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Terrorism Control in the Tourism Industry

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Abstract. In some countries, for instance Egypt, terrorists try to hurt the country income from the tourism industry by violent actions against tourists. Another example are actions of the Kurds to bring tourism down in the east of Turkey. This paper is a first attempt to model some relevant aspects of these prey–predator relations. The country tries to maximize profits from the tourism industry, where profit is defined as the difference between revenue from the tourism industry and the sum of expenditures on tourism industry investments and expenditures on enforcement associated with reducing terrorism. It turns out that, for reasonable parameter values, the optimal trajectory exhibits a cyclical strategy. The interpretation is that, after starting out with a low number of tourists and terrorists, tourism investments are undertaken to increase tourism. This attracts terrorists reducing the effect of tourism investments. Therefore, investment declines and so does the number of tourists. This makes it less attractive for terrorists to act, so we are back in the original situation, where the whole thing starts again.

Key Words. Hopf bifurcation, limit cycles, tourism industry, law enforcement.

1. Introduction

International tourism is the world largest item of trade and represents a major industry in over 100 nations. Yet, a few terrorists can have a decisive and crippling impact on travel patterns and the economies of countries.

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Terrorism, in its international and domestic forms and as practiced by revolutionary and vigilante groups, has become a fact of life since the 1980s. The reporting of terrorists activities in tourist destinations can affect adversely the level of business in tourist locations. In extreme cases, violence can undermine a country tourism industry for a shorter or longer period. There is a substantial literature on the relationships between terrorism and tourism; see e.g. Refs. 1–3.

The recurrent outbreaks of terrorism in Egypt show that the belief that the government had permanently rid the country of terrorists is not justified. The Islamic radicals began their campaign of violence in Egypt in 1992. After their success in 1993, when their own casualties were roughly half those of the police, the government pursuit seemed to have gradually limited the extremists to the southern provinces. However, the most obvious lesson of the series of bomb attacks by Islamist terrorists was that terrorism is devilishly hard to stamp out. In September 1997, the Egyptian government thought that it had defeated its terrorists. Over the past five years, the government had swept thousands of suspected Islamic militants into jail, tried them in military courts, and raided their hideouts in coves and sugarcane fields with guns blazing. After convicting 72 extremists in a mass trial, the government declared that “the heads of the terrorists have been falling, and nothing remains except a few fugitives”; see Ref. 4. However, a day later, those “few fugitives” showed what horror they could produce. Several terrorists threw flaming bottles of gasoline at a tour bus and raked the passengers with gunfire. Nine German tourists and the Egyptian driver died in this blaze just outside the Cairo Egyptian Museum.

The reaction was a wave of cancellations and an increasing protection by police forces. After a while, the hope of the terrorists to deprive the state of vital revenues from tourism was not fulfilled. The periodic ups and downs of tourism and terrorism provide an example of an interdependent oscillatory system.

Travelers have been associated always with increasing vulnerability to various types of crime. But, throughout most of history, tourists were individual victims of crime and targets for major acts of political violence. Since the late 1960s, terroristic violence has increased substantially.

The aim of this paper is to provide a theoretical foundation for the influence of terrorism on tourism and how a country should deal with that. The motivation for this is that the prey–predator relationship between tourism and terrorism need to be understood not only in terms of security and marketing, but also in terms of site development, employment policies, and enforcement management. To reach this aim, a dynamic model is formulated where the country government is the decision maker. The objective is to maximize the income generated by the tourism industry. Terrorists are

attracted by large amount of tourists. In order to reduce terrorism, the government could allocate some means to terrorism enforcement. Furthermore, the government can attract tourists by making investments in the tourism industry. Investments are more efficient in terms of attracting tourists if there is not much terrorism around. This makes it understandable that one of our results is that investment programs in the tourism industry are accompanied by large terrorism enforcement expenditures.

Our main result is that, for reasonable parameter values, the resulting optimal solution exhibits a cyclical behavior which can be explained as follows. Assume that the starting point is a country with a small tourism industry and not much terrorism around. Then, the country starts to invest in order to increase the number of tourists visiting this country. Concrete examples of tourism investments are e.g. building hotels, ski lifts, preserving nature in national parks, and so on and so forth; see Ref. 5. Increasing tourism attracts terrorism, which then grows with the amount of tourists. Eventually, the high terrorism level distracts tourists from visiting this country and also lowers the tourism industry investment climate. Therefore, tourism as well as tourism investments will drop. As a result of this, the amount of terrorists will drop too. In this way, the old situation with a small tourism industry occurs again from where the whole thing will be repeated.

The contents of the paper is as follows. In Section 2, the model is presented. In Section 3, the model is analyzed by means of the Pontryagin maximum principle. Section 4 contains economic interpretations of the results. Finally, the paper is concluded in Section 5.

2. Model

The country aim is to maximize the cash flow resulting from the tourism industry. Denoting the number of tourists by T , the revenue per unit of time is pT , where p is the (constant) revenue per tourist.⁵ The expenses with regard to tourism are twofold. First, the government undertakes investments I in order to make the country more attractive to tourists. Investment expenses are denoted by $C(I)$, where C is increasing and convex in I . $C(I)$ might also be interpreted as the service costs for the touristic infrastructure (e.g. buses, ski lifts, etc.). On the other hand, the government spends money on enforcement in order to prevent terroristic attempts. Enforcement per unit time is denoted by E , and b is the (constant) amount of money needed to activate one unit of enforcement. Assuming an infinite planning period,

⁵Note that the variable T as well as the variables I , E , N as defined below depend on the time t . We omit the time argument t for notational convenience.

denoting by r the positive rate of time preference, and noting that I and E are the control variables, the country objective function is given by

$$\max_{I,E} \int_0^{\infty} e^{-rt} [pT - C(I) - bE] dt. \quad (1)$$

The number of tourists increases with tourism investments, but the tourists are distracted by the terrorists, where N stands for the number of terrorists. Of course, terroristic activities have a negative impact on the positive effect of investments on tourism. All this is captured in the function $\gamma(I, N)$, by which the number of tourists increases per unit of time. The investment function γ measures the impact of I on the change of tourists for a given level of terrorists. It seems reasonable to assume that $\gamma(I, N)$ satisfies the inequalities below:

$$\gamma_I > 0, \quad \gamma_N < 0, \quad \gamma_{II} \leq 0, \quad \gamma_{NN} \leq 0, \quad \gamma_{IN} < 0. \quad (2)$$

The first and third inequality of (2) state that the number of tourists increase in a nonconvex way with the tourism investments for a given level of N . The second and fourth inequality mean that the tourists are distracted by the terrorists, and this effect is nondecreasing with the number of terrorists for a given level of I . The last inequality states that the positive effect on tourism of an additional unit of investment is decreasing with the number of terrorists, which makes intuitively sense.

Denoting the natural decay rate of tourism by a , $a > 0$ and constant, the development of the number of tourists over time is given by the differential equation

$$\dot{T} = \gamma(I, N) - aT. \quad (3)$$

A flourishing tourism industry attracts terrorists, so that the number of tourists has a positive effect on the number of terrorists, where we assume that the number of terrorists attracted per tourist is given by τ , $\tau > 0$ and constant. On the other hand, terrorism is affected negatively by enforcement activities. This is reflected in the function $\psi(E)$. There are decreasing returns to scale with respect to enforcement activities so that

$$\psi' > 0, \quad \psi'' < 0. \quad (4)$$

Thus, the number of terrorists over time develops as follows:

$$\dot{N} = \tau T - \psi(E). \quad (5)$$

Taking all this into account, it can be concluded that the total model is given by

$$\max_{I,E} \int_0^{\infty} e^{-rt} [pT - C(I) - bE] dt,$$

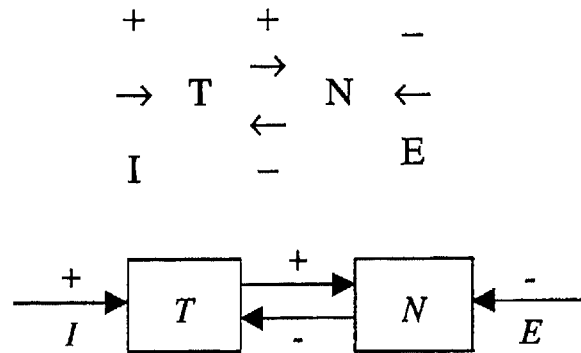


Fig. 1. State diagram of the model.

subject to the differential equations

$$\dot{T} = \gamma(I, N) - aT,$$

$$\dot{N} = \tau T - \psi(E).$$

The effects of the state and control variables on each other is schematized in Fig. 1.

In Section 2, we apply the Pontryagin maximum principle to solve this model; see, e.g., Ref. 6.

3. Solution

The Hamiltonian is

$$H = pT - C(I) - bE + \lambda_1[\gamma(I, N) - aT] + \lambda_2[\tau T - \psi(E)], \tag{6}$$

which leads to the following necessary conditions:

$$-C'(I) + \lambda_1 \gamma_I(I, N) = 0. \tag{7}$$

This equation implies that $I = I(N, \lambda_1)$, with

$$I_N = \lambda_1 \gamma_{IN} / (C'' - \lambda_1 \gamma_{II}) < 0, \tag{8}$$

$$I_{\lambda_1} = \gamma_I / (C'' - \lambda_1 \gamma_{II}) > 0. \tag{9}$$

According to (8), *ceteris paribus*⁶, tourism investments decrease with the number of terrorists, which is caused by the fact that, due to the negative sign of γ_{IN} , the efficiency of an additional unit of investment in terms of

⁶*Ceteris paribus* is a Latin expression meaning “all other things being equal”.

attracting tourists is lower when there are more terrorists around. Furthermore, (9) states that, *ceteris paribus*, if the shadow price of the number of tourists is large, the rate of investment in tourist attractions increases.

The other first-order condition is

$$-b - \lambda_2 \psi'(E) = 0, \quad (10)$$

which implies that the shadow price of the number of terrorists λ_2 is negative, which makes sense because N is a bad stock. From (10), it can be derived further that $E = E(\lambda_2)$, with

$$E_{\lambda_2} = -\psi' / \lambda_2 \psi'' < 0. \quad (11)$$

The relation (11) can be explained as follows. When λ_2 increases, this means that the terrorism shadow price becomes less negative. Hence, the harm caused by an additional terrorist decreases so that the country will cut down on enforcement expenditures.

Finally, the conditions for the development of the costate variables λ_1 and λ_2 are

$$\dot{\lambda}_1 = (r + a)\lambda_1 - p - \tau\lambda_2, \quad (12)$$

$$\dot{\lambda}_2 = r\lambda_2 - \lambda_1 \gamma_N(I, N). \quad (13)$$

Next, we examine the stability behavior of this model. To do so, let us first write the dynamic system,

$$\dot{T} = \gamma(I(N, \lambda_1), N) - \alpha T, \quad (14)$$

$$\dot{N} = \tau T - \psi(E(\lambda_2)), \quad (15)$$

$$\dot{\lambda}_1 = (r + a)\lambda_1 - p - \tau\lambda_2, \quad (16)$$

$$\dot{\lambda}_2 = r\lambda_2 - \lambda_1 \gamma_N(I(N, \lambda_1), N). \quad (17)$$

This leads to the following Jacobian:

$$J = \det \begin{bmatrix} -a & \gamma_I I_N + \gamma_N & \gamma_I I_{\lambda_1} & 0 \\ \tau & 0 & 0 & -\psi' E_{\lambda_2} \\ 0 & 0 & r + a & -\tau \\ 0 & -\lambda_1 \gamma_{NI} I_N - \lambda_1 \gamma_{NN} & -\gamma_N - \lambda_1 \gamma_{NI} I_{\lambda_1} & r \end{bmatrix}, \quad (18)$$

which equals

$$J = a(r + a)\psi' E_{\lambda_2} \lambda_1 (\gamma_{NI} I_N + \gamma_{NN}) - \tau(r + a)r(\gamma_I I_N + \gamma_N) + \tau^2 [\gamma_I \gamma_N I_N + \gamma_N^2 + \lambda_1 I_{\lambda_1} (\gamma_N \gamma_{NI} - \gamma_I \gamma_{NN})]. \quad (19)$$

Only the first term of J could be nonpositive, so that, e.g., a sufficiently large τ guarantees that J is positive.

The number K has the following form:

$$\begin{aligned}
 K = & \det \begin{bmatrix} -a & \gamma_I I_{\lambda_1} \\ 0 & r+a \end{bmatrix} + \det \begin{bmatrix} 0 & -\psi' E_{\lambda_2} \\ -\lambda_1 \gamma_{NI} I_N - \lambda_1 \gamma_{NN} & r \end{bmatrix} \\
 & + 2 \det \begin{bmatrix} \gamma_I I_N + \gamma_N & 0 \\ 0 & -\tau \end{bmatrix}, \tag{20}
 \end{aligned}$$

which can be rewritten into

$$\begin{aligned}
 K = & -a(r+a) - \lambda_1 \gamma_{NI} I_N \psi' E_{\lambda_2} - \lambda_1 \gamma_{NN} \psi' E_{\lambda_2} \\
 & - 2\tau \gamma_I I_N - 2\tau \gamma_N. \tag{21}
 \end{aligned}$$

The first term of K is negative, the second term is nonnegative, the third term is nonpositive, the fourth and fifth terms are positive. Hence, also here it holds that a sufficiently large τ guarantees a positive K , which is a necessary condition for the occurrence of stable limit cycles. In terms of the model, it holds that τ being large means that the presence of tourists attracts many terrorists.

Proposition 3.1. A necessary condition for a stable limit cycle to be optimal is that $\gamma_{IN} < 0$.

Proof. Alternatively, it holds that $\gamma_{IN} = 0$. In this case, it can be shown that the bifurcation equation $4J = K^2 + 2r^2K$ can be satisfied only if $K < 0$. However, this violates $K > 0$, which is a necessary condition for the occurrence of a limit cycle. \square

The framework is too complicated to generate analytical results. Therefore, we have to rely on numerical methods. To do so, we introduce first some specific functions,

$$\gamma(I, N) = \alpha I(N^* - N), \quad \text{where } \alpha \text{ and } N^* \text{ are positive constants.} \tag{22}$$

N^* can be interpreted as the maximal possible number of terrorists. Furthermore, we specify the functions

$$\psi(E) = (1/c)E^c, \quad \text{where } 0 < c < 1 \text{ is constant,} \tag{23}$$

$$C(I) = (1/2)hI^2, \quad \text{where } h > 0 \text{ is constant.} \tag{24}$$

Substitution of these functional forms into J and K gives

$$\begin{aligned}
 J = & \alpha^2 a(r+a) E^c \lambda_1^2 / (1-c) h \lambda_2 + \tau(r+a) r [\lambda_1 (N^* - N) \alpha^2 / h + \alpha I] \\
 & + \tau^2 [2\lambda_1 I (N^* - N) \alpha^3 / h + \alpha^2 I^2], \tag{25}
 \end{aligned}$$

$$K = -a(r+a) - \alpha^2 \lambda_1^2 E^c / h \lambda_2 (1-c) + 2\tau(N^* - N) \alpha^2 \lambda_1 / h + 2\tau \alpha I. \tag{26}$$

The first-order conditions now become

$$-hI + \lambda_1 \alpha (N^* - N) = 0, \quad (27)$$

$$-b - \lambda_2 E^{c-1} = 0. \quad (28)$$

Due to these expressions, we can rewrite J and K in the form

$$J = \alpha^2 a(r+a) E^c \lambda_1^2 / (1-c) h \lambda_2 + 2\alpha \tau (r+a) r I + 3I^2 \tau^2 \alpha^2, \quad (29)$$

$$K = -a(r+a) - \alpha^2 \lambda_1^2 E^c / h \lambda_2 (1-c) + 4\tau \alpha I. \quad (30)$$

For J as well as K , it holds that the first term is negative, while the rest is positive. Again, a sufficiently large τ guarantees that both J and K are positive.

To find out whether a stable limit cycle can be optimal, the bifurcation equation,

$$4J = K^2 + 2r^2 K, \quad (31)$$

must be satisfied. For our model, this equation has the following form:

$$\begin{aligned} &8\tau a \alpha (a+2r) I - 4\alpha^2 I^2 \tau^2 - a(r+a)(a-r)(a+2r) \\ &- [\lambda_1^2 E^c \alpha^2 / h \lambda_2 (1-c)] [\lambda_1^2 E^c \alpha^2 / h \lambda_2 (1-c) - 8\tau \alpha I + 2r^2 - 2a(r+a)] = 0. \end{aligned} \quad (32)$$

4. Discussion of a Persistent Cycle

To present a numerical example in which a stable limit cycle is optimal, we specify the functions as in (22)–(24). Making use of the parameter values

$$\begin{aligned} a &= 0.067, & \tau &= 0.089, & p &= 0.315, & b &= 3.370, \\ c &= 0.714, & h &= 2.000, & \alpha &= 0.124, & N^* &= 2.110, \end{aligned}$$

and choosing the discount rate r as the bifurcation parameter, the Jacobian evaluated at the steady state possesses two purely imaginary eigenvalues for the critical value

$$r_{\text{crit}} = 0.0545149.$$

The steady state is given by

$$(T, N, I, E) = (0.2774, 1.0378, 0.1398, 0.0035).$$

According to the computer code BIFDD (see Ref. 7), stable cycles occur for $r < r_{\text{crit}}$. Making use of the boundary-value problem solver COLSYS (Ref. 8), a stable cycle was computed for $r = 0.0545$. The period of the cycle

is approximately

$$t_{\text{per}} = 265.023.$$

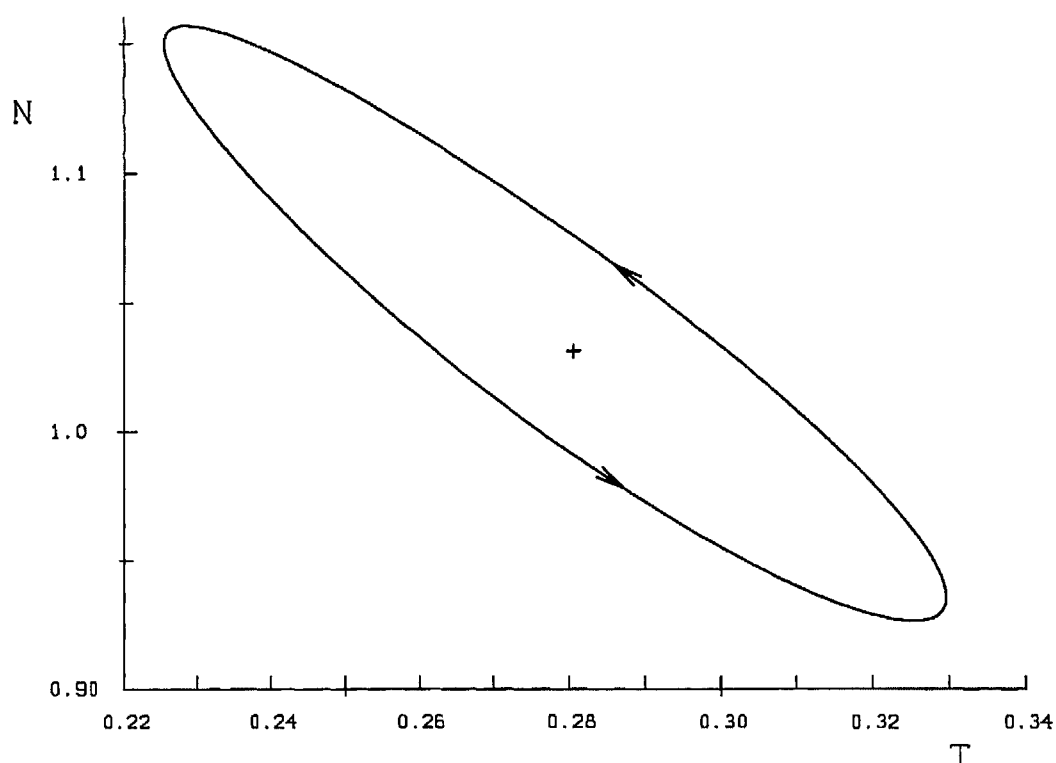


Fig. 2. Phase portrait in the (T, N) -plane.

Figures 2–4 show the projection of the cycle in the 2D state space and in two state-control spaces, respectively. Figure 5 shows the time paths of the two control variables (E, I) and the two state variables (N, T) .

Table 1 shows which ones of the variables N, I, T, E increase or decrease within a full period. The eight time points $t_i, i = 1, 2, \dots, 8$, mark the extrema of the four variables. According to that, we are able to identify the following four regimes.

Regime 1. Decline. Let us start with a situation in which the terrorism booms and there are few tourists (e.g., Egypt just after the Luxor outrage). According to the state dynamics of T , a high number of terrorists makes investments inefficient, and T will be kept small. The law enforcement rate increases from a relatively low level, which prevails since there are only few tourists around to be protected.

After a short while, the investment I reaches a minimum and increases afterward to attract tourists. After a certain delay, the number of tourists

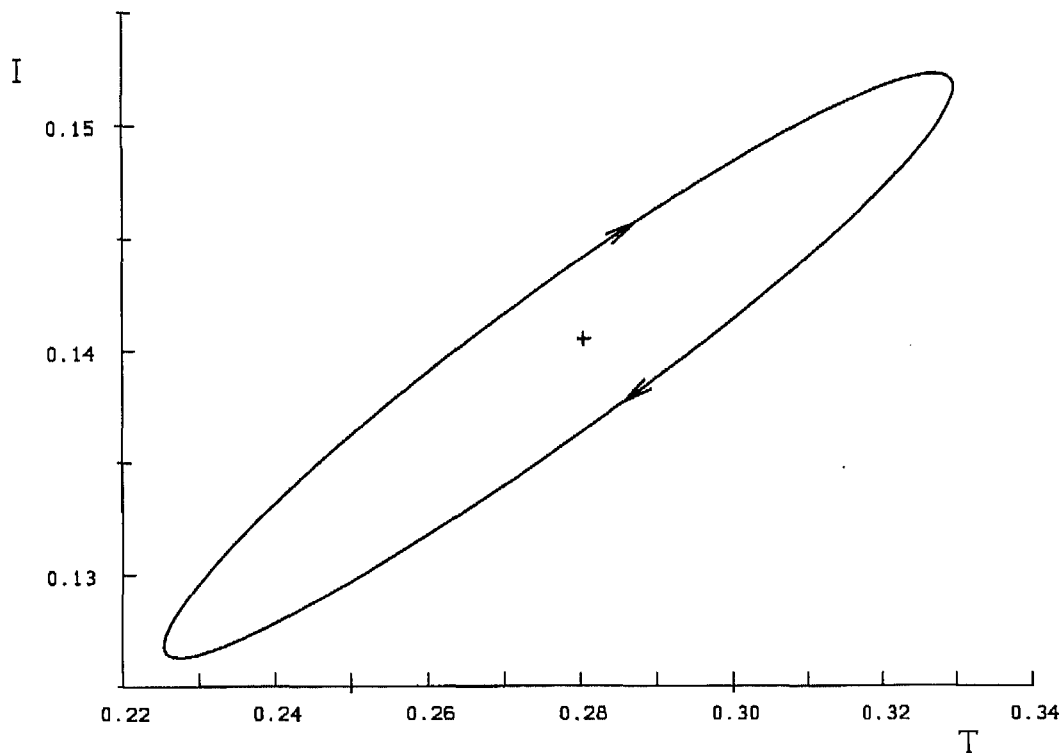


Fig. 3. Phase portrait in the (T, I) -plane.

reaches its minimum. During the whole period, the law enforcement rate increases.

Regime 2. Recovery. The transition from the phase of decline to recovery is characterized by an at first slight increase of tourists. Clearly, this occurs because the control I still increases, while N further decreases. Again, it is the investment function $\gamma(I, N)$ which drives the process. After a while, the number of terrorists is low enough that the enforcement can be reduced. The second part of the recovering phase is characterized by increasing I and T , but by decreasing E and N . This regime ends by minimal terrorists activities.

Regime 3. Prosperity. The following phase is characterized by many tourists, high investment, few terrorists, and sufficient protection measures. E can be reduced, N increases slightly, T still increases, and I peaks in this regime.

Regime 4. Saturation. After the touristic boom, both I and T decline (being still relatively high). The increasing terrorism is a bad omen, which calls for a change in the trend of the enforcement rate.

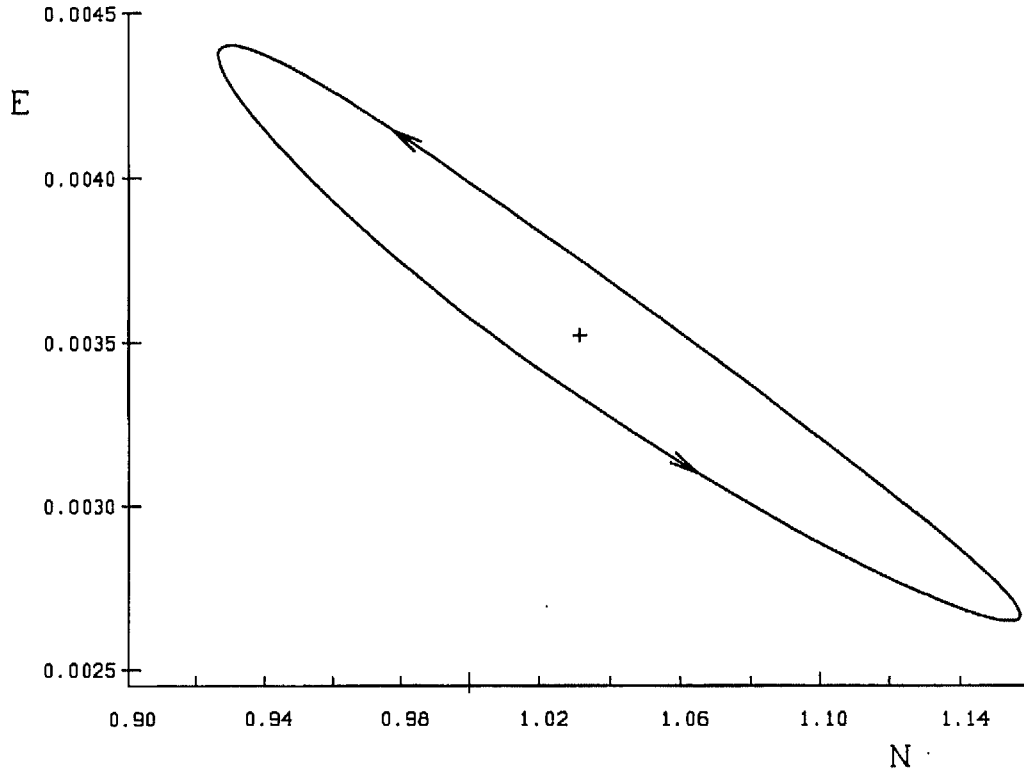


Fig. 4. Phase portrait in the (N, E) -plane.

While the length of the various subintervals (t_i, t_{i+1}) within one period is governed by the selection of the parameters and might be changed with them, the sequence of the maxima and minima is robust against changes in the parameter values. The solution of the model is driven to a persistent cycle by the assumption $\gamma_{IN} < 0$. In particular, the specification

$$\gamma(I, N) = \alpha I(N^* - N)$$

Table 1. Signs of the time derivatives of the variables $N(t)$, $I(t)$, $T(t)$, $E(t)$ during one full period.

Regime	Type	Start time	\dot{N}	\dot{I}	\dot{T}	\dot{E}	End time
R1	Decline	t_1 N_{\max}	-	-	-	+	t_2 I_{\min}
		t_2 I_{\min}	-	+	-	+	t_3 T_{\min}
R2	Recovery	t_3 T_{\min}	-	+	+	+	t_4 E_{\max}
		t_4 E_{\max}	-	+	+	-	t_5 N_{\min}
R3	Boom	t_5 N_{\min}	+	+	+	-	t_6 I_{\max}
		t_6 I_{\max}	+	-	+	-	t_7 T_{\max}
R4	Saturation	t_7 T_{\max}	+	-	-	-	t_8 E_{\min}
		t_8 E_{\min}	+	-	-	+	t_1 N_{\max}

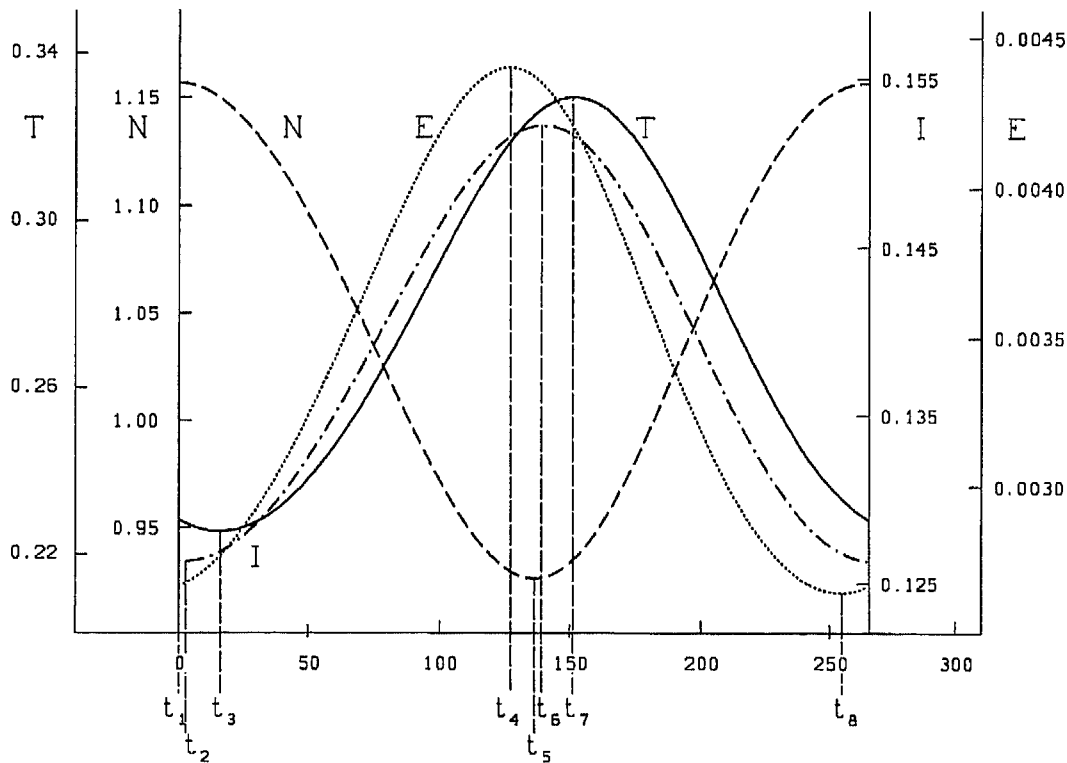


Fig. 5. Time paths of the states and controls of the persistent cycle during one period.

means that the negative effect of terrorism on tourism is largest when I is large. Hence, the decision maker incentive to reduce N is largest when I is large. Thus, E and I are complements, no substitutes. The managerial implication is that investment programs in tourism must be accompanied by large enforcement expenditures in order to make the effect of I on T as large as possible.

5. Conclusions

The main issue of this paper was to establish the fact that periodic investment and enforcement programs may be optimal under certain parameter constellations. Moreover, the order of the peaks makes economic sense. Large investments make it attractive for tourists to enter. This implies that the tourism industry generates large revenues. Terrorists want to damage the country economically, so they come into action. Therefore, in order to preserve the fact that tourism investments make it more attractive for tourists to enter, it is optimal to accompany tourism investments by enforcement expenditures.

The use or threat of violence as a means to achieve political ends is an old form of political expression. In the 1970s, terrorism has become a familiar phenomenon due to the mass media. Following the American raids on Libya and terrorists attacks on several European airports, approximately 1.8 million Americans changed their plans for foreign travel in 1986. Terroristic attacks or threats of violence can have a tremendous economic impact on the tourism industry. The purpose of the present paper was to analyze the interaction of terrorism and tourism in a simple prey-predator framework. An intertemporal optimization approach was used to study the optimal design of the touristic infrastructure as well as efficient law enforcement policies.

The framework that we considered is rather simple. The advantage of our approach is that results are clear and easy to interpret. But one drawback is, for instance, that in our model investment expenditures influence only the current inflow of tourists, and thus have no effect on the touristic development in the future. This could be repaired by introducing the state variable "touristic infrastructure", which increases with investments and decreases with depreciation (see Ref. 5), and replace "tourism investments" by "touristic infrastructure" in the state equation of the number of tourists. The resulting model will contain three state variables which implies that it will be harder to generate results. Therefore, alternatively, instead of being a state variable, the number of terrorists could be modeled as a function of tourists (increasing) and enforcement expenditures (decreasing).

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