# A beautiful theorem needs no application

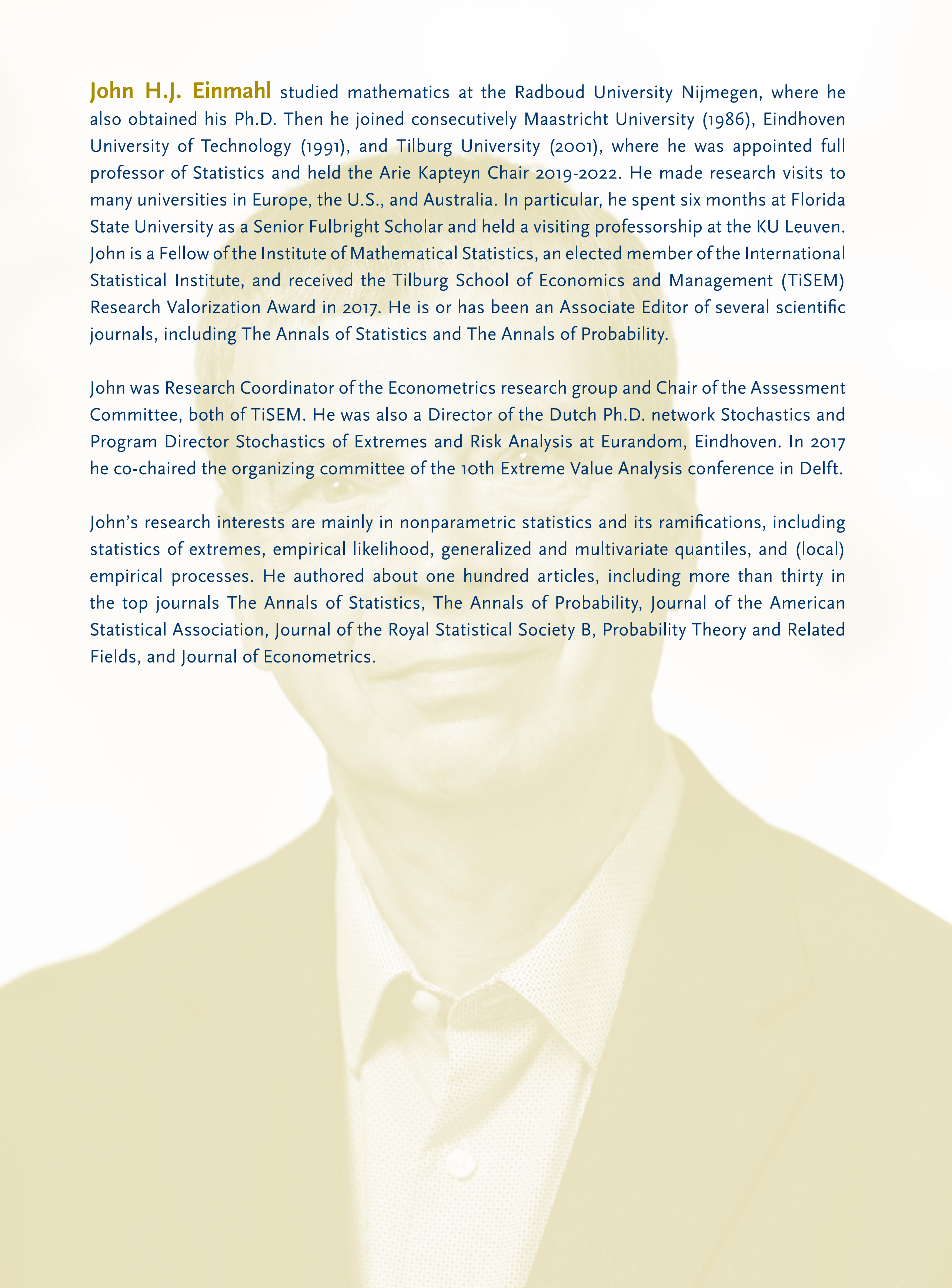
Valedictory address, delivered by

Prof. dr. John H.J. Einmahl

on September 8, 2023

Tilburg University

Understanding Society



**John H.J. Einmahl** studied mathematics at the Radboud University Nijmegen, where he also obtained his Ph.D. Then he joined consecutively Maastricht University (1986), Eindhoven University of Technology (1991), and Tilburg University (2001), where he was appointed full professor of Statistics and held the Arie Kapteyn Chair 2019-2022. He made research visits to many universities in Europe, the U.S., and Australia. In particular, he spent six months at Florida State University as a Senior Fulbright Scholar and held a visiting professorship at the KU Leuven. John is a Fellow of the Institute of Mathematical Statistics, an elected member of the International Statistical Institute, and received the Tilburg School of Economics and Management (TiSEM) Research Valorization Award in 2017. He is or has been an Associate Editor of several scientific journals, including The Annals of Statistics and The Annals of Probability.

John was Research Coordinator of the Econometrics research group and Chair of the Assessment Committee, both of TiSEM. He was also a Director of the Dutch Ph.D. network Stochastics and Program Director Stochastics of Extremes and Risk Analysis at Eurandom, Eindhoven. In 2017 he co-chaired the organizing committee of the 10th Extreme Value Analysis conference in Delft.

John's research interests are mainly in nonparametric statistics and its ramifications, including statistics of extremes, empirical likelihood, generalized and multivariate quantiles, and (local) empirical processes. He authored about one hundred articles, including more than thirty in the top journals The Annals of Statistics, The Annals of Probability, Journal of the American Statistical Association, Journal of the Royal Statistical Society B, Probability Theory and Related Fields, and Journal of Econometrics.

# A beautiful theorem needs no application

Prof. dr. John H.J. Einmahl

**Valedictory address,**
delivered at Tilburg University on Friday 8 September 2023.

**www.tilburguniversity.edu/nl**

# A beautiful theorem needs no application

Prof. dr. John H.J. Einmahl

# Introduction

## Dear rector magnificus, dean, head of department, coauthors, colleagues, former Ph.D. students, students, family and friends

My fields are statistics and probability theory. Maybe when you hear the word probability, you think of dice. Here they are and I will let them roll. This roll will be important because the outcome will determine the contents of this lecture. In statistics it is important to reduce variability, therefore I glued the two dice together, with a 1 and a 6 on the hidden faces. I have prepared a text for this last lecture and I will present it if a throw of the two dice yields a sum of 7. If it is less than 7, then I will read instead the first chapter of the book of Laurens de Haan and Ana Ferreira and we will probably not make it to the reception, because we also have to solve the 18 exercises at the end. If it is more than 7, then I will read the second chapter which is slightly shorter but has 19 exercises and they are more difficult. So again the reception is at risk. Let the dice roll and see what we obtain. By the way, these are ordinary dice. It is all fair. I am not a magician.

Here we go ...

The outcome is 7. So we go for my prepared text. That is a relief. It would have been very stressful to solve these difficult exercises in a limited time.

This is my last lecture and it will be about teaching and research, which have been the main tasks during my career. Teaching is naturally very important at a university. One way to show this is that the students evaluate the teacher by filling out forms about the quality of the lecture. Today, there are no evaluation forms, so maybe this lecture is not so important or maybe it is too late to improve my teaching. Also, there will be no exam after this lecture. Therefore I don't need to explain things in detail. It is more like permanent education: you get the credits, in this case the drinks afterwards, just for participation. When teaching, it is very important to get the attention of the audience. This I tried to accomplish by the glued dice act, but I am not sure that this attention will be kept for the remaining part of this lecture.

Maybe it helps if I tell you how it all began. I recall that mathematics was quickly my thing although I didn't know the word mathematics in the beginning, of course. Here it helped that I wasn't good in other things, like singing, music, drawing, gym classes, and also not a great soccer talent. On the first day at primary school, I was anxious because I had skipped kindergarten and thought that I was lagging behind, as the headmaster had suggested to my father. When the teacher asked us to draw two little circles to check if we understood the number 2, I was disappointed and I said I can recite the 37 times table. The teacher said: “That is too difficult, what about the table of 9?” and I replied that this is very easy because the digits always add up to 9. This is a little anecdote, which is exemplary for my interest in mathematics. It is long ago, but I think it is true. When I was in high school, I got even more interested in mathematics and I began tutoring students in lower grades. I also recall that when in 5$^{th}$ and 6$^{th}$ grade of high school I had to choose my field of study for university and there was a little booklet with all the possibilities in it. I put a lot of effort into the process of making the right choice, but in hindsight, it was clear that there was only one choice possible, namely mathematics. I went to an open day in Eindhoven and liked it, but thought it was a bit too applied. So I started in 1975 in Nijmegen, for a much more theoretical mathematics program.

# Research and Teaching

After my studies I was happy and privileged to get a Ph.D. position, again in Nijmegen. The project was on multivariate empirical processes, which also became the title of my thesis. Formally the project was on statistics, but actually it was more probability theory. My supervisor Frits Ruymgaart gave me on the first day a recent preprint by himself and Jon Wellner and said something like: “This probability inequality is not optimal, go ahead and improve it and get the optimal result”. At first, I had no idea what I was reading and how I ever could improve it, but later, after a lot of hard work, I obtained the desired result. It is the so-called basic inequality in my thesis. It became clear then that I liked mathematical research very much, much more than studying for exams, as I did before. As said, my thesis was on empirical processes, univariate and multivariate, and mainly on the so-called almost sure behavior. Let me tell you a bit more about these empirical processes. Assume you have 5 data points, say insurance losses. They are 3, 4, 7, 8, and 25 keuros. Now what the so-called empirical measure does is very easy, it gives probabilities 1/5 to these 5 numbers. So if you want to estimate the probability that the next claim is 4 keuros it is 1/5, and if you want to estimate the probability that it is 6 keuros it is 0, because 6 is not in the data. It is maybe surprising that this simple object is a very good estimator and that it led to many papers in scientific journals, including some of mine. Frequent co-authors on the subject were Frits Ruymgaart, David Mason, Paul Deheuvels, and a bit later I also joined forces with Jan Beirlant and Martien van Zuijlen. Here are two figures from my Statistics for Econometrics class: the first one shows as an example the distribution function of the standard normal distribution, the smooth dashed curve, and the empirical distribution function of a random sample of size 400 from this standard normal distribution. The empirical distribution function is obtained from the empirical measure by adding the probability estimates, which are now 1/400, of all the data points below a given point on the horizontal axis. At the smallest data point it is 1/400, at the second smallest 2/400, and so on. We see that both curves are about the same and this remarkably happens almost all the time when you repeat the experiment. The corresponding formal statement is sometimes called the fundamental theorem of statistics.

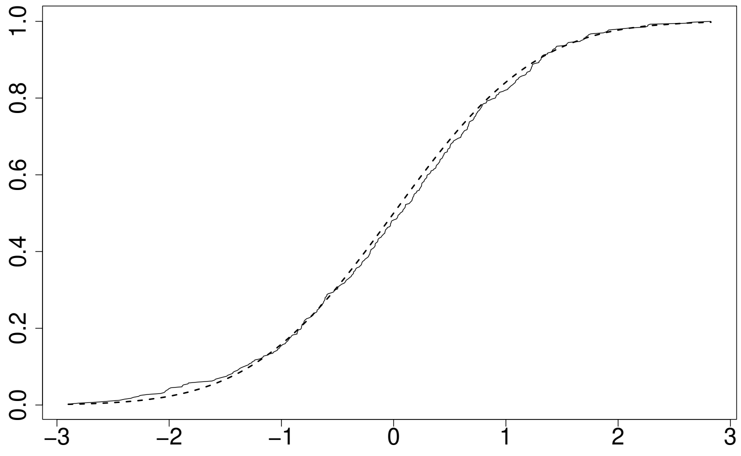



*Figure 1. The standard normal distribution function (dashed) and the empirical distribution function, based on a random sample of size 400.*

But when curves are about the same it is a bit boring, so you should look at their difference and scale it properly, in this case multiply with a factor of 20, which is $\sqrt{400}$. Then we obtain the empirical process and it leads to the following figure for the same 400 data. It looks very different, though it contains the same information: you still can find all the data. If you repeat this you might get a completely different figure.

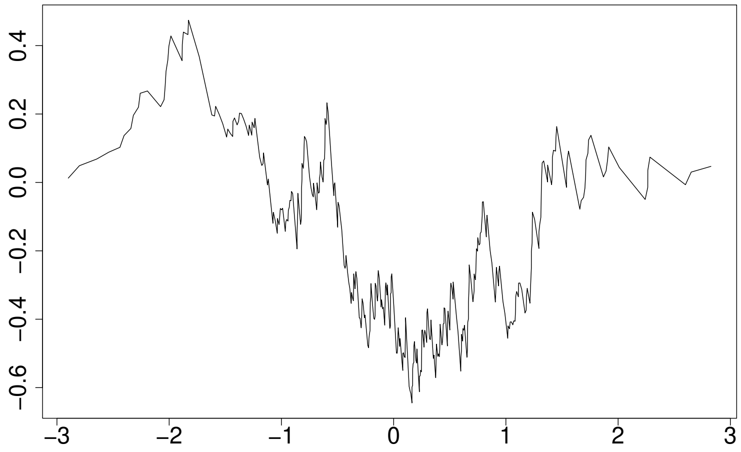



*Figure 2. The empirical process corresponding to Figure 1.*

This randomness of functions is on the basis for the study of empirical processes, but I return now to my Ph.D. research on this subject. At an early stage, I obtained an interesting result and my supervisor Frits Ruymgaart kindly mentioned it at the end of his talk at an Oberwolfach conference. David Mason was in the audience, but was not able to talk to him afterwards, because Frits had to leave early. I didn't know about this and to my surprise I got a bit later a letter from David, saying that he heard about my result and that he liked it, but that he could add something to it. We teamed up and the joint paper ended up in the last issue ever of the Zeitschrift fur Wahrscheinlichkeitstheorie und verwandte Gebiete in 1985, before it changed its name to Probability Theory and Related Fields. Needless to say that I was very happy with the paper and the collaboration with David Mason, who later became my other supervisor. Another paper with David, which ended up in the Annals of Probability, is about quantile processes. The empirical quantiles are obtained by inverting the empirical distribution function, that is interchanging the horizontal and vertical axes. In the insurance loss example the quantiles at the values 1/5, 2/5, 3/5, 4/5, and 1, are 3, 4, 7, 8, and 25 keuros, respectively. This gives another way of looking at the same data. Quantile processes complement empirical processes. Empirical quantiles are measured in the units of the data, like keuros, whereas empirical distribution functions represent the more abstract probabilities. From an applied as well as theoretical point of view, both processes have their merits. In particular, many nonparametric estimators or test statistics can be conveniently expressed in these processes, and thus the results obtained for empirical and quantile processes can be used to establish the behavior of these estimators and test statistics. To be precise, in many of my papers the empirical and quantile processes are weighted, which makes the results more general and more suitable for statistical applications, and it is much more fun (that is, challenging) to establish the results. Also, some of the results were extended to the censored case. In particular, two papers with Paul Deheuvels ended up in the Annals of Probability.

Quantiles are essentially the ordered data. Since the 5 losses in my example are already ordered, we obtain the quantiles without effort. However, this ordering is something, which can be done easily for numbers, but not for pairs of numbers. Recall the losses in keuros: 3, 4, 7, 8, and 25. Now assume that together with the losses we have some expenses for the insurance company in order to determine the losses. Then we obtain pairs of losses and expenses, for instance (3, 1) (the loss is 3 and the expense is 1), (4, 0), (7,3), (8, 5), and (25, 2). These pairs cannot be ordered in a canonical way. If you order the losses as we did, then the expenses are not ordered, and if we order the expenses, the losses are not ordered. Here is what we did. Assume you have 1000 pairs instead of 5 and imagine a picture of those 1000 pairs as a two-dimensional data cloud. Now look for the smallest circle that contains, at least 40%, that is 400 of the 1000 data. The area of that circle is the generalized quantile at 40%, or 0.4. Do the same for other numbers between 0 and 1. This gives you an example of the generalized quantile function. This has been elaborated in a paper with David Mason in the Annals of Statistics in 1992, long ago, but more recently this paper got renewed attention from researchers in statistical learning and machine learning. Also, this idea of generalized quantiles and the results obtained in the paper laid the foundation for several other papers I wrote on diverse statistical topics.

Beginning December 1997, I spent half a year at Florida State University in Tallahassee as a Senior Fulbright Scholar. The goal of the visit was to work with Ian McKeague on empirical likelihood. We wrote then two papers together and much later a third one. We worked on an extension of empirical likelihood, which we called localized empirical likelihood. Let me give you a simple example of empirical likelihood. Recall the 5 insurance losses: 3, 4, 7, 8, and 25 keuros. As we saw before, the empirical measure assigns probability 1/5 to each of these 5 values. Now assume it is known that the probability of ending up below 5 keuros is 1/2. Then the probability of at least 5 keuros is also 1/2. Now empirical likelihood tells you to look at the amounts below 5, that is 3 and 4, and give them equal probability of in total 1/2, so they both get 1/4. Similarly, 7, 8, and 25 get in total 1/2, so individually they get a probability of 1/6. That's all. For localized empirical likelihood we specify the probability (which was 1/2) not only for 5 keuros, but for every amount, e.g. the probability is 3/4 for being below 10 keuros, and so on. This is a simple example and again many papers have been written on the subject. The what-we-thought to be main paper ended up in the Annals of Statistics, but the other paper on nonparametric hypothesis testing, in the journal Bernoulli, got more attention from colleagues, for instance, through citations. Talking about probabilities, in particular small probabilities: in Florida, every day there was a small probability of getting eaten by an alligator. There were very many of them and they were everywhere. We liked them, but fortunately they didn't like us.

Maybe you now think what is all this theoretical research good for? Is the taxpayers' money well spent? Does this have applications to real-life problems? Actually, for me the mathematical theory with its theorems and proofs is the main reason for spending so much time on this research. The intellectual challenge of solving problems, that are often motivated by applications, is the main fun of being a researcher in mathematical statistics and probability theory. Understanding things, finding the truth, and seeing the beauty of the results and the underlying proofs is my main motivation for my research. That is what I try to say with the slightly provocative title: A beautiful theorem needs no application. Often when discussing my research work with non-specialists, we quickly move to talking about applications and I cannot convey how much I like the theory and the methods. Today I take the liberty to at least express my strong interest in these. I refrain from saying more about beauty in mathematics and how nicely things can fit together in proofs, since this is highly subjective and difficult to describe. If you now still think of the taxpayers' money, the good news is that I can assure you that there are numerous, important applications of the research I sketched and will sketch in the sequel, and that I am very much interested in several of these applications. I will address this a bit later.

But recall that the other part of my job was teaching, so let's talk about teaching now. To give you an idea, the total number of students I had, probably would not fit in the Willem II stadium. But this is simply due to the fact that I taught for a very long time, some 50 years if you count generously. I have taught for 22 years here at Tilburg University, so I can confine myself easily to the Tilburg period. However, the most interesting teaching anecdotes relate to my time at Maastricht University where I worked for 5 years, directly after obtaining my Ph.D. In those 5 years the student number of the then small university increased by a factor of 3, which led to all kinds of logistic problems, which were not funny at all then, but which afterward led to a kind of feeling of togetherness with my colleagues. As an example, we ended up with 400 students for an exam in a completely empty sports hall, so no tables, no chairs. It turned out that they had put them in another sports hall more than 5 kilometers away. This caused a lot of stress, in particular for the students. But, I should talk about my teaching here in Tilburg. I have taught students in Econometrics and OR, including the research master, but I have also done the so-called service teaching for other fields of study within our school TiSEM. In general, I enjoyed the teaching, especially since I experienced it as a fair share, not too much. The main course that I taught is Statistics for Econometrics, in the $2^{nd}$ year of the bachelor's. I taught it for the 22 years I have been here, maybe for too long. In particular, I enjoyed the collaboration for many years with Feico Drost on this course. The course was always received as difficult, which is maybe true, but it is also because this course doesn't have an assignment, which typically would lower the bar. But if you pass this course, I think it gives you a strong basis for the continuation of your studies, and beyond. Despite the not-so-high passing rates the students were in general positive about the course and about our explanations. I even got a sign of recognition from the students, through the Lecturer of the Year Awards 2019 of the study association Asset Econometrics. It says "Most likely to get a theorem named after him".

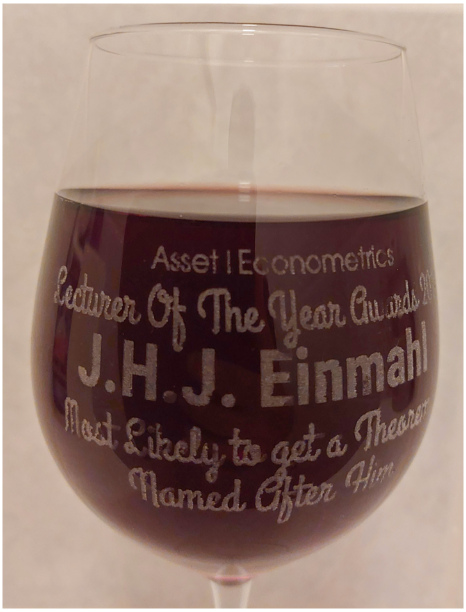



*Figure 3. Lecturer of the Year Awards 2019.*

I have not fulfilled their prophecy yet and got at best stuck at a probability inequality instead of a theorem. That's why I can't completely retire and have to continue working after today. As far as I remember in the early years of this century we had around 45 students in EOR and this number quadrupled in recent years. That is a large increase over time, which we noticed in particular when we had to grade exams. Here also higher passing rates would have helped. Let me give you another example of what we taught in the course instead of only talking about it. The subject is called sufficiency. Assume you have data, a large sheet full of 0's and 1's, 10 million of them. 1 means you have a certain property, 0 you don't. A lot of space is needed to store these data. Now the theory says that under natural assumptions it is good enough to know only the number of 1's (you only need 1 number of at most 8 digits instead of 10 million digits) and that continuing with this number of 1's yields better estimators. Isn't this beautiful? You tidy up the place. Not only you have more space, but also profit from it in the quality of the statistical procedures. I could enthusiastically tell you more about this, but let's return to research.

We were discussing research on empirical likelihood and the Florida alligators, in the year 1998. Let's go back, a bit more than 10 years, from there. It must have been 1987, a year after I obtained my Ph.D., and I worked at Maastricht University, as I told you. Suddenly, I got a phone call from Laurens de Haan whom I had seen a couple of times before, but we didn't have scientific contact then. He had questions about statistics of extremes. When studying empirical and quantile processes, I had worked jointly with David Mason on tail versions of these processes. This simply means that when you have uniformly-(0,1) distributed random variables, you zoom in at the processes near the origin. In other words, you blow up the vanishing tail of the process both in horizontal and vertical directions such that it stays alive, when the sample size grows. I think that Laurens learned about these results and realized that they can be important in the statistical theory of extremes. Obviously, tails of distributions and extreme observations are linked. At the beginning of the phone call I was happy and felt honored, but during the call my enthusiasm disappeared, since I understood less and less of what he was explaining to me. However, at the end he told me that much of the explanation was not directly needed and that the question boiled down to theory that I was familiar with, in particular to the joint work with David Mason. That is essentially how our collaboration started 36 years ago and how I became a coauthor, together with Laurens and Arnold Dekkers, of the moment estimator paper, which appeared in the Annals of Statistics in 1989. Now, Laurens and I, with other coauthors, have almost 12 joint papers. The twelfth one is not finished yet. That is a reason why I, and also Laurens, cannot retire.

At the end of the eighties of the previous century, extreme value statistics was a relatively small field and in particular only a few people with a background in empirical process theory were working in the field. The links between both subfields of mathematical statistics were also not so obvious then. My solid background in empirical and quantile processes turned out to be very helpful when trying to establish new theory in extreme value statistics, and even nowadays it is.

My research on extremes can be split up into univariate work and work in the multivariate or functional case. In the first case, the data are numbers expressed in, for instance, euros, seconds, or meters, in the second case the data are pairs or more generally tuples of numbers or even functions, e.g. when something is recorded continuously over time. After the univariate moment estimator paper in 1989, I mainly moved to multivariate extreme value statistics, but I returned to the univariate case much later. I would like to mention the paper on censoring and extremes with Amelie Fils-Villetard and Armelle Guillou 2008, one with Laurens de Haan and Chen Zhou, another frequent coauthor, in 2016 about heteroscedastic extremes, that is, data coming from somewhat different distributions, and two very recent ones with Yi He, where the distributions can be very different. I have not told you so far what extreme value statistics is about. Actually one of the goals is to make statistical inference in the far tail of the distribution, where you have no data, that is, to produce sensible statements about values we haven't seen before. When I will discuss applications this will become much more clear, but let me mention now already that a very classical problem regarding extremes is determining the height of the Dutch sea dikes. In general, in univariate extreme value statistics we talk about light tails and heavy tails. Let me explain this a little, without being very precise. In athletics, the world record holder on the 100 meters dash is Usain Bolt with a time of 9.58 seconds and a corresponding speed of 37.6 km/h. Adding 10% to this speed and converting back to time yields 8.71 seconds on the 100 meters. How likely is it that the next world record is lower than 8.71 seconds? It is very, very unlikely! That is roughly what we call a light tail. Now consider the insured loss of the natural disasters world record holder, Hurricane Katrina. This loss is 99 billion US dollars (indexed to 2022). How likely is it that the next world record exceeds this amount with 10%? This is rather likely and certainly not very unlikely. That is what we call a heavy tail. In univariate extreme value statistics, a large part of the literature is on heavy tails, but light-tailed distributions can also be very interesting. Extreme value theory also makes clear that the normal distribution is of limited use and hence has a misleading name!

Remember that there is also multivariate and functional extreme value statistics. I wrote quite a few papers in these areas and would like to mention two related papers, which are actually for bivariate data, on the estimation of the so-called spectral measure of an extreme value distribution. The first one in 2001 with Laurens de Haan and Vladimir Piterbarg and the second one in 2009 with Johan Segers. Both were a tour de force and I am still pleased with those results. The first one basically solved the problem and the second one substantially improved the estimator. Also for functional data, I would like to mention two papers, one with Tao Lin in 2006 where we study one-dimensional estimators (think of tail heaviness) jointly in a functional setting, and the other one, again with Johan Segers in 2021, where we study tail dependence in a functional setting. Indeed, as you might guess, Johan Segers is also a frequent coauthor. All four papers ended up in the Annals of Statistics. Tao Lin was a Ph.D. student of Laurens de Haan and me, who graduated in Tilburg in 2002 and it so happened that his promotion was the 1000$^{th}$ at Tilburg University in the 75 years of existence at that moment. Also, I mentioned several times the journals where the papers have been published. You noticed then the switch from probability journals to statistics journals, though both fields have a large overlap. You might also say that I gradually moved from more theoretical research to application-oriented research.

Now I halve talked in some detail about research in extreme value statistics, let's go back again to statistics teaching but link it to heavy tails. Assume for simplicity you have only one observation from a Pareto distribution starting at 0, with some positive shape parameter. Based on this observation you would like to do hypothesis testing, more precisely you would like to test the null hypothesis $\mu \leq 100$ against the alternative $\mu > 100$, where $\mu$ is the mean of the distribution. This is a perfect problem for the course Statistics for Econometrics. Now it is easy to find the optimal test for this problem at the 5% level, namely reject the null hypothesis if the observation is greater than or equal to 18.4, which is much less than 100. Hence the interesting finding is that when we observe, say, 30, this proves in the usual statistical way that the expectation $\mu$ exceeds 100. This might be surprising. We learn here that you have to be careful with heavy tails and maybe also with expectations.

# Valorization

So far, I have hardly talked about applications of the research in the subfields of probability theory and statistics I worked in. Maybe you are eager to learn about these applications, but note that it is hard to find areas of human activity or interest where statistics does not play a role. But this is too general, so let me be a bit more specific and consider only those fields of applications related to the work I discussed. Here I don't strive for completeness but only present a selection. Linked to the research areas of our School, I would like to mention Risk management in particular in finance and insurance, e.g., problems concerning systemic risk, but also the Analysis of auctions. More related to physics or engineering are applications in the Analysis of extreme wind speeds, Seismology, Hydrology and in particular flooding, Hydroclimatology, and Corrosion engineering. Although the quantities measured here are not in euros or dollars, at the end they are often converted to euros or dollars, e.g., when considering losses caused by hurricanes or earthquakes. In this way also these applications fit well within the scope of our school TiSEM. Some other applications are in Anomaly detection, Seed dispersal of plants, Bibliometrics, Genetics and Bioinformatics, and Aviation safety.

There are at least two projects I have worked on that are really applied in the sense that the application is more important than the method. Though other applications as above can be more important, it has been democratically established that both projects are very interesting. I don't consider the corresponding papers my best papers, but they got much, much more attention than the papers I am most happy with and then I started to like them a bit more. The two main papers of the projects are both in JASA in the Applications and Case Studies part. Note that we have shifted gears. Referring to the title, the message of this part of the lecture is more like: A beautiful application needs no theorem. The first project is on ultimate athletics records with Jan Magnus and a follow-up paper with Sander Smeets, who was a QFAS student here at Tilburg University. The paper was a short version of his master's thesis. Because of this research, he got the university prize for the student who performed best in the media. He was, e.g., interviewed about his work by Dione de Graaff on the Dutch Sports radio channel. The questions we considered in the papers are of the type "How fast can we run the 100 meters?", where the word "we" should be understood properly. The other, more recent project is with Jesson Einmahl and Laurens de Haan on limits to human life span. Here the question is "How long can we live?". Normally, at best, my papers are read by a relatively small number of colleagues, but the results of these papers made it to thousands of newspapers and news sites all over the globe and we got quite a few requests for interviews for radio and television. I recall talking from our kitchen table for the BBC radio about athletics records and to Vicky Davila from Colombian radio about extreme ages. Note that both projects are applications of univariate extreme value theory, where the distributions have light tails.

I would like to tell you a bit about the ultimate record for the 100 meters dash for men. For women, we have a similar analysis, but it is less interesting since the world record there is from 1988 and in those days the doping control was not organized very well. The analysis of the 100 meters men was done first for the paper with Jan Magnus. Then we refined the analysis and used better data in the paper with Sander Smeets and finally in 2012 when I was invited for a session related to the London Olympic Games at a Statistics conference in the UK, we used the same refined analysis but updated the data once again. This is a picture from an article about our 2012 update in the Dutch Metro News, a hard copy of which was then available for free at, for instance, railway stations. I now use it sometimes in teaching material, since it not only depicts Usain Bolt, but also a very basic formula from extreme value theory.

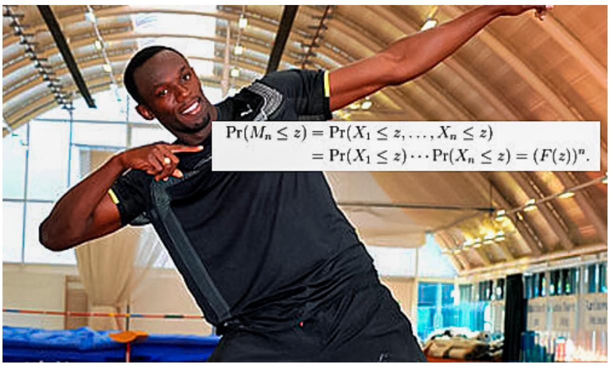



*Figure 4. Usain Bolt "recommends" Extreme Value Theory.*

But remember the question was "How fast can we run the 100 meters?". Here you see a figure with 14 dots which are the most recent 14 world records.

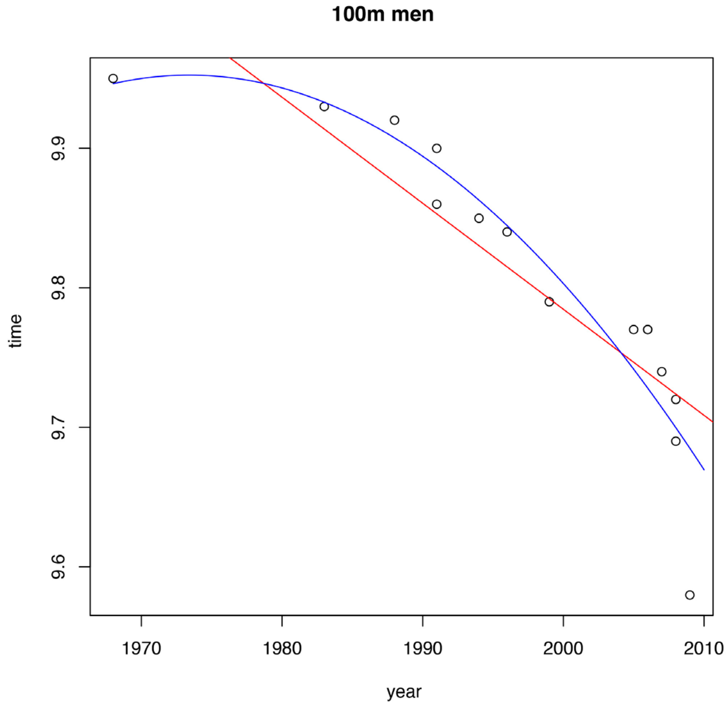



*Figure 5. World records on the 100 meters dash for men.*

On the left-upper corner, there is the 9.95 seconds of Jim Hines in 1968 and then when time passes by, the world record becomes lower until the present world record of 9.58 seconds of Usain Bolt in the right-lower corner, run in 2009. Now you want to predict the world record in 2010 or 2012 and what people do and you should *not* do is fit a straight line or a curved line through the records. It is easy to see that in 2010 or a bit later the predicted world record (red or blue) is above the 9.58 seconds of Bolt. That is, you say that the ultimate performance in 2010 cannot be faster than 9.67 seconds (blue), but from the dot in the right-lower corner you see that this is not true. Therefore you shouldn't follow this approach. You should use extreme value statistics and use much more data than only the 14 consecutive world records. We considered 100 meters times from 1991, when modern doping control was introduced, until July 2012, two months before the aforementioned presentation in the UK. For every athlete we take the personal best. We begin with the world record, then the second personal best, and so on. In this way, we were able to obtain 1034 personal bests, ranging from the world record of 9.58 seconds to number 1034 with a time of 10.30 seconds. Sometimes I call this the worst time, but note that it is pretty fast. From a statistical point of view, it is better to have more than 1000 100-meters times instead of only 14. Now by methods from extreme value statistics, we estimate the so-called extreme-value index to be –0.15, which indicates a light tail or actually no tail at all. This value goes into the endpoint estimation procedure as seen in the next figure.

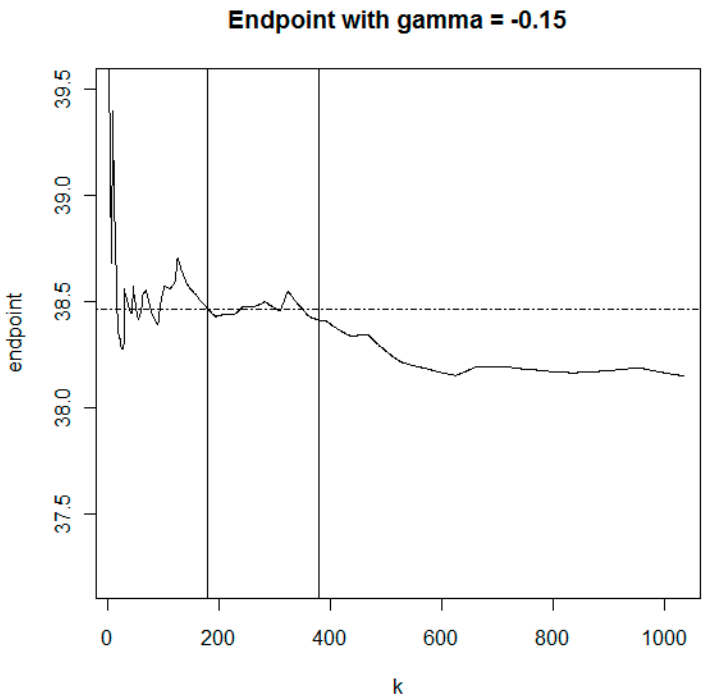



*Figure 6. Estimated endpoint in km/h for 100-m; the dashed-dotted horizontal line represents the final endpoint estimate.*

Now averaging the endpoint estimator over the indicated window leads to a speed of 38.47 km/h, which corresponds to a time of 9.36 seconds, 2.3% faster than the present world record. More precisely our statement back then in 2012 was: nobody can run the 100 meters faster than 9.36 seconds in the near future. Now in 2023, we see that this is still the case. Clearly, our estimate will change when we have more data available and also, as in all statistical procedures, there is uncertainty involved in the estimation, but if you want one number, just like the media, it is 9.36 seconds. If tomorrow somebody runs 9.35 seconds, then I will come back to you with more precise statements about the uncertainty.

When I taught a full course of 14 lectures, it sometimes happened that at the last lecture I finished 5 minutes early and then I apologized to the students saying that they paid tuition for 90 minutes but that the lecture lasted only 85 minutes. Since you didn't pay tuition for today's lecture, I wouldn't need to apologize when finishing early, but let me use all the allocated time and tell you a final story. At several Ph.D. defenses here in this auditorium, I have talked about Erdős and Erdős numbers, but you have heard enough about mathematics and mathematicians now, so at the end of this lecture let us move to Hollywood, although I never had the ambition of becoming an actor. Even as a teacher I didn't act too much. Now apparently Kevin Bacon is a prolific actor who played in many movies with many other actors and as such he is the Erdős of the movie industry. Kevin Bacon himself has by definition Bacon number 0. If you played in a movie with Bacon, you have Bacon number 1. If you played in a movie with someone who has Bacon number 1, and you don't have Bacon number 0 or 1, then you have Bacon number 2, and so on. Now why is this relevant here? Long ago, I appeared in a school television movie called "Is het toeval?" with Michiel Huisman, who then was at the beginning of his career. It was about probability and chance and specifically the ultimate athletics records we just discussed. He came to the university here with a whole team and it took hours to record a short movie clip. This is another reason why I have no ambition to become an actor. Here is a "still" from the movie clip.


*Figure 7. Michiel Huisman in our coffee room for the recording of "Is het toeval?".*

Later Michiel Huisman became a famous actor and he obtained a Bacon number of 2: he connects to Bacon through, for instance, Brad Pitt. Hence this gives me a Bacon number of 3. People from network theory will tell you that this is not very special, and I agree.

There are at least one million people with Bacon number 3. Nevertheless, maybe TiSEM or Tilburg University should consider incorporating the Bacon number in the assessment of research visibility and valorization.

# Words of Thanks

Let me finish with the usual but important words of thanks. I would like to begin with the students. I have been privileged to teach many, smart students. I thank them for choosing our programs, in particular the Bachelor's Econometrics and Operations Research. The students make universities, including all scientific research there, possible.

Within the department, I would like to thank especially the various heads of departments and the secretaries for keeping the department going and for supporting me in many ways. I also would like to thank the colleagues in the department with whom I worked constructively and pleasantly together in teaching and research.

I thank Tilburg University and especially the Tilburg School of Economics and Management for providing me with ample research time, the opportunity to carry out my own research program, and in particular the strong research support through the Arie Kapteyn chair 2019-2022.

I am very grateful to my former Ph.D. students and important coauthors Juan-Juan Cai and Yi He for organizing the successful international scientific workshop, preceding this lecture, in Oisterwijk, bringing in many top scholars. If feel honored. Thank you, Juan and Yi.

In particular, I would like to thank all my co-authors, including my Ph.D. students. Several of these coauthors I have mentioned already when describing the research. I would like to add now by name Estate Khmaladze, Ingrid Van Keilegom, Umut Can, Roger Laeven, and Anna Kiriliouk. Collaborating with my coauthors was the core of my work. Collaborating on research in mathematical statistics is simply great fun. In various ways, we built on projects and enjoyed the little and larger steps we made. Often it seemed very difficult when it was fresh and later when we better understood it, it looked simpler and sometimes beautiful. Thank you very much, coauthors.

I would like to highlight one of them. Highly learned de Haan, beste Laurens, it means much to me that you are here. I learned a lot from you and it was great to work with you on fascinating statistical problems for most of my academic life. You infected me with the extreme value virus and I am glad that I didn't have antibodies. I hope this farewell lecture doesn't change much and that our pleasant cooperation continues until we touch the endpoint of the life span distribution. Thank you very much!

I also thank my children Ivana and Jesson for their patience with me, when this mathematician wanted to know things precisely and didn't answer questions before they were carefully formulated. I also recall that when they were in high school they found out that it was my job to create my own problems and then to solve them, which they thought was rather silly.

Finally, I would like to thank my spouse, who among many other things made this beautiful painting that brightens my office. Lieve Fia, it has been a long and great journey that we made together. The journey was much more than moving of course, but you moved to Nijmegen when I started my Ph.D. and later we moved together to Maastricht, Eindhoven, and then Tilburg, not to mention the various longer stays abroad, where you did the homeschooling of the children at some occasions. You strongly supported this moving, since we deemed it relevant for my career and happiness. You always had confidence in me, even in the early days when I was a master's student and getting a job was not at all obvious. Thank you very much for all your support and love and for sharing your life with me since then.

*I have spoken.*
*Ik heb gezegd.*

# Colophon

*design*
Beelenkamp ontwerpers, Tilburg
*photography cover*
Maurice van den Bosch
*layout and printing*
Studio | powered by Canon