Consumers have a nested utility structure with Cobb Douglas preferences over sectors, that is overall utility is given by

$$\int_0^1 \alpha(j) \ln Q(j) dj \quad (1)$$

where  $Q(j)$  is a utility index for sector  $j$ , with  $\alpha(j) \geq 0$  for all  $j \in [0, 1]$  and  $\int_0^1 \alpha(j) dj = 1$ . The within sector utility for industry  $j$  is given by

$$Q(j) = \int_0^{N_j} v(q_j(i)) di$$

where  $q_j(i)$  is the output level of firm  $i$  in industry  $j$  and the function  $v(\cdot)$  satisfies  $v(0) = 0, v'(q) > 0$  and  $v''(q) \leq 0$ . If  $v''(q) < 0$  we say consumers have a taste for variety, while products are perfect substitutes if  $v''(q) = 0$ . Let  $Y$  denote aggregate income that is spent on consumption in this economy. Then the Cobb Douglas structure gives us that expenditure in market  $j$  is given by  $E_j = \alpha(j)Y$ . This assumption allows us to consider each industry in isolation without worrying about spillover effects to other industries (we come back to this in section 5). Below we focus on one industry  $j$  and drop the subscript  $j$  where this does not cause confusion.

Now turn to the firm side in the industry. Each firm produces one and only one product. Each firm has a constant returns to scale technology. Let  $n$  denote the productivity of a firm, that is the marginal cost of producing an additional unit equals  $\frac{1}{n}$ . We assume that  $n$  is distributed on  $[n_0, n_1]$  with density function  $f(\cdot)$  and distribution function  $F(\cdot)$ . Further, in order to enter the industry each firm has to pay a sunk entry cost  $\gamma \geq 0$ .

The mechanism design problem is to determine the menu of contracts which maximizes utility or consumer surplus under the restriction that total expenditure in the industry equals  $E$ . In particular, the planner offers combinations  $(R(n), q(n))$  of revenue  $R(n)$  and output levels  $q(n)$ . Firms announce their efficiency level in such a way that they get the combination of revenue and output level that maximizes their profits.<sup>5</sup> Consider incentive compatibility and individual rationality in turn.

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<sup>5</sup>Using the revelation principle (see for instance Fudenberg and Tirole (1991: chapter 7)) we can indeed focus, without loss of generality, on such a direct mechanism where firms announce their type.



Once a firm of type  $n$  enters the market, it announces its efficiency level  $\tilde{n}$  to maximize profits, that is

$$\pi(n) = \max_{\tilde{n}} \left\{ R(\tilde{n}) - \frac{q(\tilde{n})}{n} \right\}$$

As shown by Fudenberg and Tirole (1991: 258-261) and Guesnerie and Laffont (1984) a necessary and sufficient condition for truthful revelation in this case is that  $R(\cdot)$  and  $q(\cdot)$  are nondecreasing in  $n$ . If  $R(\cdot)$  and  $q(\cdot)$  are strictly increasing in  $n$ , we have full separation of types.<sup>6</sup> Given that we have truthful revelation, we find (using the envelope theorem) that

$$\pi'(n) = \frac{q(n)}{n^2} \quad (2)$$

Next consider individual rationality. Firms enter the market if and only if their profits exceed the entry cost  $\gamma$ ,  $\pi(n) \geq \gamma$ . Since quantities  $q(n)$  are nonnegative, equation (2) implies that profits are nondecreasing in  $n$ . Hence, if type  $n$  enters the market, all types  $n' > n$  enter as well. Let  $n_w$  denote the least efficient firm that enters the market. Then the profits for firm  $n$  can be written as follows.

**Lemma 1** *Consider an incentive compatible menu of choices  $(R(\cdot), q(\cdot))$  for firms. If firm  $n_w > n_0$  is the least efficient firm to enter the market,<sup>7</sup> then the profits for firm  $n \geq n_w$  equal*

$$\pi(n) = \gamma + \int_{n_w}^n \frac{q(t)}{t^2} dt \quad (3)$$

The proof of the lemma is straightforward. First, since firms enter the market freely, it must be the case that  $\pi(n_w) \geq \gamma$ . Next, the case where  $\pi(n_w) > \gamma$  can be ruled out because in that case firms with efficiency  $n_w - \varepsilon$  (for  $\varepsilon > 0$  but small) would enter the market as well,<sup>8</sup> contradicting that  $n_w$  is the least efficient firm to enter the market. Hence we have  $\pi(n_w) = \gamma$ . Second, equation (2) with  $\pi(n_w) = \gamma$  is a differential equation with a boundary condition, and it is routine to verify that (3) is the solution.

We first assume that the competition authority's goal is maximization of consumer surplus and lateron we consider other objectives. Thus we can formulate the optimization problem by the competition authority as

$$\max_{q(\cdot), \pi(\cdot), n_w} \int_{n_w}^{n_1} v(q(n)) f(n) dn$$

<sup>6</sup>We will ignore this monotonicity condition in the derivation of the outcome and check lateron that the solution indeed satisfies this condition.

<sup>7</sup>If  $n_w = n_0$  the result still holds if the competition authority puts enough weight on consumer surplus (as compared to producer surplus) because that will imply  $\pi(n_w) = \gamma$  (instead of  $\pi(n_w) > \gamma$ ).

<sup>8</sup>To see this, note that  $R(n_w) - \frac{q(n_w)}{n_w} > \gamma$  implies that  $R(n_w) - \frac{q(n_w)}{n_w - \varepsilon} > \gamma$  for  $\varepsilon > 0$  small enough.

subject to

$$\begin{aligned}\pi'(n) &= \frac{q(n)}{n^2} \\ \pi(n_w) &= \gamma \\ \int_{n_w}^{n_1} \left[ \pi(n) + \frac{q(n)}{n} \right] f(n) dn &= E\end{aligned}$$

Note that now we index firms by their efficiency level (i.e. not by their identity  $i \in [0, N]$  as above). The first and second constraint have been discussed above. The last constraint is that total expenditure,  $\int_{n_w}^{n_1} R(n) f(n) dn$ , in the market is equal to  $E$ ,<sup>9</sup> where  $R(n) = \pi(n) + \frac{q(n)}{n}$ . Let  $\lambda$  denote the Lagrange multiplier on this budget constraint, then this optimization problem can be formulated as follows.

**Lemma 2** *The competition authority's optimization problem can be written as*

$$\max_{q(\cdot), n_w} \int_{n_w}^{n_1} [v(q(n)) - \lambda MC(n) q(n)] f(n) dn - \lambda (1 - F(n_w)) \gamma$$

subject to

$$\int_{n_w}^{n_1} MC(n) q(n) f(n) dn + (1 - F(n_w)) \gamma = E$$

where  $MC(n)$  denotes firm  $n$ 's industry marginal costs

$$MC(n) \equiv \frac{1}{n} \left[ 1 + \frac{1 - F(n)}{f(n)n} \right] \quad (4)$$

The lemma shows that the competition authority's optimization problem has a very simple structure. It maximizes utility,  $v(q(n))$ , minus total variable costs of production,  $MC(n) q(n)$ , minus total entry costs,  $(1 - F(n_w)) \gamma$ , where the costs are priced with the shadowprice,  $\lambda$ , of expenditure  $E$ . The definition of industry marginal costs in (4) is borrowed from the auction literature's concept marginal revenue.<sup>10</sup> To see why industry marginal costs for firm  $n$ ,  $MC(n)$ , exceed private marginal cost for firm  $n$ ,  $\frac{1}{n}$ , consider the industry cost of raising output for firms with efficiency  $n$  with one unit, i.e.  $\Delta q(n) = 1$ . That implies that total costs for these firms rises with  $\frac{\Delta q(n)}{n} f(n) = \frac{f(n)}{n}$ . In addition to this, it becomes more attractive for firms with  $n' > n$  to mimic firm  $n$ . In order to keep incentive compatibility, equation (2) implies that the profits of

<sup>9</sup>Since we have assumed non-satiation ( $v'(q) > 0$  for all  $q \geq 0$ ), total expenditure will never be less than  $E$ .

<sup>10</sup>See, for instance, Bulow and Roberts (1989) and Bulow and Klemperer (1996)). Their motivation for calling it (industry) marginal costs is the following. Let  $x(n)$  denote the 'quantity' of firms with efficiency greater than  $n$ , that is  $x(n) = 1 - F(n)$ . Then marginal costs at the industry level can be defined as  $\frac{d[\frac{1}{n}x(n)]}{dx(n)}$ . Writing this derivative as  $\frac{d[\frac{1}{n}x(n)]}{dn} \left( \frac{dx(n)}{dn} \right)^{-1}$  it is routine to verify that this expression equals that for  $MC(n)$  in equation (4).

the firm just above  $n$  has to rise with  $\Delta\pi = \frac{\Delta q(n)}{n^2} = \frac{1}{n^2}$ . Further, to maintain incentive compatibility for all firms above  $n$ , the profits of these firms have to rise as well with  $\Delta\pi$ . Since there are  $(1 - F(n))$  firms above  $n$ , the total costs of maintaining incentive compatibility equal  $\Delta\pi(1 - F(n)) = \frac{1 - F(n)}{n^2}$ . Hence the total increase in industry costs in response to  $\Delta q(n) = 1$  equal

$$f(n) \frac{1}{n} \left( 1 + \frac{1 - F(n)}{f(n)n} \right) = f(n) MC(n)$$

This explains the intuition why industry marginal costs exceed marginal costs  $\frac{1}{n}$  at the firm level. By letting firm  $n$  produce an additional unit, an informational rent (or virtual surplus) is created for types  $n' > n$ . The industry marginal costs  $MC(n)$  takes this informational rent into account as well.

The following proposition characterizes the solution to the maximization problem above. That is, it characterizes the optimal competition outcome.

**Proposition 1** *Assume that  $MC(n)$  is non-increasing in  $n$ ,  $n_0 = 0, \gamma > 0, v''(\cdot) < 0$  and  $\lim_{q \rightarrow +\infty} v'(q) = 0$ . Then  $q(\cdot)$ ,  $n_w$  and  $\lambda$  are determined by the following three equations*

$$v'(q(n)) = \lambda MC(n) \quad (5)$$

for all  $n \geq n_w$ ,

$$v(q(n_w)) = \lambda [q(n_w) MC(n_w) + \gamma] \quad (6)$$

$$\int_{n_w}^{n_1} MC(n) q(n) f(n) dn + (1 - F(n_w)) \gamma = E \quad (7)$$

The proposition shows the trade off that a competition authority faces. On the one hand, the concavity of the utility function  $v(\cdot)$  implies that consumers like variety and hence  $n_w$  should be low. However, a low value of  $n_w$  is costly not only because the entry cost  $\gamma$  has to be incurred for a bigger number of firms but also because entry by low efficiency firms creates rents for more efficient firms ( $MC(n)$  goes up as  $n$  falls). Hence to keep a balanced budget, lower values of  $n_w$  lead to lower values of  $q(n)$  for all  $n > n_w$ . In other words, the trade off here is between variety (number of goods) and the quantity of each good on offer.

The intuition for the equations (5)-(7) is as follows. The first equates the marginal utility of output  $q(n)$  with its marginal cost (in terms of the shadow price of expenditure). Equation (6) says that the benefit of having the least efficient firm  $n_w$  in the industry,  $v(q(n_w))$ , equals the cost of having it in the industry  $\gamma$  plus the cost of producing  $q(n_w)$ ,  $\lambda [q(n_w) MC(n_w) + \gamma]$ . Put differently, the planner is indifferent whether  $n_w$  enters the industry or not. Finally, total industry costs (total variable production costs plus entry costs) should equal total expenditure  $E$ .

The intuition for the assumptions made in the proposition is the following. A sufficient condition for  $MC(n)$  to be decreasing in  $n$  is that the distribution of efficiency satisfies the monotone-hazard-rate condition,  $\frac{d}{dn} \left( \frac{f(n)}{1 - F(n)} \right) \geq 0$ . This

is a standard condition in mechanism design problems. If the efficiency distribution is such that industry marginal costs are rising in  $n$  over some interval, we get that more efficient types produce less than less efficient types. This contradicts incentive compatibility of the solution.<sup>11</sup> The condition that  $n_0 = 0$  ensures that we have an interior solution for  $n_w$ . Clearly, with  $\gamma > 0$  it is never optimal to let a firm which cannot produce any output enter an industry, so  $n_w > n_0 = 0$ . If  $n_0 > 0$ , equation (6) needs to be adjusted to allow for a corner solution.<sup>12</sup> The condition that utility  $v(\cdot)$  is strictly concave, rules out that goods are perfect substitutes. If goods are perfect substitutes, the solution to the planner's optimization problem is not well defined. In principle, in that case it is optimal to have only the most efficient firms enter the industry,  $n_w = n_1$ . However, the firms with efficiency  $n_1$  have zero mass and hence industry production equals zero.<sup>13</sup> Assuming that consumers value variety ( $v''(\cdot) < 0$ ) avoids this technical complication. Finally, assuming that  $v'(+\infty) = 0$  ensures that we do not need to worry about corner solutions in equation (5).<sup>14</sup>

The outcome in proposition 1 can be represented by two curves in  $(n_w, \lambda)$  space as illustrated in figure 1. Equation (6) is upwardsloping in  $(n_w, \lambda)$  space, with slope  $\left. \frac{\partial \lambda}{\partial n_w} \right|_{(6)} = \lambda \frac{-MC'(n_w)}{MC(n_w) + \gamma} > 0$ . We call this the *entry condition* (EC): as the marginal value of income,  $\lambda$ , decreases then less efficient firms can enter as well and  $n_w$  falls. We call equation (7) the *budget constraint* (BC). This curve is downwardsloping, with  $\left. \frac{\partial \lambda}{\partial n_w} \right|_{(7)} = f(n_w) \frac{q(n_w)MC(n_w) + \gamma}{\int_{n_w}^{n_1} \frac{dq(n)}{d\lambda} MC(n)f(n)dn} < 0$  because equation (5) implies  $\frac{dq(n)}{d\lambda} < 0$ . As the marginal value of income decreases, high efficiency firms' production goes up and hence to satisfy the budget constraint (7) money has to be saved on production and entry costs by eliminating inefficient firms from the industry ( $n_w$  goes up).

With an upward sloping EC and downward sloping BC curve in  $(n_w, \lambda)$  space, we have three possibilities as illustrated in figure 1. Panel a describes the case with an interior solution for  $n_w$  as characterized in proposition 1. In panel b we find that at the point  $(n_0, \lambda_{BC})$  it is the case that  $v(q(n_0)) > \lambda_{BC} [q(n_0)MC(n_0) + \gamma]$ . Thus here we get the corner solution  $n_w = n_0$ . Finally, panel c of figure 1 describes a situation where the industry closes down

<sup>11</sup>Interestingly, in this case the solution to the competition authority's optimization problem involves bunching or rationing. This means that firms of different efficiency levels get the same output and revenue combinations. Put differently, optimal competition in that case involves (over some range of types  $n$ ) the weakest competition possible: more efficient firms are not rewarded by higher market shares (but they do have higher profits). This line of research is not further pursued in this paper.

<sup>12</sup>In particular, it would read  $v(q(n_w)) \geq \lambda [q(n_w)MC(n_w) + \gamma]$  with strict inequality only if  $n_w = n_0$ . For instance, it is routine to verify that  $v''(q) < 0, \gamma = 0$  and  $n_0 > 0$  imply  $n_w = n_0$ .

<sup>13</sup>Formally, the solution can be characterized as  $n_w = n_1 - \varepsilon$  with  $\varepsilon > 0$  as small as possible. The value of  $\varepsilon$  has to be as close to zero as possible in order to have only the most productive firms in the industry and hence reduce both industry marginal costs and entry costs.

<sup>14</sup>Note that we do not need the assumption that  $\lim_{q \rightarrow 0} v'(q) = +\infty$  to rule out corner solutions. If  $v'(0)$  is finite, and equation (5) would imply that  $q(\hat{n}) = 0$  for some  $\hat{n}$ , then equation (6) would imply that  $n_w > \hat{n}$ .

as  $n_w > n_1$ . This case can be ruled out by the assumption that the inverse of the function  $v'(\cdot)$  is finite valued on  $\mathfrak{R}_+$ .<sup>15</sup>

### 3 Implications for competition policy

This section interprets the results in proposition 1 in terms of optimal competition and considers some simple examples. To do this, we introduce a formal definition of competition in this context. We identify competition by its output reallocation effect (see, for instance, Boone (2001) or Vickers (1995)). The idea is that a rise in competition reallocates output from less efficient to more efficient firms. From this point of view, the ultimate uncompetitive outcome is rationing (each firm produces the same output level)<sup>16</sup> not monopoly. In fact, a highly concentrated industry is associated here with competition that is so intense that less efficient firms cannot enter. More formally, we define competition as follows.

**Definition 1** *Consider an industry characterized by its efficiency distribution  $f(n)$  on an interval  $[n_0, n_1]$ . Comparing two industry outcomes, denoted  $I$  and  $II$ , we say that outcome  $I$  is more competitive than  $II$  if there exists  $\bar{n} \in \langle n_0, n_1 \rangle$  such that*

$$\begin{aligned} q^I(n) &> q^{II}(n) \text{ for all } n > \bar{n} \\ q^I(n) &\leq q^{II}(n) \text{ for all } n < \bar{n} \text{ and} \\ q^I(n) &< q^{II}(n) \text{ for all } n < \bar{n} \text{ with } q^{II}(n) > 0 \end{aligned}$$

This definition says that industry outcome  $I$  is more competitive than  $II$  if there is a pivotal efficiency level  $\bar{n}$  such that all firms above  $\bar{n}$  produce more in outcome  $I$  than in  $II$  while the firms below  $\bar{n}$  produce less in outcome  $I$  than in  $II$ . Clearly, if  $q^{II}(n) = 0$  for low efficiency levels a rise in competition cannot reduce these output levels further. Figure 2 illustrates this definition graphically. In panels a and b the outcome in industry  $I$  is more competitive than in  $II$ . Note however that the definition does not imply a complete ordering of industry outcomes in terms of which outcome is more competitive. This is illustrated in figure 2c where the outcomes  $I$  and  $II$  cannot be ranked. Yet, for our purposes here that turns out not to pose any problems. That is, although we use this weak criterion, we can rank the outcomes in terms of competition intensity where we want to do so. The following example illustrates the definition by comparing a Cournot and Bertrand outcome, where the Bertrand outcome is generally seen as more competitive.

<sup>15</sup>To see this, note that this implies  $q(n)$  as determined by equation (5) with  $\lambda = \lambda_{BC} > 0$  is finite valued. Therefore the left hand side of equation (7) goes to zero as  $n_w$  goes to  $n_1$  and equation (7) cannot hold at the point  $(n_1, \lambda_{BC})$ . This rules out the case where the BC curve lies everywhere above the EC curve.

<sup>16</sup>Note that incentive compatibility excludes the case where more efficient firms produce lower output levels than less efficient firms.

**Example 1** Consider an industry with two firms producing perfect substitutes where the demand curve is given by  $p = 1 - q_1 - q_2$ . Assume that the constant marginal costs of firm 1 equal  $c_1 = 0$  while the marginal costs of 2 equal  $c_2 = c < \frac{1}{2}$ . The Cournot outcome in this industry features output levels  $q_1^C = \frac{1+c}{3}, q_2^C = \frac{1-2c}{3}$  and price level  $p^C = \frac{1+c}{3}$ . The Bertrand outcome has  $q_1^B = 1 - c, q_2^B = 0$  and price level  $p^B = c$ . Using definition 1 we say that the Bertrand outcome is more competitive than the Cournot outcome, because  $q_1^B > q_1^C$  and  $q_2^B < q_2^C$ .

Using this example we can also illustrate why variables like concentration and industry profits are not useful to measure competition. First, the switch from Cournot to Bertrand competition is a move from duopoly to monopoly. Hence competition goes up and concentration as well. Next, consider industry profits. Under Cournot competition, industry profits equal  $\Pi^C = \frac{1+c}{3} \left( \frac{1+c}{3} - 0 \right) + \frac{1-2c}{3} \left( \frac{1+c}{3} - c \right)$  and under Bertrand competition  $\Pi^B = (1 - c)c$ . It is routine to verify that  $\Pi^B > \Pi^C$  for  $c \in \left( \frac{2}{7}, \frac{1}{2} \right)$ .

Boone (2001) shows that for well known parametrizations of competition, an increase in competition always features the output reallocation effect. The intuition is that in a more competitive environment firms are punished more harshly (in terms of output) for a fall in efficiency. Although this notion of competition is, in general, not directly related to price cost margins, in the case considered here there is a clear relationship as shown below.

The following result gives a straightforward way to determine how competitive the optimal competition outcome is.

**Lemma 3** *The shadow price of expenditure  $\lambda$  is an inverse measure of competition.*

This result follows immediately from equations (5) and (7). As  $\lambda$  falls, output  $q(n)$  rises for all active firms (since  $v''(\cdot) < 0$ ) and hence equation (7) implies that  $n_w$  rises. Hence, using definition 1, the outcome with lower  $\lambda$  is more competitive. As a further motivation for using  $\lambda$  as a measure competition consider the following comparative static results which are partial results since we vary  $\lambda$  as an exogenous variable.<sup>17</sup>

**Lemma 4** *The profits of the least efficient firm increase with  $\lambda$*

$$\frac{\partial \pi(n_w)}{\partial \lambda} > 0$$

If  $\frac{v'(q_1)}{v'(q_2)}$  can be written as a function of  $\frac{q_1}{q_2}$  only, then a rise in  $\lambda$  raises price cost margins

$$\frac{\partial \left( \frac{R(n) - \frac{q(n)}{n}}{R(n)} \right)}{\partial \lambda} > 0$$

for all  $n \geq n_w$ .

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<sup>17</sup>More precisely,  $q(n)$  is determined by (5) and  $n_w$  is determined by the budget constraint (7).

Hence a fall in  $\lambda$  reduces the profits of the least efficient firm by reducing its production level through the reallocation effect. A rise in competition (and a fall in  $\lambda$ ) does not necessarily reduce all firms' profits since the most efficient firms may gain from more aggressive competitive interaction in the industry. Further, a fall in  $\lambda$  reduces price cost margins for all active firms which is another motivation to view a fall in  $\lambda$  as a rise in competition. The condition that  $\frac{v'(q_1)}{v'(q_2)}$  can be written as a function of  $\frac{q_1}{q_2}$  only is, for instance, satisfied if the utility function is given by  $v(q) = \frac{1}{\alpha}q^\alpha$  (implying a CES utility function at the industry level).

Using figure 1a, we can derive the effects of entry cost  $\gamma$  and expenditure  $E$  on the optimal intensity of competition  $\lambda$  and entry  $n_w$ . As expenditure goes up, the budget constraint shifts to the left: for given intensity of competition more firms can enter. Hence the effect of a rise in  $E$  is a fall in both  $\lambda$  and  $n_w$ . If consumers are willing to spend more, the competition authority can afford to have both more intense competition and more varieties.

A rise in entry cost  $\gamma$  shifts both curves to the right. For given intensity of competition,  $\lambda$ , both the entry condition and the budget constraint indicate that less firms can enter the industry (that is,  $n_w$  goes up). The effect of  $\gamma$  on  $\lambda$  depends on relative slopes of the two curves.

**Lemma 5** *The effect of a rise in the amount  $E$  spent in the industry on the optimal competition benchmark is*

$$\frac{d\lambda}{dE} \leq 0, \frac{dn_w}{dE} < 0$$

and the effect of a rise in entry cost  $\gamma$  is

$$\begin{aligned} \frac{dn_w}{d\gamma} &> 0 \\ \frac{d\lambda}{d\gamma} &\begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ if and only if } [1 - F(n_w)] - f(n_w) [\gamma + q(n_w) MC(n_w)] \frac{\partial n_w}{\partial \gamma} \begin{matrix} \leq \\ \geq \end{matrix} 0 \end{aligned}$$

where  $\frac{\partial n_w}{\partial \gamma} = \frac{1}{q(n_w)[-MC'(n_w)]} > 0$  as determined by equation (6).

The first result can be seen as a formalization of the following two forms of an infant industry argument. First, an industry that starts off in a country has a relatively low share of total income  $Y$  spent on its goods (i.e.  $\alpha(j)$  is low in equation (1)). Hence, ceteris paribus the efficiency distribution in the industry and the entry cost, such a starting industry should be relatively less competitive than a mature industry which attracts a higher share of total income  $Y$ . In other words, the optimal competition benchmark is indeed tighter ( $\lambda$  is lower) for mature industries than for infant industries. In this sense, for given competitive behavior in the market, a competition authority should pay more attention to the mature than the infant industry. Similarly, considering an industry moving from maturity into decline in the sense that the amount of money spent in the industry decreases over time, the optimal competition

outcome becomes less intense. Hence competition reducing mergers should be viewed more favourably by a competition authority in a declining industry than in a mature industry.

Second, comparing the same industry (with the same share of total income spent in the industry) in two countries where one country is more developed than the other in terms of income  $Y$ , we find that the less developed country has a less competitive optimal competition benchmark than the more developed country. This shows that the claim that developing countries should have the same competition standards as developed western countries is, in general, not correct.<sup>18</sup>

The interpretation of the condition for  $\frac{d\lambda}{d\gamma}$  is the following. As  $\gamma$  goes up, the effect on  $\lambda$  is determined by the overall effect on the budget constraint. On the one hand, as  $\gamma$  goes up with 1 expenditure on entry costs for all types  $n > n_w$  goes up. This is the term  $[1 - F(n_w)]$ . On the other hand, equation (6) implies that a rise in  $\gamma$  increases  $n_w$  because a higher efficiency is needed to pass the (now) tougher entry condition. Clearly, the rise in  $n_w$  reduces total expenditure. The higher the savings due to the rise in  $n_w$ , i.e. the higher  $f(n_w) [\gamma + q(n_w) MC(n_w)] \frac{\partial n_w}{\partial \gamma}$ , the more likely that more intense competition can be afforded,  $\frac{d\lambda}{d\gamma} < 0$ . If total costs rise with the increase in  $\gamma$ , competition becomes less intense in order to lower active firms' output levels and keep the budget constraint satisfied.

The condition for the sign of  $\frac{\partial n_w}{\partial \gamma}$  suggests that for low  $\gamma$  the effect is positive because  $n_w$  is small and hence  $[1 - F(n_w)]$  is big. Then as  $\gamma$  is raised  $[1 - F(n_w)]$  falls and the effect becomes negative. More formally this can be described as follows.

**Corollary 1** *Assume  $n_0 > 0$ ,  $v''(\cdot) < 0$  and  $\lim_{q \downarrow 0} v'(q) = +\infty$ . Then there exist  $\gamma_*, \gamma^*$  with  $\gamma_* < \gamma^*$  such that*

$$\frac{d\lambda}{d\gamma} > 0 \text{ for } \gamma < \gamma_*$$

and

$$\frac{d\lambda}{d\gamma} < 0 \text{ for } \gamma > \gamma^*$$

The intuition for this result is as follows. If  $\gamma$  is low, most firms can enter and the post entry game can be rather competitive without damaging variety much. If  $\gamma$  is big, duplication of entry costs is too expensive and hence not too many firms should be attracted to the industry. Consequently, the post entry game should be rather competitive. It is only when entry costs are somewhere in between that weakening competition makes sense. The goal of softening competition is to attract more firms into the industry (at the expense of lower output per firm) which raises welfare because consumers value variety.

<sup>18</sup>Yet, it does not necessary follow that the reduction in competition in developing countries should take the protectionist form of introducing import tariffs for competing foreign goods.



Another interpretation of this result is in terms of the uncertainty surrounding the industry. Consider the following modification of the model above. A firm invests  $\gamma$  in R&D to invent a new product and enter the industry. With a probability  $(1 - s)$  this investment is not successful and the firm earns nothing. With probability  $s$  the new product is successfully introduced and the firm earns profits as stated above. It is routine to verify that results of this new model are the same as the results derived above with sunk costs  $\frac{\gamma}{s}$ . Hence a fall in  $s$  (making R&D more risky for firms) may initially call for a fall in competition. Eventually for low values of  $s$  further reductions in  $s$  should raise competition. This goes against the Schumpeterian intuition that more risk in the R&D process should lead to more monopoly power for firms. The reason is that for low  $s$  the R&D is unlikely to be successful and it is efficient to limit the number of firms undertaking R&D by raising competition. An in-depth analysis of the effects of R&D on optimal competition is beyond the scope of this paper and left for future research.

The U-shaped relation between competition and entry cost is illustrated in the following example.

**Example 2** Assume that efficiency has a uniform distribution on  $[0, n_1]$ . That is,  $f(n) = \frac{1}{n_1}$  and  $F(n) = \frac{n}{n_1}$ . It follows that  $MC(n) = \frac{n_1}{n^2}$ . If we assume that  $v(q) = \frac{1}{\alpha}q^\alpha$ , then equation (5) implies that  $q(n)$  is determined by

$$q(n) = \lambda^{-\frac{1}{1-\alpha}} \left( \frac{n^2}{n_1} \right)^{\frac{1}{1-\alpha}}$$

Routine manipulation of equations (6) and (7) yields

$$\frac{\alpha}{1+\alpha} \left( \frac{n_w}{n_1} \right)^{-\frac{2\alpha}{1-\alpha}} - \frac{1+2\alpha}{1+\alpha} \frac{n_w}{n_1} + 1 = \frac{E}{\gamma}$$

$$\lambda = \left( \frac{1-\alpha}{\alpha\gamma} \right)^{1-\alpha} \left( \frac{n_w^2}{n_1} \right)^\alpha$$

Finally, industry profits can be written as

$$\Pi = \alpha\gamma \left[ \frac{1-\alpha}{2(1+\alpha)} \left( \frac{n_1}{n_w} \right)^{\frac{2\alpha}{1-\alpha}} - \frac{1}{1+\alpha} \frac{n_w}{n_1} + \frac{1}{2} \left( \frac{n_w}{n_1} \right)^2 \right]$$

Figure 3 plots  $\Pi$ ,  $n_w$  and  $\lambda$  as a function of  $\gamma$ . We have chosen the following parameter values for this graph:  $\alpha = \frac{1}{2}$ ,  $E = 100$ ,  $n_1 = 10$  and  $\gamma \in [1, 500]$ . Note that we find indeed the inverse-U relation between  $\gamma$  and  $\lambda$  suggested above. Hence, as  $\gamma$  rises competition becomes less intense in the optimal competition benchmark while for higher values of  $\gamma$  a rise in  $\gamma$  intensifies competition in the benchmark.

The example given in the introduction also follows from this example. In industry I the entry cost equals  $\gamma_I = 31$ , then it follows that  $n_w = 3.52$ ,  $\lambda = 0.20$

and  $\Pi = 18.21$ . In industry II the entry cost  $\gamma_{II} = 150$  and thus the optimal competition outcome features  $n_w = 7.25, \lambda = 0.19$  and  $\Pi = 7.23$ . The outcome presented in the introduction has been derived with  $\lambda^m = 1$  and hence  $q^m(n) = \frac{n^4}{100}$ . To guarantee incentive compatibility we assume  $\pi'(n) = \frac{q^m(n)}{n^2} = \left(\frac{n}{n_1}\right)^2$ . Finally,  $n_w^m$  is then determined by  $\int_{n_w^m}^{n_1} \left[\pi^m(n) + \frac{q^m(n)}{n}\right] \frac{1}{n_1} dn = 100$  where  $\pi^m(n) = 150 + \int_{n_w^m}^n \left(\frac{t}{n_1}\right)^2 dt$ . Although industry profits equal  $\Pi^m = 0.72$  in the market outcome, which is below  $\Pi = 7.23$  in the optimal competition benchmark for industry II, it is clearly the case that the market outcome is less competitive

As another illustration, the next example considers the optimal competition benchmark for the case where the efficiency distribution is a Pareto distribution.

**Example 3** Assume that the efficiency distribution on  $[n_0, +\infty)$  (with  $n_0 > 0$ ) takes the Pareto form:  $f(n) = \frac{1}{\phi} n_0^{\frac{1}{\phi}} n^{-\frac{1+\phi}{\phi}}$  and  $F(n) = 1 - \left(\frac{n_0}{n}\right)^{\frac{1}{\phi}}$ . Then industry marginal costs take the following simple form

$$MC(n) = \frac{1 + \phi}{n}$$

Further, assume that the function  $v(\cdot)$  takes the form  $v(q) = \frac{1}{\alpha} q^\alpha$  for  $\alpha > 0$ . To ensure that  $n_w \in \mathfrak{R}$ ,<sup>19</sup> we impose the condition that

$$\phi < \frac{1 - \alpha}{\alpha}$$

Then equation (5) implies that

$$q(n) = \left(\frac{n}{\lambda(1+\phi)}\right)^{\frac{1}{1-\alpha}} \quad (8)$$

The budget constraint (7) can be written as

$$\lambda = \frac{\int_{n_w}^{+\infty} \left(\frac{n}{\lambda(1+\phi)}\right)^{\frac{1-\alpha}{1-\alpha}} \frac{1}{\phi} n_0^{\frac{1}{\phi}} n^{-\frac{1}{\phi}-1} dn}{E - \left(\frac{n_0}{n_w}\right)^{\frac{1}{\phi}} \gamma}$$

or equivalently

$$\lambda^{\frac{1}{1-\alpha}} = \frac{(1+\phi)^{\frac{-\alpha}{1-\alpha}} \frac{1}{\phi} n_0^{\frac{1}{\phi}} n_w^{-\left(\frac{1}{\phi} - \frac{\alpha}{1-\alpha}\right)}}{E - \left(\frac{n_0}{n_w}\right)^{\frac{1}{\phi}} \gamma} \quad (9)$$

<sup>19</sup>Since the Pareto distribution is unbounded at the top it is tempting to produce output only with the firms that have  $\frac{1}{n} = 0$ . This solution, however, is not optimal if consumers value variety. The condition  $\frac{1-\alpha}{\alpha} > \phi$  says that consumers value variety more than the tail of the efficiency distribution is thick for high  $n$ . Hence it is optimal to have  $n_w \in \mathfrak{R}$  and  $\frac{1}{n_w} > 0$ .

Finally, equation (6) can be written as

$$\lambda^{\frac{1}{1-\alpha}} = \frac{1-\alpha}{\alpha\gamma} \left( \frac{n_w}{1+\phi} \right)^{\frac{\alpha}{1-\alpha}} \quad (10)$$

Combining equations (9) and (10) yields

$$\frac{n_w}{n_0} = \left( \frac{1-\alpha\phi}{1-\frac{\alpha}{1-\alpha}\phi} \frac{\gamma}{(1-\alpha)E} \right)^{\phi} \quad (11)$$

Substituting this into (10) yields the following expression for  $\lambda$

$$\lambda^{\frac{1}{1-\alpha}} = \frac{1-\alpha}{\alpha\gamma} \left( \frac{n_0}{1+\phi} \left( \frac{1-\alpha\phi}{1-\frac{\alpha}{1-\alpha}\phi} \frac{\gamma}{(1-\alpha)E} \right)^{\phi} \right)^{\frac{\alpha}{1-\alpha}} \quad (12)$$

Finally, equation (8) can now be written as

$$q(n) = \left( \frac{n}{1+\phi} \right)^{\frac{1}{1-\alpha}} \frac{\alpha\gamma}{1-\alpha} \left( \frac{1+\phi}{n_0} \left( \frac{1-\frac{\alpha}{1-\alpha}\phi}{1-\alpha\phi} \frac{(1-\alpha)E}{\gamma} \right)^{\phi} \right)^{\frac{\alpha}{1-\alpha}} \quad (13)$$

Using this example it becomes relatively straightforward to illustrate the comparison between the optimal competition benchmark and the Cournot market outcome.

## 4 Comparing optimal competition with the market outcome: Cournot case

In this section we compare the optimal competition outcome with the market outcome in case of Cournot competition. We show that three effects determine the difference between the optimal and market outcome: monopoly power effect, appropriability effect and rent creation effect. These effects work in opposite directions and hence it is not a priori clear whether the optimal competition outcome is more competitive than the Cournot outcome. Looking at the example of a CES utility function and Pareto efficiency distribution we show that for  $\gamma$  sufficiently high the Cournot outcome is too competitive.

### 4.1 General case

First, we derive the demand function that a firm faces under Cournot competition. Consumers maximize utility under the budget constraint that they are willing to spend  $E$  in this market, that is they solve

$$\max \int_{n_0}^{n_1} v(q(n)) f(n) dn$$

$$\text{subject to } \int_{n_0}^{n_1} p(n) q(n) f(n) dn = E$$

where  $p(n)$  denotes the price of a good produced by a firm with efficiency  $n$ .<sup>20</sup> Let  $\mu$  denote the Lagrange multiplier for the budget constraint, then the first order condition for  $q(n)$  can be written as

$$v'(q(n)) = \mu p(n)$$

Hence, each firm faces a demand curve of the form

$$p(q) = \frac{v'(q)}{\mu}$$

and firms view  $\mu$  as an exogenous parameter that their output decisions do not affect. The Lagrange multiplier  $\mu$  is determined by the budget constraint which can be written as

$$\int_{n_c}^{n_1} \frac{v'(q(n))}{\mu} q(n) f(n) dn = E$$

where  $n_c$  denotes the least efficient firm that still produces under Cournot competition (to be determined below). Note that under Cournot competition firms are right that they cannot affect  $\mu$  in the sense that their own direct effect on the budget constraint is negligible and they conjecture that  $\frac{dq(n)}{dq(i)} = 0$  for  $n \neq i$ . Hence, a firm in the market with efficiency  $n$  chooses output level  $q(n)$  to solve

$$\max_q \left( p(q) - \frac{1}{n} \right) q$$

and its profits equal

$$\pi(n) = \max_q \left\{ \frac{v'(q)q}{\mu} - \frac{q}{n} \right\}$$

The first order condition for  $q(n)$  can be written as

$$v''(q(n))q(n) + v'(q(n)) = \frac{\mu}{n} \quad (14)$$

Comparing this equation to equation (5) which determines output of firm  $n$  under optimal competition we see two differences. First, on the left hand side the Cournot outcome features the term  $v''(q(n))q(n) < 0$  which does not appear in the optimal competition benchmark. This is the *monopoly power effect* that tends to reduce output in the private outcome as compared to the optimal competition benchmark. Firms take into account that increasing output  $q$  tends to reduce the price at which they sell ( $\frac{dp(q)}{dq} = \frac{v''(q)}{\mu} < 0$ ). On the right hand side of (14) we find the private marginal cost  $\frac{1}{n}$  instead of the industry marginal cost  $MC(n)$  in (5). Since  $MC(n) > \frac{1}{n}$  this tends to raise output in the

<sup>20</sup>Note that to simplify notation we have used that firms with the same efficiency level choose the same price-output combination in a (symmetric) Cournot equilibrium.

private outcome above output in the optimal competition benchmark. We call this the *rent creation effect*. Low efficiency firms tend to produce too much in the private outcome thereby raising the rents that accrue to more efficient firms. In order to understand the overall effect (including the effect on  $\mu$ ) we need to derive the least efficient firm  $n_c$  that still enters in the Cournot outcome. This is determined as the efficiency level at which a firm generates enough profits to pay for the entry cost  $\gamma$ ,

$$\frac{v'(q(n_c))}{\mu} q(n_c) - \frac{q(n_c)}{n_c} = \gamma$$

Writing this as

$$v'(q(n_c)) q(n_c) = \mu \left[ \frac{q(n_c)}{n_c} + \gamma \right] \quad (15)$$

and comparing this equation with (6) we see two effects appearing. On the right hand side we have again the rent creation effect:  $MC(n) > \frac{1}{n}$ : by entering a firm creates additional rents for firms with higher efficiency as they can now mimic this firm. Hence in the private outcome there tends to be too much entry. However, comparing the left hand sides of these two equations, we see the *appropriability effect* which tends to lead to insufficient entry in the private outcome since  $v'(q)q < v(q)$  for a strictly concave function  $v(\cdot)$ . The intuition is that a social planner sees the utility created by a firm,  $v(q)$ , as the incentive to enter while a firm looks at the revenue which is generated by entering,  $v'(q)q$ . Since firms cannot appropriate the whole consumer surplus in this model, the private incentive to enter falls short of the social incentive.

Hence when comparing the optimal competition outcome with the private (Cournot) outcome along both the output and the entry dimension, we see effects pulling in opposite directions. On the one hand, the rent creation effect leads to excess entry and production in the private outcome as compared to the optimal outcome. On the other hand, the monopoly power effect leads to output levels that are too low and the appropriability effect leads to insufficient entry in the private outcome as compared to the optimum. Hence it is impossible to derive an unambiguous comparison of the Lagrange multipliers  $\mu$  and  $\lambda$ . Thus we cannot say, in general, whether the private outcome is more or less competitive than the optimal competition benchmark. To get further intuition on this issue we consider the special case with  $v(q) = \frac{1}{\alpha}q^\alpha$  and the Pareto efficiency distribution introduced in example 3.<sup>21</sup>

<sup>21</sup>The reason why this combination of utility function and efficiency distribution makes the comparison particularly easy is (as we will see below) that they lead to fixed mark ups of prices over private marginal costs  $\frac{1}{\alpha}$ . In the private outcome this mark up is determined by  $\alpha$  and in the optimal competition benchmark by  $\phi$ .

## 4.2 CES utility and Pareto efficiency distribution

Assuming that  $v(q) = \frac{1}{\alpha}q^\alpha$ ,  $F(n) = 1 - \left(\frac{n_0}{n}\right)^\phi$  for  $n > n_0$  with  $\phi < \frac{1-\alpha}{\alpha}$  we can write output in the Cournot outcome as

$$q^c(n) = \left(\frac{\alpha n}{\mu}\right)^{\frac{1}{1-\alpha}}$$

Using this, equation (15) can be written as

$$\mu^{\frac{1}{1-\alpha}} = \frac{1-\alpha}{\gamma} (\alpha n_c)^{\frac{\alpha}{1-\alpha}} \quad (16)$$

and the budget constraint can be written as

$$\mu^{\frac{1}{1-\alpha}} = \frac{1}{E} \alpha^{\frac{\alpha}{1-\alpha}} \int_{n_c}^{+\infty} n^{\frac{\alpha}{1-\alpha}} f(n) dn \quad (17)$$

Standard manipulation of these equations yields the following comparison with the optimal competition benchmark derived in example 3.

**Proposition 2** *Let  $\bar{\gamma}$  and  $\tilde{\gamma}$  denote resp.  $\bar{\gamma} = E(1 - \alpha(1 + \phi)) > 0$  and  $\tilde{\gamma} = E \frac{1-\alpha(1+\phi)}{1-\alpha\phi} > \bar{\gamma}$ . Then we find that if*

- $\gamma \in [0, \bar{\gamma})$  then  $n_c = n_w = n_0$  and  $q^c(n) < q^o(n)$  for all  $n \in [n_0, n_1]$ ;
- $\gamma = \bar{\gamma}$  then  $n_c = n_w = n_0$  and  $q^c(n) = q^o(n)$  for all  $n \in [n_0, n_1]$ ;
- $\gamma \in (\bar{\gamma}, \tilde{\gamma})$  then  $n_c > n_w = n_0$  and  $q^c(n) > q^o(n)$  for all  $n \in [n_c, n_1]$ ;
- $\gamma > \tilde{\gamma}$  then  $n_c > n_w > n_0$  and  $q^c(n) > q^o(n)$  for all  $n \in [n_c, n_1]$ .

In words, if  $\gamma$  is big enough that some firms do not enter in the market outcome ( $n_c > n_0$ ) then the Cournot outcome is more competitive than the optimal competition benchmark. This can be seen by using definition 1 with  $\bar{n} = n_c$ . The intuition is the rent creation effect: firms produce too much output thereby generating excess rents for high efficiency firms. Both the high output and the excess rents cost money and therefore there is no budget left for entering firms. Hence the market outcome here is biased towards quantity at the expense of variety.

Hence from a social point of view, competition authorities should not only intervene when competition is too soft in an industry, but there may also be a call for action when competition is too intense. Hence without recourse to dynamic models with incomplete information as used in the literature on entry deterrence and predation, we have formalized here the idea that competition in the market may be too intense.

Further, there is a knife edge case ( $\gamma = \bar{\gamma}$ ) where Cournot competition is actually optimal competition. That is, there is no mechanism (operating under the information constraint assumed here) that can improve upon the Cournot outcome. Cournot competition maximizes consumer welfare.

Finally, when entry costs are rather low ( $\gamma < \bar{\gamma}$ ) then Cournot competition is not competitive enough. The intuition is that Cournot competition in this case where all firms enter ( $n_c = n_0$ ) gives away excess profits in the sense that  $\pi(n_0) > \gamma$ . The optimal competition outcome, in contrast, has as a condition  $\pi(n_w) = \gamma$ , as shown in lemma 1. Therefore there is not enough budget left in the Cournot outcome to have output levels as high as in the optimal outcome.

Note how this result contrasts with the conventional wisdom that competition authorities need not worry too much about industries with low entry costs. This intuition is based on a Cournot model with symmetric firms. In that case, as the entry cost goes to zero, the Cournot outcome converges to the perfect competition outcome. Above we take seriously the idea that not all firms are equally efficient. Then with a CES utility function and Pareto distribution of firms' efficiency levels we find that Cournot competition becomes too slack exactly when the entry cost is low. This is not to say that a Pareto efficiency distribution is necessarily a realistic assumption. The point is that with this assumption and a CES utility function it is straightforward to capture the idea that the market outcome can be too competitive. Unfortunately, we are not aware of empirical results on the efficiency distribution of firms in an industry (this in contrast to the public economics literature on wage and income distributions).

## 5 Extensions

This section considers three simple extensions of the model above. First, we analyze how different objectives for the competition authority affect the optimal competition benchmark. Second, we generalize the utility function in equation (1) to the case where expenditure per market is not fixed but depends on relative prices. Third, we use the optimal competition benchmark to derive under which conditions regulating a monopolist is better than opening up the industry for competition.

### 5.1 Different objectives

As surveyed by Motta (2003), historically there have been a number of objectives specified for competition policy. Above we have focused on consumer welfare, but other possibilities are total welfare, employment, protection of the environment, supporting national champions or defending small firms. To illustrate how the optimal competition benchmark is affected by different objectives for the competition authority, we consider an objective function that puts weight on firms' profits and employment in the industry.

In this way, we show that a competition authority that wants to maximize total welfare instead of consumer surplus tends to soften competition, as is indeed often claimed. However, the idea that a competition authority that takes industry employment into account should soften competition, is not correct in the framework here.

We write the objective function of the competition authority now as

$$\int_{n_w}^{n_1} \left[ v(q(n)) + \zeta(\pi(n) - \gamma) + \xi \frac{q(n)}{n} \right] f(n) dn$$

where  $\zeta \geq 0$  is the weight attached to producer surplus and  $\xi \geq 0$  is the weight attached to employment. Note that we assume here that output in this industry is produced with labor only and that the wage equals one. Hence employment by a firm with efficiency  $n$  equals  $\frac{q(n)}{n}$ . Maximizing this objective function under the budget constraint (7) and using  $\lambda$  as the Lagrange multiplier for this budget constraint, we can write the equations determining  $q(n)$ ,  $n_w$  and  $\lambda$  as follows

$$v'(q(n)) = \lambda MC(n) - \xi \frac{1}{n} - \zeta \frac{1 - F(n)}{n^2 f(n)} \quad (18)$$

$$v(q(n_w)) = \left( \lambda MC(n_w) - \xi \frac{1}{n_w} - \zeta \frac{1 - F(n_w)}{n_w^2 f(n_w)} \right) q(n_w) + \lambda \gamma \quad (19)$$

$$\int_{n_w}^{n_1} MC(n) q(n) f(n) dn + (1 - F(n_w)) \gamma = E \quad (20)$$

With these equations we can derive the following results. We need upperbounds on  $\zeta$  and  $\xi$  to make sure that the system determined by (19) and (20) remains stable.

**Proposition 3** *Assume that the density function  $f(\cdot)$  satisfies the monotone-hazard-rate condition. Then there exist  $\bar{\zeta}, \bar{\xi} > 0$  such that a rise in  $\zeta < \bar{\zeta}$  makes the optimal competition outcome less competitive and a rise in  $\xi < \bar{\xi}$  makes the optimal competition outcome more competitive.*

In other words, when a competition authority's goal moves from consumer welfare to total welfare (a rise in  $\zeta$ ) it softens competition in an industry. This reduces output levels for high efficiency firms and hence allows less efficient firms to enter the industry. The entry of these less efficient firms creates rents for the more efficient firms. Thus total producer surplus rises. Note that this is a comparative static exercise within optimal competition outcomes. As illustrated in example 1 (for  $c \in (\frac{2}{7}, \frac{1}{2})$ ), a more competitive market outcome can raise industry profits. However, a more competitive 'optimal competition' outcome always features lower industry profits.

If a competition authority increases the weight on employment then it makes competition more intense in an industry. This is contrary to common wisdom where employment considerations are used to defend soft competition in an industry. The intuition for the result here is as follows. To maximize employment subject to the budget constraint, the competition authority has to minimize rents. That is, output should be produced by firms with low  $MC(n)$  and these are high efficiency firms. Having low efficiency firms enter the industry and produce output has as an advantage that these firms directly generate high employment for low output levels (as  $\frac{1}{n}$  is high for these firms), however the rent



creation effect overturns this. Therefore employment increases with competition.

The result that employment increases with competition does depend on the assumptions made here, in particular the one shot game nature of the model. If, for instance, one would consider a dynamic model where firms experience cost shocks from one period to the next and where the labor market is described by a search and matching model as in Mortensen and Pissarides (1999) the following effect works in the opposite direction. If competition intensifies, firms' output levels respond more strongly to cost shocks and therefore the inflow in and outflow from unemployment increases. This effect tends to raise unemployment.

## 5.2 Related markets

In equation (1) we made the convenient assumption that consumers have Cobb Douglas preferences over industries. This implies that expenditure in each industry is fixed and there are no interindustry effects. In this section we consider a more general utility set up which does allow for such cross over effects. To illustrate the main difference with the analysis above, we ask the question how does deregulation in one market (modelled through a fall in entry cost in that market) affect the optimal competition outcome in another market?

To analyze this point in the most simple set up, we assume that there are two industries in the economy, denoted by 1 and 2. As in section 2, let  $Q_i$  ( $i = 1, 2$ ) denote the utility derived in sector  $i$ . Now instead of equation (1), we assume that overall utility is given by

$$U(Q_1, Q_2)$$

where the function  $U(., .)$  is increasing and concave. From the analysis above we know that, for given  $E_i$ , utility  $Q_i$  equals

$$Q_i = \max_{q_i(\cdot), n_{wi}} \int_{n_{wi}}^{n_{1i}} v_i(q_i(n)) f_i(n) dn - \lambda_i \left[ \int_{n_{wi}}^{n_{1i}} MC_i(n) q_i(n) f_i(n) + [1 - F_i(n_{wi})] \gamma_i - E_i \right]$$

where  $f_i(\cdot)$  (resp.  $F_i(\cdot)$ ) denotes the density (distribution) function of efficiency in sector  $i$  with support  $[n_{0i}, n_{1i}]$ ,  $n_{wi}$  is the least efficient firm to enter in sector  $i$ ,  $v_i(\cdot)$  is the per product utility function in sector  $i$ ,  $\lambda_i$  denotes the Lagrange multiplier for the budget constraint in sector  $i$ ,  $\gamma_i$  the entry cost in this sector and  $MC_i(n) = \frac{1}{n} \left[ 1 + \frac{1 - F_i(n)}{f_i(n)n} \right]$  the industry marginal costs of firm  $n$  in sector  $i$ . From proposition 1 we know that  $Q_i$  is determined by the entry cost in sector  $i$  and the expenditure in sector  $i$ . However, unlike the analysis above,  $E_i$  is an endogenous variable now, since it is no longer exogenously fixed by  $E_i = \alpha_i Y$ . In particular,  $E_i$  is determined by the following optimization problem.

$$\max_{E_1, E_2} U(Q_1(E_1, \gamma_1), Q_2(E_2, \gamma_2)) \text{ subject to } E_1 + E_2 = Y \quad (21)$$

The following result derives sufficient conditions under which deregulation in sector 2 (in the sense that  $\gamma_2$  falls) leads to more intense optimal competition in sector 1.

**Proposition 4** *If the following inequalities hold*

$$\frac{\partial^2 U(Q_1, Q_2)}{\partial Q_1 \partial Q_2} \geq 0 \text{ and}$$

$$[1 - F_2(n_{w2})] - \frac{f_2(n_{w2}) [\gamma_2 + q_2(n_{w2}) MC_2(n_{w2})]}{q_2(n_{w2}) [-MC_2'(n_{w2})]} \geq 0$$

*then a fall in  $\gamma_2$  makes the optimal competition outcome in sector 1 more competitive.*

The intuition for this result is as follows. The first inequality implies that the goods in sectors 1 and 2 are complementary. As the utility  $Q_2$  derived from sector 2 goes up (as it does after a fall in  $\gamma_2$ ), the marginal utility derived from sector 1 rises as well. This tends to raise the amount of money  $E_1$  spent in sector 1. Lemma 5 implies that higher  $E_1$  intensifies competition in sector 1. Next, using again lemma 5, the second inequality implies that the fall in  $\gamma_2$  reduces the marginal value of expenditure,  $\lambda_2$ , in sector 2. This tends to reduce  $E_2$  and hence raises  $E_1 = Y - E_2$  which, as above, intensifies competition in sector 1. Hence under these conditions deregulation in sector 2 should lead to a more competitive outcome in sector 1 as well.

### 5.3 Regulation vs liberalization

An important policy question is whether a certain industry should be regulated or liberalized. For a discussion of the trade offs in this case, see for instance Armstrong, Cowan and Vickers (1994). Comparing the regulated outcome with the industry outcome after competition has been introduced, there is a clear benchmark for the equilibrium under regulation but not for the market outcome. For instance, Armstrong, Cowan and Vickers (1994) compare the optimal regulation outcome with Cournot and Bertrand competition in the industry. But this cannot give sufficient conditions for regulation to dominate liberalization. To illustrate, if regulation leads to higher welfare than both Cournot and Bertrand competition, there may still be another way of organizing the liberalized market outcome that does better than regulation.

Here the optimal competition outcome defined above yields the desired benchmark for the liberalized outcome. If one can show that the regulated outcome yields higher welfare than the optimal competition outcome then this is a sufficient condition for regulation to be optimal. Indeed no market outcome in this case will generate higher welfare than the regulated outcome. The following example illustrates this by focusing on the trade off between the bigger capacity for the regulated monopolist and the higher efficiency in the market outcome.

**Example 4** *This example works with different specifications of utility and cost structures than the ones assumed above. The assumptions made here are closer to the ones usually made in the optimal regulation literature. In particular, we assume here that goods are homogenous, which seems appropriate when considering markets like electricity or water. The utility of consuming  $q \in \mathfrak{R}_+$  units equals  $v(q) = vq$  with the scalar  $v > 0$ . Further, the cost of spending an amount  $E$  in this industry equals  $\lambda E$ , where  $\lambda > 0$  denotes the opportunity cost of spending money in this industry (instead of on other goods in the economy). We assume here that  $\lambda$  is exogenously given. Finally, in the case of the regulated monopolist, the cost structure of the firm is*

$$c(q) = \frac{1}{n}q + \frac{1}{2}\phi q^2$$

with  $\phi > 0$  common knowledge, but the efficiency level  $n$  cannot be observed by the regulator. We assume that  $n$  is uniformly distributed on  $[0, n_1]$ . The regulator offers a menu of revenue and output combinations  $(R(n), q(n))$  and the firm reports efficiency level  $\tilde{n}$  to maximize its profits

$$\pi(n) = \max_{\tilde{n}} \left[ R(\tilde{n}) - \frac{1}{n}q(\tilde{n}) - \frac{1}{2}\phi(q(\tilde{n}))^2 \right]$$

*Incentive Compatibility and Individual Rationality imply*

$$\begin{aligned} \pi'(n) &= \frac{q(n)}{n^2} \\ \pi(n) &\geq 0 \end{aligned}$$

*We assume that the regulator maximizes consumer surplus and hence leaves no rents for the least efficient type to participate. Therefore we have*

$$\pi(n) = \int_0^n \frac{q(t)}{t^2} dt$$

*Since expenditure on a type  $n$  firm can be written as*

$$R(n) = \pi(n) + \frac{1}{n}q(n) + \frac{1}{2}\phi(q(n))^2$$

*consumer welfare under regulation equals*

$$W^R = \max_{q(\cdot)} \int_0^{n_1} \left[ vq(n) - \lambda \left( \pi(n) + \frac{1}{n}q(n) + \frac{1}{2}\phi(q(n))^2 \right) \right] \frac{1}{n_1} dn$$

*Using integration by parts as in the proof of lemma 2, it is routine to derive that*

$$q(n) = \begin{cases} 0 & \text{for } n < n_w \\ \frac{v}{\lambda\phi} - \frac{n_1}{\phi n^2} & \text{for } n \in [n_w, n_1] \end{cases}$$

where  $n_w = \sqrt{\frac{n_1\lambda}{v}}$ . Substituting this into the expression for  $W^R$  we find that

$$W^R = \frac{v}{2n_1\phi} \left[ \frac{vn_1}{\lambda} + 2 - \frac{1}{3} \frac{\lambda}{vn_1} - \frac{8}{3} \sqrt{\frac{n_1v}{\lambda}} \right]$$

In this example we take liberalization of the market to mean the following. The monopolist is split up in smaller firms and other firms are allowed to enter the market. The cost function of a firm now takes the form

$$c(q) = \frac{1}{n}q + \frac{1}{2}\phi(1 + \delta)q^2$$

where the distribution of costs after liberalization is uniform on  $[0, n_1(1 + \rho)]$  with  $\delta, \rho > 0$ . This captures the following two effects of liberalization. On the one hand,  $\delta > 0$  formalizes the idea that after liberalization the firms are smaller and hence have smaller capacity than the regulated monopolist. On the other hand,  $\rho > 0$  implies that, after liberalization, increased competition and entry by new firms lead to a gain in expected efficiency. It is routine to verify that the optimal competition outcome after liberalization has welfare equal to

$$W^L = \frac{v}{2n_1(1 + \rho)\phi(1 + \delta)} \left[ \frac{vn_1(1 + \rho)}{\lambda} + 2 - \frac{1}{3} \frac{\lambda}{vn_1(1 + \rho)} - \frac{8}{3} \sqrt{\frac{n_1(1 + \rho)v}{\lambda}} \right]$$

Hence a sufficient condition for regulation to be preferable over any market outcome is  $W^R > W^L$ , which can be written as

$$\delta > \frac{-2\rho - \frac{1}{3} \frac{\lambda}{vn_1} \left( \frac{1}{1 + \rho} - (1 + \rho) \right) - \frac{8}{3} \sqrt{\frac{n_1(1 + \rho)v}{\lambda}} (1 - \sqrt{1 + \rho})}{(1 + \rho) \left[ \frac{vn_1}{\lambda} + 2 - \frac{1}{3} \frac{\lambda}{vn_1} - \frac{8}{3} \sqrt{\frac{n_1v}{\lambda}} \right]}$$

In words, the economy of scale advantage of the monopolist,  $\delta$ , needs to be sufficiently big for regulation to be optimal. One can check that the right hand side of this inequality falls with  $\rho$  and  $\frac{v}{\lambda}$ . If  $\rho$  is small, there is hardly a gain in efficiency due to liberalization and hence regulation is more likely to be optimal. Similarly, if  $\frac{v}{\lambda}$  is big, this industry is important compared to the rest of the economy. This implies that output will be high and hence the economies of scale of the monopolist are important for welfare.

## 6 Conclusion

Instead of analyzing how a change in the number of firms in an industry or how a change in firms' conduct affects welfare, this paper asks 'How competitive should an industry be?'. To answer this question we have set up a mechanism design problem. The solution of this problem is called the optimal competition outcome. Hence we do not compare the way in which output depends on firms' efficiency under different equilibrium configurations (say, Cournot and Bertrand Nash equilibrium) but expand the domain to any incentive compatible output function. Surprisingly, this domain expansion simplifies the analysis. The solution can be characterized in a simple two dimensional diagram determining the optimal number of firms and the optimal conduct of firms in terms of aggressiveness of interaction. The main reason why this works is a new way to identify

competition in a mechanism design problem. This gain in simplicity comes at a cost. How should one interpret the optimal competition outcome since it is not necessarily an equilibrium outcome of a game between firms? This section summarizes the results of our paper keeping this caveat in mind.

First, we have shown that under some conditions Cournot competition can be optimal competition. Hence in that case an equilibrium outcome is in fact the best possible outcome. This turns out to be a knife edge result, and the more interesting implication here is that the market outcome can be too competitive compared to the optimal outcome. In such a case, a merger that makes competition less intense in the industry is not necessarily bad.

Second, the optimal competition outcome gives an upperbound on what any market outcome can achieve. This is useful when the market outcome is compared to the possibility of regulating an industry. If the regulated outcome leads to higher welfare than the optimal competition outcome, this is a sufficient condition for regulation to dominate liberalization of an industry.

Third, although the optimal competition outcome may not be implementable it still gives a competition authority an idea of the best possible outcome it can achieve by intervening in an industry. If the gain in welfare from moving from the current outcome to the optimal outcome in an industry is small, there is little use for intervention in the industry. In that case, the scarce resources of the competition authority can be used more productively in scrutinizing other industries.

Finally, the framework introduced here allows us to do comparative static exercises to see how entry costs, industry expenditure and the competition authority's objective function affect the optimal intensity of competition in an industry. We showed, for instance, that for given conduct of firms there is more need for intervention in industries where more money is spent by consumers.

Using the framework of optimal competition introduced here, we see the following areas for future research. First, optimal competition is derived under an information constraint for the competition authority. We view this outcome as a benchmark which is not necessarily implementable for a competition authority. More restrictions can be added to the problem to derive an outcome which is implementable by competition authorities. Second, above we have looked in a very simple way at effect of R&D on optimal competition. This comes back to the debate in the endogenous growth literature and recent empirical literature on whether intense competition or monopoly power leads to more innovation. Using the framework here one could analyze which parameters of the R&D process call for more (or less) intense competition to stimulate innovation. Other questions that one can analyze in this framework are the following. Should industries where the majority of firms are run by professional managers (instead of owners) be more competitive than industries with firms run by their owners? In other words, under which conditions should competition be intensified to alleviate the contractual problems of a principal agent relationship. Finally, if the demand side of a market is very concentrated does this justify less intense competition in the industry itself? This comes back to the idea that buyer power can be used as a justification for a merger in the industry supplying these buyers to

create countervailing power.

### References

- Aghion, P., N. Bloom, R. Blundell, R. Griffith and P. Howitt, 2002, Competition and innovation: an inverted U relationship, NBER working paper no. 9269.
- Armstrong, M. , S. Cowan and J. Vickers, 1994, *Regulatory reform: Economic analysis and British experience*, MIT Press.
- Blanchard, O. and F. Giavazzi, 2001, Macroeconomic effects of regulation and deregulation in goods and labor markets, NBER working paper no. 8120.
- Bolton, P., J.F. Brodley and M.H. Riordan, 2000, Predatory pricing: strategic theory and legal policy, *The Georgetown Law Journal*, Vol. 88: 2239-2330.
- Boone, J., Competition, CEPR discussion paper no. 2636.
- Bulow, J. and P. Klemperer, 1996, Auctions versus negotiations, *American Economic Review* Vol. 86 (1), 180-194.
- Bulow, J. and J. Roberts, 1989, The simple economics of optimal auctions, *Journal of Political Economy* Vol. 97 (5), 1060-1090.
- Dixit, A. and J. Stiglitz, 1977, Monopolistic competition and optimum product diversity, *American Economic Review* 67: 297-308.
- Fudenberg, D. and J. Tirole, 1991, *Game theory*, MIT Press.
- Gilbert, R. and P. Klemperer, 2000, An equilibrium theory of rationing, *Rand Journal of Economics* 31 (1), pp. 1-21.
- Guesnerie, R. and J.-J Laffont, 1984, A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm, *Journal of Public Economics*, 25, pp. 329-369.
- Mankiw, G. and M. Whinston, 1986, Free entry and social inefficiency, *Rand Journal of Economics* 17: 48-58.
- Moldovanu, B. and A. Sela, 2001, The optimal allocation of prizes in contests, *American Economic Review*, Vol. 91 (3), pp. 542-558.
- Mortensen, D. and C. Pissarides, 1999, New developments in models of search in the labor market, in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, Volume 3, North-Holland, Amsterdam.
- Motta, M. , 2003, *Competition policy: theory and practice*, Cambridge University Press.
- Reiss, P. and F. Wolak, 2002, Structural econometric modeling: rationales and examples from industrial organization, draft chapter for Handbook of Econometrics.
- Vickers, J., Entry and competitive selection, Mimeo Oxford University.

## 7 Appendix: Proofs of results

### Proof of Lemma 2

The maximization problem under the expenditure constraint can be written as

$$\max_{q(\cdot), \pi(\cdot), n_w} \int_{n_w}^{n_1} \left\{ v(q(n)) f(n) - \lambda \left[ \pi(n) + \frac{q(n)}{n} \right] f(n) \right\} dn + \lambda E$$

where  $\lambda$  is the Lagrange multiplier. Now substituting  $\pi(n) = \gamma + \int_{n_w}^n \frac{q(t)}{t^2} dt$  into this expression and using partial integration, in particular  $\int_{n_w}^{n_1} \left( \int_{n_w}^n \frac{q(t)}{t^2} dt \right) f(n) dn = \int_{n_w}^{n_1} \frac{q(n)}{n^2} (1 - F(n)) dn$ , this can be written as

$$\int_{n_w}^{n_1} \{v(q(n)) - \lambda q(n) MC(n)\} f(n) dn - \lambda(1 - F(n_w)) \gamma + \lambda E$$

when maximizing with respect to  $q(\cdot)$  and  $n_w$  the last term,  $\lambda E$ , can be dropped without loss of generality. Finally,  $\lambda$  is determined by the expenditure constraint which can be written as

$$\int_{n_w}^{n_1} MC(n) q(n) f(n) dn + (1 - F(n_w)) \gamma = E$$

Q.E.D.

#### Proof of Proposition 1

The Euler equation for  $q(\cdot)$  can be written as

$$v'(q(n)) = \lambda MC(n)$$

Differentiating  $\int_{n_w}^{n_1} \{v(q(n)) - \lambda q(n) MC(n)\} f(n) dn - \lambda(1 - F(n_w)) \gamma$  with respect to  $n_w$  yields

$$v(q(n_w)) = \lambda [q(n_w) MC(n_w) + \gamma]$$

Finally, as noted in the proof of lemma 2,  $\lambda$  is determined by the expenditure constraint  $\int_{n_w}^{n_1} MC(n) q(n) f(n) dn + (1 - F(n_w)) \gamma = E$ . Q.E.D.

#### Proof of Lemma 4

Using equation (3) for  $\pi(n)$  we can write

$$\frac{\partial \pi(n)}{\partial \lambda} = -\frac{q(n_w)}{n_w^2} \frac{\partial n_w}{\partial \lambda} + \int_{n_w}^n \frac{1}{t^2} \frac{\partial q(t)}{\partial \lambda} dt$$

From equation (5) it follows that  $\frac{\partial q(t)}{\partial \lambda} < 0$  and from equation (7) that  $\frac{\partial n_w}{\partial \lambda} < 0$ , hence  $\frac{\partial \pi(n)}{\partial \lambda} > 0$  for  $n$  close to  $n_w$ .

The price cost margin for firm  $n$  can be written as

$$\frac{R(n) - \frac{q(n)}{n}}{R(n)} = \frac{\pi(n)}{\pi(n) + \frac{q(n)}{n}} = \frac{1}{1 + \frac{q(n)}{n\pi(n)}}$$

Hence the price cost margin is increasing in  $\lambda$  if and only if

$$\frac{\partial \left( \frac{n\pi(n)}{q(n)} \right)}{\partial \lambda} > 0$$

Using equation (3) for  $\pi(n)$ , we can write this as

$$\begin{aligned} \frac{\partial \left( \frac{n}{q(n)} \left[ \gamma + \int_{n_w}^n \frac{q(t)}{t^2} \right] \right)}{\partial \lambda} &= n \left\{ \frac{-\gamma}{(q(n))^2} \frac{\partial q(n)}{\partial \lambda} + \int_{n_w}^n \frac{\partial \left( \frac{q(t)}{q(n)} \right)}{\partial \lambda} \frac{1}{t^2} dt - \frac{q(n_w)}{q(n)} \frac{1}{n_w^2} \frac{\partial n_w}{\partial \lambda} \right\} \\ &> 0 \end{aligned}$$

From equation (5) it follows that  $\frac{\partial q(n)}{\partial \lambda} < 0$ . Further, the assumption that  $\frac{v'(q_1)}{v'(q_2)}$  is a function of  $\frac{q_1}{q_2}$  only, together with  $\frac{v'(q(n))}{v'(q(t))} = \frac{MC(n)}{MC(t)}$  (from equation (5)) implies that  $\frac{q(t)}{q(n)}$  is not affected by  $\lambda$ . Finally,  $\frac{\partial n_w}{\partial \lambda} < 0$  from equation (7).

Hence we have shown that  $sign \left[ \frac{\partial \left( \frac{R(n) - \frac{q(n)}{n}}{R(n)} \right)}{\partial \lambda} \right] = sign \left[ \frac{\partial \left( \frac{-\frac{n}{q(n)} [\gamma + \int_{n_w}^n \frac{q(t)}{t^2}] \right)}{\partial \lambda} \right] >$

0. Q.E.D.

**Proof of Lemma 5**

Linearizing equations (6) and (7) with respect to  $n_w, \lambda, E$  and  $\gamma$  we get

$$\begin{pmatrix} q(n_w) MC(n_w) + \gamma & \lambda q(n_w) MC'(n_w) \\ \int_{n_w}^{n_1} MC(n) f(n) \frac{dq(n)}{d\lambda} dn & -f(n_w) [q(n_w) MC(n_w) + \gamma] \end{pmatrix} \begin{pmatrix} d\lambda \\ dn_w \end{pmatrix} = \begin{pmatrix} -\lambda d\gamma \\ -(1 - F(n_w)) d\gamma + dE \end{pmatrix}$$

Writing  $\Delta$  for the determinant of the matrix on the left hand side, we find that

$$\begin{aligned} \Delta &= (q(n_w) MC(n_w) + \gamma) f(n_w) (-[q(n_w) MC(n_w) + \gamma]) \\ &\quad - \left( \int_{n_w}^{n_1} MC(n) f(n) \frac{dq(n)}{d\lambda} dn \right) (\lambda q(n_w) MC'(n_w)) \\ &< 0 \end{aligned}$$

because  $\frac{dq(n)}{d\lambda} < 0$  and  $MC'(n_w) \leq 0$ .

Inverting the matrix, we can write this as

$$\begin{pmatrix} d\lambda \\ dn_w \end{pmatrix} = \frac{1}{-\Delta} \begin{pmatrix} -f(n_w) [q(n_w) MC(n_w) + \gamma] & -\lambda q(n_w) MC'(n_w) \\ -\int_{n_w}^{n_1} MC(n) f(n) \frac{dq(n)}{d\lambda} dn & q(n_w) MC(n_w) + \gamma \end{pmatrix} \times \begin{pmatrix} \lambda d\gamma \\ (1 - F(n_w)) d\gamma - dE \end{pmatrix}$$

Hence we find that

$$\begin{aligned} \frac{d\lambda}{dE} &= \frac{\lambda q(n_w) MC'(n_w)}{-\Delta} \leq 0 \\ \frac{dn_w}{dE} &= -\frac{q(n_w) MC(n_w) + \gamma}{-\Delta} < 0 \\ \frac{d\lambda}{d\gamma} &= \lambda \frac{-f(n_w) [q(n_w) MC(n_w) + \gamma] - q(n_w) MC'(n_w) (1 - F(n_w))}{-\Delta} \end{aligned}$$

and thus

$$\frac{d\lambda}{d\gamma} \geq 0 \text{ if and only if } [1 - F(n_w)] - \frac{f(n_w) [\gamma + q(n_w) MC(n_w)]}{q(n_w) [-MC'(n_w)]} \geq 0$$



Finally, the effect of  $\gamma$  on  $n_w$  is

$$\frac{dn_w}{d\gamma} = \frac{-\lambda \int_{n_w}^{n_1} MC(n) f(n) \frac{dq(n)}{d\lambda} dn + [q(n_w) MC(n_w) + \gamma] (1 - F(n_w))}{-\Delta} > 0$$

Q.E.D.

**Proof of Corollary 1**

Note that the assumptions  $n_0 > 0, v''(\cdot) < 0$  and  $\lim_{q \downarrow 0} v'(q) = +\infty$  together with equations (5) and (6) imply that for  $\gamma = 0$  we get the corner solution  $n_w = n_0$ . To see this, first note that equation (5) implies that  $q(n) > 0$  irrespective of the value of  $\lambda \in \mathfrak{R}_+$ . Then dividing the equations (5) and (6), we get for  $\gamma = 0$  that

$$v'(q(n_w)) = \frac{v(q(n_w))}{q(n_w)}$$

which contradicts the strict concavity of the utility function  $v(\cdot)$ . Hence it must be the case that equation (6) yields a corner solution, that is

$$v(q(n_0)) > \lambda [q(n_0) MC(n_0)]$$

By continuity we find that  $n_w = n_0$  for small values of  $\gamma > 0$  as well. That is, there exists  $\gamma_* > 0$  such that  $\frac{\partial n_w}{\partial \gamma} = 0$  for  $\gamma \in [0, \gamma_*)$ . The expression for  $\frac{d\lambda}{d\gamma}$  in lemma 5 then implies that  $\frac{d\lambda}{d\gamma} > 0$ .

Now consider the case where  $\gamma$  becomes so big that  $n_w$  approaches  $n_1$ . Then the expression  $[1 - F(n_w)]$  in the equation for  $\frac{d\lambda}{d\gamma}$  in lemma 5 approaches 0 and hence we find  $\frac{d\lambda}{d\gamma} < 0$  for  $\gamma$  big enough. Q.E.D.

**Proof of Proposition 2**

To prove this proposition, we have to be a bit more careful with corner solutions for  $n_w$  and  $n_c$ . Therefore we restate the conditions for the optimal case and the Cournot case taking corner solutions explicitly into account.

First, consider the optimal competition outcome. The solution for  $n_w$  is given by equation (11) and we find the corner solution  $n_w = n_0$  if the right hand side of this equation is smaller than 1. In that case the value of  $\lambda$  is given by the budget constraint (9) (not by the first order condition for  $n_w$ , that is (10)). So we find the following two cases:

Case (O1):  $\gamma \leq \tilde{\gamma}$ . Then we find that

$$\begin{aligned} n_w &= n_0 \\ \lambda^{\frac{1}{1-\alpha}} &= \frac{(1+\phi)^{-\frac{\alpha}{1-\alpha}} n_0^{\frac{\alpha}{1-\alpha}} \frac{1}{1-\frac{\alpha}{1-\alpha}\phi}}{E-\gamma} \\ q^o(n) &= n^{\frac{1}{1-\alpha}} \frac{1}{1+\phi} n_0^{-\frac{\alpha}{1-\alpha}} \left(1 - \frac{\alpha}{1-\alpha}\phi\right) (E-\gamma) \end{aligned}$$

Case (O2):  $\gamma > \tilde{\gamma}$ . Then we find that  $n_w, \lambda$  and  $q^o(n)$  are determined by equations (11)-(13).

Next consider the Cournot outcome. Solving equations (16) and (17) for  $n_c$  we find that

$$\frac{n_c}{n_0} = \left( \frac{\gamma}{E} \frac{1}{1 - \alpha - \alpha\phi} \right)^\phi$$

Clearly,  $\gamma \leq \bar{\gamma}$  implies that  $n_c$  equals the corner solution  $n_c = n_0$ . In this case  $\mu$  is determined by the budget constraint (17) (that is, not by the free entry condition (16)). Thus we have the following two cases:

Case (C1):  $\gamma \leq \bar{\gamma}$ . Then we find that

$$\begin{aligned} n_c &= n_0 \\ \mu^{\frac{1}{1-\alpha}} &= \frac{1}{E} \alpha^{\frac{\alpha}{1-\alpha}} n_0^{\frac{\alpha}{1-\alpha}} \frac{1}{1 - \frac{\alpha}{1-\alpha}\phi} \\ q^c(n) &= \alpha n^{\frac{1}{1-\alpha}} E n_0^{-\frac{\alpha}{1-\alpha}} \left( 1 - \frac{\alpha}{1-\alpha}\phi \right) \end{aligned}$$

Case (C2):  $\gamma > \bar{\gamma}$ . Then we find that

$$\begin{aligned} \frac{n_c}{n_0} &= \left( \frac{\gamma}{E} \frac{1}{1 - \alpha - \alpha\phi} \right)^\phi \\ \mu^{\frac{1}{1-\alpha}} &= \frac{1-\alpha}{\gamma} \alpha^{\frac{\alpha}{1-\alpha}} n_0^{\frac{\alpha}{1-\alpha}} \left( \frac{\gamma}{E} \frac{1}{1 - \alpha - \alpha\phi} \right)^{\frac{\alpha\phi}{1-\alpha}} \\ q^c(n) &= \alpha n^{\frac{1}{1-\alpha}} \frac{\gamma}{1-\alpha} n_0^{-\frac{\alpha}{1-\alpha}} \left( (1 - \alpha - \alpha\phi) \frac{E}{\gamma} \right)^{\frac{\alpha\phi}{1-\alpha}} \end{aligned}$$

Combining the four cases above, we get the following three cases (I)-(III) for  $\gamma$ :

(I)  $\gamma < \bar{\gamma} (< \tilde{\gamma})$ : then we find that  $n_w = n_c = n_0$ . Further, dividing the expressions for  $q^o(n)$  and  $q^c(n)$  under (O1) and (C1) one gets

$$\frac{q^o(n)}{q^c(n)} = \frac{E - \gamma}{E} \frac{1}{\alpha(1 + \phi)}$$

It is routine to verify that  $\gamma < \bar{\gamma}$  implies  $\frac{q^o(n)}{q^c(n)} > 1$ .

If  $\gamma = \bar{\gamma}$  then it follows that  $n_w = n_c = n_0$  and  $q^o(n) = q^c(n)$ .

(II)  $\bar{\gamma} < \gamma \leq \tilde{\gamma}$ : then we find that  $n_w = n_0$  and  $n_c > n_0$ . Then the expressions for output under (O1) and (C2) imply

$$\frac{q^o(n)}{q^c(n)} = \frac{1}{\alpha(1 + \phi)} \frac{(1 - \alpha - \alpha\phi) \left( \frac{E}{\gamma} - 1 \right)}{\left( (1 - \alpha - \alpha\phi) \frac{E}{\gamma} \right)^{\frac{\alpha\phi}{1-\alpha}}}$$

We will now show that  $\bar{\gamma} < \gamma < \tilde{\gamma}$  implies  $\frac{q^o(n)}{q^c(n)} < 1$  for all  $n \geq n_c$ . Note that  $\frac{q^o(n)}{q^c(n)} < 1$  can be written as

$$\alpha(1 + \phi) \left( (1 - \alpha - \alpha\phi) \frac{E}{\gamma} \right)^{\frac{\alpha\phi}{1-\alpha}} - (1 - \alpha - \alpha\phi) \frac{E}{\gamma} + (1 - \alpha - \alpha\phi) > 0$$

To ease notation, define the variable  $x$  as  $x \equiv (1 - \alpha - \alpha\phi) \frac{E}{\gamma}$ , where the restrictions on  $\gamma$  (i.e.  $\bar{\gamma} < \gamma < \tilde{\gamma}$ ) imply that  $1 - \alpha\phi < x < 1$ . Hence we need to show that

$$g(x) > 0 \text{ for all } x \in \langle 1 - \alpha\phi, 1 \rangle$$

where

$$g(x) = \alpha(1 + \phi) x^{\frac{\alpha\phi}{1-\alpha}} - x + (1 - \alpha - \alpha\phi)$$

This follows from the following observations:

$$\begin{aligned} g(1 - \alpha\phi) &> 0 \\ g(1) &= 0 \\ g'(1) &< 0 \\ g''(x) &< 0 \end{aligned}$$

where we have used the assumed inequality  $\phi < \frac{1-\alpha}{\alpha}$  several times. For instance, proving that  $g(1 - \alpha\phi) > 0$  boils down to showing that

$$(1 + \phi)(1 - \alpha\phi)^{\frac{\alpha\phi}{1-\alpha}} > 1 \tag{22}$$

This inequality can be seen as a function  $\phi$  where  $\phi$  lies in the interval  $\langle 0, \frac{1-\alpha}{\alpha} \rangle$ . Hence we need to show that

$$h(\phi) > 0 \text{ for all } \phi \in \langle 0, \frac{1-\alpha}{\alpha} \rangle$$

where

$$h(\phi) = \ln(1 + \phi) + \frac{\alpha\phi}{1-\alpha} \ln(1 - \alpha\phi)$$

This follows from the following observations

$$\begin{aligned} h(0) &= 0 \\ h\left(\frac{1-\alpha}{\alpha}\right) &= 0 \\ h'(0) &> 0 \\ h'\left(\frac{1-\alpha}{\alpha}\right) &< 0 \\ h''(\phi) &< 0 \end{aligned}$$

Coming back to the function  $g(\cdot)$  above. We see that the function  $g(\cdot)$  crosses the  $x$ -axis at  $x = 1$  and  $g(\cdot)$  is decreasing at that point. Further,  $g(\cdot)$  is concave and strictly positive at  $x = 1 - \alpha\phi$ . This implies that  $g(x)$  is strictly positive for all  $x \in \langle 1 - \alpha\phi, 1 \rangle$ .

(III)  $\gamma > \tilde{\gamma}$ : then we find that  $n_w, n_c > n_0$ . In particular, using the expressions for  $n_w$  and  $n_c$  under (O2) and (C2),

$$n_w = n_0 \left( \frac{\gamma}{E} \frac{1 - \alpha\phi}{1 - \alpha - \alpha\phi} \right)^\phi < n_0 \left( \frac{\gamma}{E} \frac{1}{1 - \alpha - \alpha\phi} \right)^\phi = n_c$$

Using the expressions for output under (O2) and (C2) we find that

$$\frac{q^o(n)}{q^c(n)} = \frac{1}{1+\phi} \left( \frac{1}{1-\alpha\phi} \right)^{\frac{\alpha\phi}{1-\phi}} < 1$$

where the inequality follows from equation (22) proved above. Q.E.D.

**Proof of Proposition 3**

Linearizing equations (19) and (20) with respect to  $n_w, \lambda, \zeta$  and  $\xi$  and writing the result in matrix notation, we get

$$\begin{aligned} & \begin{pmatrix} (MC(n_w)q(n_w) + \gamma) & \left( \lambda MC'(n_w) + \xi \frac{1}{n_w^2} + \zeta \eta(n_w) \right) q(n_w) \\ \left| \frac{\partial VC(n_w)}{\partial \lambda} \right| & (MC(n_w)q(n_w) + \gamma) f(n_w) \end{pmatrix} \begin{pmatrix} d\lambda \\ dn_w \end{pmatrix} \\ = & \begin{pmatrix} \frac{q(n_w)}{n_w} d\xi + \frac{1-F(n_w)}{n_w^2 f(n_w)} q(n_w) d\zeta \\ \frac{\partial VC(n_w)}{\partial \zeta} d\zeta + \frac{\partial VC(n_w)}{\partial \xi} d\xi \end{pmatrix} \end{aligned}$$

where

$$\begin{aligned} \eta(n_w) & \equiv -\frac{d\left(\frac{1-F(n_w)}{n_w^2 f(n_w)}\right)}{dn_w} > 0 \\ \left| \frac{\partial VC(n_w)}{\partial \lambda} \right| & \equiv \int_{n_w}^{n_1} MC(n) \left( -\frac{\partial q(n)}{\partial \lambda} \right) f(n) dn > 0 \\ \frac{\partial VC(n_w)}{\partial \zeta} & \equiv \int_{n_w}^{n_1} MC(n) \frac{\partial q(n)}{\partial \zeta} f(n) dn > 0 \\ \frac{\partial VC(n_w)}{\partial \xi} & \equiv \int_{n_w}^{n_1} MC(n) \frac{\partial q(n)}{\partial \xi} f(n) dn > 0 \end{aligned}$$

The sign of  $\eta(n_w)$  follows from the assumption of a monotone hazard rate, the signs of  $\left| \frac{\partial VC(n_w)}{\partial \lambda} \right|$ ,  $\frac{\partial VC(n_w)}{\partial \zeta}$  and  $\frac{\partial VC(n_w)}{\partial \xi}$  follow from equation (18). Denoting the determinant of the matrix above by  $\Delta$ , we find that

$$\Delta = (MC(n_w)q(n_w) + \gamma)^2 f(n_w) - \left| \frac{\partial VC(n_w)}{\partial \lambda} \right| \left( \lambda MC'(n_w) + \xi \frac{1}{n_w^2} + \zeta \eta(n_w) q(n_w) \right) > 0$$

where the inequality follows for values of  $\xi, \zeta > 0$  that are close enough to 0. In particular, since  $MC'(n_w) < 0$  there exist  $\bar{\xi}, \bar{\zeta} > 0$  such that

$$\lambda MC'(n_w) + \xi \frac{1}{n_w^2} + \zeta \eta(n_w) q(n_w) < 0 \quad (23)$$

for  $\xi < \bar{\xi}$  and  $\zeta < \bar{\zeta}$ .

Hence we find that

$$\begin{aligned} \begin{pmatrix} \frac{d\lambda}{d\zeta} \\ \frac{dn_w}{d\zeta} \end{pmatrix} & = \frac{1}{\Delta} \begin{pmatrix} \begin{pmatrix} MC(n_w)q(n_w) \\ +\gamma \end{pmatrix} f(n_w) & - \begin{pmatrix} \lambda MC'(n_w) + \xi \frac{1}{n_w^2} \\ +\zeta \eta(n_w) \end{pmatrix} q(n_w) \\ - \left| \frac{\partial VC(n_w)}{\partial \lambda} \right| & MC(n_w)q(n_w) + \gamma \end{pmatrix} \\ & \times \begin{pmatrix} \frac{1-F(n_w)}{n_w^2 f(n_w)} q(n_w) \\ \frac{\partial VC(n_w)}{\partial \zeta} \end{pmatrix} \end{aligned}$$

Using the inequalities derived above and the inequality assumed in (23), it is straightforward to derive that

$$\frac{d\lambda}{d\zeta} > 0$$

The sign of  $\frac{dn_w}{d\zeta}$  turns out to be ambiguous, however we do not need the expression for  $\frac{dn_w}{d\zeta}$  as shown below.

Differentiating equation (18) with respect to  $\zeta$  we get

$$v''(q(n)) \frac{dq(n)}{d\zeta} = MC(n) \frac{d\lambda}{d\zeta} - \frac{1-F(n)}{n^2 f(n)}$$

This can be written as

$$\frac{dq(n)}{d\zeta} = \frac{\left(1 - \frac{d\lambda}{d\zeta}\right) \frac{1-F(n)}{n f(n)} - \frac{d\lambda}{d\zeta}}{n(-v''(q(n)))}$$

Since  $\frac{d\lambda}{d\zeta} > 0$ , we find that  $\frac{dq(n)}{d\zeta} < 0$ . The monotone hazard rate condition implies that  $\frac{1-F(n)}{n f(n)}$  falls with  $n$ . Hence there are two possibilities. First, it is possible that  $\frac{dq(n)}{d\zeta} > 0$  for low  $n$ . Second, it is possible that  $\frac{dq(n)}{d\zeta} < 0$  for all  $n$ . Then the budget constraint (20) implies that  $\frac{dn_w}{d\zeta} < 0$ . In both cases definition 1 implies that a rise in  $\zeta$  makes the outcome less competitive.

Now we turn to the effect of  $\xi$ . Solving the equations above in matrix notation for  $\frac{d\lambda}{d\xi}$  and  $\frac{dn_w}{d\xi}$ , one gets

$$\begin{pmatrix} \frac{d\lambda}{d\xi} \\ \frac{dn_w}{d\xi} \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} \begin{pmatrix} MC(n_w)q(n_w) \\ +\gamma \\ -\left|\frac{\partial VC(n_w)}{\partial \lambda}\right| \end{pmatrix} f(n_w) & - \begin{pmatrix} \lambda MC'(n_w) + \xi \frac{1}{n_w^2} \\ +\zeta \eta(n_w) \\ MC(n_w)q(n_w) + \gamma \end{pmatrix} q(n_w) \end{pmatrix} \\ \times \begin{pmatrix} \frac{q(n_w)}{n_w} \\ \frac{\partial VC(n_w)}{\partial \xi} \end{pmatrix} \end{pmatrix}$$

Hence  $\frac{d\lambda}{d\xi}$  can be written as

$$\frac{d\lambda}{d\xi} = \omega \frac{\frac{q(n_w)}{n_w}}{MC(n_w)q(n_w) + \gamma} + (1 - \omega) \frac{\frac{\partial VC(n_w)}{\partial \xi}}{\left|\frac{\partial VC(n_w)}{\partial \lambda}\right|} \quad (24)$$

where

$$\omega = \frac{(MC(n_w)q(n_w) + \gamma)^2 f(n_w)}{(MC(n_w)q(n_w) + \gamma)^2 f(n_w) - \left|\frac{\partial VC(n_w)}{\partial \lambda}\right| \left(\lambda MC'(n_w) + \xi \frac{1}{n_w^2} + \zeta \eta(n_w)q(n_w)\right)}$$

The inequality in equation (23) implies that  $\omega \in (0, 1)$ . Using this, we will now argue that  $\frac{d\lambda}{d\xi} \in (0, 1)$  by showing that  $\frac{d\lambda}{d\xi}$  in equation (24) can be seen

as the weighted average of two terms which are each between 0 and 1. Using the definition of marginal costs  $MC(n)$ , it is clear that  $\frac{\frac{q(n_w)}{n_w}}{MC(n_w)q(n_w)+\gamma} \in \langle 0, 1 \rangle$ . Next, consider the fraction  $\frac{\frac{\partial VC(n_w)}{\partial \xi}}{\left| \frac{\partial VC(n_w)}{\partial \lambda} \right|}$ . Equation (18) implies that

$$\frac{\frac{\partial q(n)}{\partial \xi}}{\left| \frac{\partial q(n)}{\partial \lambda} \right|} = \frac{\frac{1}{n}}{MC(n)} \leq 1 \text{ for all } n \in [n_0, n_1]$$

Hence we find that

$$\frac{\frac{\partial VC(n_w)}{\partial \xi}}{\left| \frac{\partial VC(n_w)}{\partial \lambda} \right|} = \frac{\int_{n_w}^{n_1} \frac{\partial q(n)}{\partial \xi} MC(n) f(n) dn}{\int_{n_w}^{n_1} \left( -\frac{\partial q(n)}{\partial \lambda} \right) MC(n) f(n) dn} < 1$$

So indeed  $\frac{d\lambda}{d\xi}$  is the weighted average (with weights  $\omega$  and  $1 - \omega$ ) of two terms  $\left( \frac{\frac{q(n_w)}{n_w}}{MC(n_w)q(n_w)+\gamma} \text{ and } \frac{\frac{\partial VC(n_w)}{\partial \xi}}{\left| \frac{\partial VC(n_w)}{\partial \lambda} \right|} \right)$  in between 0 and 1. Therefore  $\frac{d\lambda}{d\xi} \in \langle 0, 1 \rangle$ .

Differentiating equation (18) with respect to  $\xi$  we get

$$v''(q(n)) \frac{dq(n)}{d\xi} = MC(n) \frac{d\lambda}{d\xi} - \frac{1}{n}$$

This can be written as

$$\frac{dq(n)}{d\xi} = \frac{\left( 1 - \frac{d\lambda}{d\xi} \right) - \frac{1-F(n)}{nf(n)} \frac{d\lambda}{d\xi}}{n(-v''(q(n)))}$$

Hence it follows from  $\frac{d\lambda}{d\xi} \in \langle 0, 1 \rangle$  that

$$\frac{dq(n_1)}{d\xi} > 0$$

Now we have two possibilities (again using the fact that  $\frac{1-F(n)}{nf(n)}$  is decreasing in  $n$  by the monotone hazard rate property). First, it can be the case that  $\frac{dq(n)}{d\xi} < 0$  for low values of  $n$ . Second, it can be the case that  $\frac{dq(n)}{d\xi} > 0$  for all  $n$ , but then the budget restriction (20) implies that  $\frac{dn_w}{d\xi} > 0$ . In both cases definition 1 implies that competition rises with  $\xi$ . Q.E.D.

#### Proof of proposition 4

Since we are interested in how outcomes vary with entry costs and expenditure in a sector, we write the optimal value of the following optimization problem as a function of  $\gamma_i$  and  $E_i$

$$\begin{aligned} Q_i(E_i, \gamma_i) &= \max_{q_i(\cdot), n_{wi}} \int_{n_{wi}}^{n_{1i}} v_i(q_i(n)) f_i(n) dn \\ &\quad - \lambda_i \left[ \int_{n_{wi}}^{n_{1i}} MC_i(n) q_i(n) f_i(n) + [1 - F_i(n_{wi})] \gamma_i - E_i \right] \end{aligned} \quad (25)$$

Using the budget constraint of the sectoral allocation problem,  $E_1 + E_2 = Y$ , we can write the overall optimization problem as

$$\max_{E_2} U(Q_1(Y - E_2, \gamma_1), Q_2(E_2, \gamma_2))$$

and hence the first order condition for  $E_2$  can be written as

$$-U'_1 \frac{\partial Q_1}{\partial E_1} + U'_2 \frac{\partial Q_2}{\partial E_2} = 0$$

where  $U'_i = \frac{\partial U(Q_1, Q_2)}{\partial Q_i}$ . The spillover effect of a fall in  $\gamma_2$  to sector 1 works via the amount of money spent in sector 1,  $E_1 = Y - E_2$ . Hence we use the first order condition for  $E_2$  to derive  $\frac{dE_2}{d\gamma_2}$  as follows

$$\left( -\frac{\partial \left[ -U'_1 \frac{\partial Q_1}{\partial E_1} + U'_2 \frac{\partial Q_2}{\partial E_2} \right]}{\partial E_2} \right) \frac{dE_2}{d\gamma_2} = -U''_{12} \frac{\partial Q_1}{\partial E_1} \frac{\partial Q_2}{\partial \gamma_2} + U''_{22} \frac{\partial Q_2}{\partial E_2} \frac{\partial Q_2}{\partial \gamma_2} + U'_2 \frac{\partial^2 Q_2}{\partial \gamma_2 \partial E_2}$$

where  $U''_{ij} = \frac{\partial^2 U(Q_1, Q_2)}{\partial Q_i \partial Q_j}$ . Note that the bracketed expression on the left hand side is positive because the second order condition for  $E_2$  implies that  $\frac{\partial \left[ -U'_1 \frac{\partial Q_1}{\partial E_1} + U'_2 \frac{\partial Q_2}{\partial E_2} \right]}{\partial E_2} < 0$ . Using the definition of  $Q_i$  in equation (25), we can derive the following expressions

$$\begin{aligned} \frac{\partial Q_2(E_2, \gamma_2)}{\partial \gamma_2} &= -\lambda_2 (1 - F_2(n_{w2})) \\ \frac{\partial Q_2(E_2, \gamma_2)}{\partial E_2} &= \lambda_2 \\ \frac{\partial^2 Q_2(E_2, \gamma_2)}{\partial E_2 \partial \gamma_2} &= \frac{\partial \lambda_2}{\partial \gamma_2} \end{aligned}$$

Therefore we find that

$$\text{sign} \left( \frac{dE_2}{d\gamma_2} \right) = \text{sign} \left( U''_{12} \lambda_1 \lambda_2 (1 - F(n_{w2})) - U''_{22} \lambda_2^2 (1 - F(n_{w2})) + U'_2 \frac{\partial \lambda_2}{\partial \gamma_2} \right)$$

By the assumption that  $U''_{12} \geq 0$  and the concavity of  $U$  (which implies  $U''_{22} < 0$ ) we find that the first two terms on the right hand side are positive. Finally, the other inequality assumed in the proposition implies (see lemma 5) that  $\frac{\partial \lambda_2}{\partial \gamma_2} \geq 0$ . Hence we find that  $\frac{dE_2}{d\gamma_2} > 0$ . This implies that a fall in  $\gamma_2$  leads to a fall in  $E_2$  and therefore a rise in  $E_1 = Y - E_2$ . Again using lemma 5, we find that a rise in  $E_1$  makes the optimal competition outcome more competitive in sector 1. Q.E.D.





