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Abstract

In this paper we analyze the performance of internationally investing U.S.-based mutual funds, correcting for market frictions such as short sell constraints and transaction costs using a variety of performance measures. We first of all show that for a number of funds Jensen's α is significantly positive if market frictions are ignored. Subsequently we show that the evidence of outperformance is robust to measuring performance with respect to an international asset pricing model with three country portfolios and a currency portfolio as the factor mimicking portfolios. As is well known by now these performance measures can alternatively be interpreted as tests of the hypothesis that the supposed factor mimicking portfolios span the efficient frontier of these portfolios and the mutual fund, i.e. of the hypothesis that mean-variance agents can not improve their risk return trade-off by also investing in the mutual fund.

An important shortcoming of the standard CAPM and APT based performance measures is the underlying assumption of frictionless markets. Recent papers of Luttmer [1996] and De Roon, Nijman and Werker [1997] have developed tests for mean-variance spanning to take frictions such as transaction costs and short selling restrictions into account. In this paper we show that for a large number of internationally investing mutual funds in our sample the hypothesis of mean-variance efficiency of the benchmark portfolios can not be rejected if market frictions are taken into account, whereas the performance measures that do not take market frictions into account suggest that the fund outperforms the benchmark assets. Furthermore, we show that incorporating load fees for mutual funds substantially affects the diversification benefits that investors can realize by including internationally investing mutual funds in their portfolio.

1 Introduction

The empirical literature on performance evaluation of mutual funds concentrates on the question whether fund managers have special abilities in composing a portfolio (e.g., Jensen [1968]), which could provide investors with superior returns. The issue of performance measurement is closely related to the question whether investors can improve their portfolio's risk-return trade-off when additional assets are taken into account (see, e.g. Jobson and Korkie [1988], Chen and Knez [1996]). Performance measurement requires a pricing model in order to define outperformance of efficient benchmark portfolios. On the contrary, a test for a shift in the mean-variance frontier by including additional assets can do without a pricing model since it only starts with the assumption that the investor already holds an efficient portfolio of the benchmark assets. This implies that performance evaluation of mutual funds is equivalent to a test for diversification benefits under the assumption that investors hold efficient combinations of benchmark assets that correspond to the pricing model used. If no mean-variance optimizing investor can significantly extend the investment set by considering a set of additional assets then there is mean-variance spanning, as defined by Huberman and Kandel [1987]. If there is only a particular group of investors that cannot extend the efficient set by adding an additional asset then there is intersection. In the latter case the mean-variance frontier of the original assets in portfolio and the mean-variance frontier of the extended portfolio have one point in common.

An important shortcoming of many tests for mean-variance spanning proposed in the literature is the supposed absence of market frictions. When buying assets, investors are confronted with transaction costs. In particular, investors considering international diversification have to deal with high transaction costs. For these investors internationally diversified mutual funds are an alternative for obtaining a highly diversified portfolio (see Cumby and Glenn [1989]). However, mutual funds have operating expenses such as management fees, administrative costs, advisory fees and marketing costs which are deducted from the fund's assets. Although in performance evaluation studies the difference between returns before expenses and returns after expenses is taken into account (see, e.g. Malkiel [1995]), the load fees charged by some of the mutual funds are usually ignored.

Only recently, tests for mean-variance spanning have been extended to take frictions, such as transaction costs and short selling restrictions, into account (see Hansen, Heaton and Luttmer [1995], Luttmer [1996] and DeRoon, Nijman and Werker [1998]). In this paper we employ a sample of internationally investing mutual funds, and we will show that for a large number of the funds the hypothesis of mean-variance spanning will not be rejected if short selling restrictions are incorporated, whereas in the case without market frictions, mean-variance spanning is rejected for most of these funds. Furthermore, we will show that incorporating load fees for mutual funds substantially affects the diversification benefits that investors can realize by including internationally investing mutual funds in their portfolio. This paper extends the one from DeRoon, Nijman and Werker [1997] in its application to mutual fund performance evaluation. Moreover, we incorporate market frictions in conditional performance evaluation, and we show how to test two-sided inequality restrictions that arise in mean-variance spanning tests where transaction costs are incorporated.

The remainder of this paper is organized as follows. In Section 2 we show the relationship between performance evaluation and testing for diversification benefits by adding an asset to the initial portfolio. Furthermore, we present our sample of internationally investing mutual funds, and discuss some previous empirical results on performance measurement of mutual funds. In Section 3 we show how market frictions such as short sales restrictions and transaction costs can be incorporated in performance evaluation. Section 4 presents the tests for mean-variance spanning in case of a frictionless market as well as when market frictions are incorporated. The empirical results show that the possible diversification benefits by including internationally investing mutual funds are seriously affected by short selling constraints on some of the assets under consideration. In conditional performance measurement studies some predetermined information variables, that can be used to predict stock returns, are explicitly taken into account in evaluating mutual fund performances. In Section 5 we present the tests as well as the empirical results for potential diversification benefits of mutual funds in a frictionless as well as a market with frictions when we allow for time-varying expected returns. Finally, Section 6 concludes.

2 Performance Analysis of US based International Mutual Funds in a Frictionless Market

For a risk-averse optimizing agent, the decision to invest in an internationally diversified mutual fund depends on the question whether a fund manager is able to extend the investor's efficient set of assets. The superior risk-return trade-off that a mutual fund potentially provides due to timing or selection ability of the fund manager, will be the motive to add a fund to the initial portfolio of assets. Suppose a mean-variance investor considers to extend his initial efficient set of K assets by adding a set of N internationally investing mutual funds. The gross return vector r_{t+1}, represents the returns of the mutual funds after operating expenses. The gross returns for the K benchmark assets are denoted by the vector R_{t+1}. In a frictionless market, where the Law of One Price holds, there exists a stochastic discount factor M_{t+1} such that

$$E[M_{t+1}R_{t+1}|\, I_t] = \iota_K, \tag{1}$$

where ι_K is a K-vector of ones and I_t is the public information set available at time t. In the Sections 2, 3 and 4, we assume that the expected returns on the assets and the corresponding (co)variances are constant over time. Extensions to the conditional version of (1) will be implemented in Section 5, following Ferson and Schadt [1996].

Since we consider a mean-variance optimizing investor, a stochastic discount factor M_{t+1} is a linear function of the K asset returns. Moreover, as shown by Hansen and Jagannathan [1991], the mean-variance stochastic discount factor $m\,(v)_{t+1}$ given by

$$m\,(v)_{t+1} = v + \alpha(v)'(R_{t+1} - E[R_{t+1}]), \tag{2}$$

with

$$\alpha(v) = Var[R_{t+1}]^{-1}(\iota_K - vE[R_{t+1}]),$$

has the lowest variance of all stochastic discount factors with expectation v, that price R_{t+1} correctly. It is straightforward to show that the zero beta rate[1] corresponding to the mean-variance investor's optimal portfolio is equal

[1] The zero beta rate of a portfolio can be obtained as the intercept of the line tangent to the mean-variance frontier in the point where the investor's optimal portfolio is located.

to $1/v$, i.e. the inverse of the expectation of the stochastic discount factor. A mean-variance optimizing investor cannot extend the efficient set by investing in the mutual fund under consideration if the stochastic discount factor given in (2) also prices the mutual fund's return r_{t+1} correctly (see, e.g. Bekeart and Urias [1996], DeRoon, Nijman and Werker [1996]). This can be seen easily if we recognize that (1) can be interpreted as the first order conditions of an investors portfolio problem.

As mentioned, the question whether there is a shift in the efficient frontier by extending the investment set is closely related to performance measurement (see, e.g. Chen and Knez [1996]). A well known measure of the performance of a mutual fund is the generalized Jensen [1968] measure. It can be obtained as the intercept in a regression of the excess return[2] of the mutual fund on the excess returns of some benchmark portfolios and a constant. However, in order to evaluate the performance of a mutual fund a pricing model is required that specifies the set of K efficient benchmark portfolios that span the mean-variance frontier. For instance, under the assumption that the CAPM is the pricing model, the so-called market portfolio with return R^m_{t+1} and the risk free deposit are the efficient benchmark portfolios.

Since a minimum variance stochastic discount factor is linear in the returns of the benchmark assets, it is straightforward to show that the performance of a fund relative to the set of benchmark assets can be measured by

$$\lambda(v) = E[m(v)_{t+1} r_{t+1}] - 1 = v\alpha_J(v), \tag{3}$$

where $\alpha_J(v)$ is the generalized Jensen measure. A similar relationship can be derived for a multifactor pricing model, implying a stochastic discount factor $m(v)_{t+1}$ that is linear in the factor mimicking portfolios (see, e.g. Fama [1996]). The performance measure (3) indicates that an investor can improve the risk-return trade-off by buying the mutual fund if $\lambda(v) > 0$, and selling the fund, i.e. taking a short position, if $\lambda(v) < 0$. The case where $\lambda(v) = 0$ corresponds to no diversification benefits by including this fund into the portfolio. If $\lambda(v) = 0$ for exactly one value of v, this is equivalent with intersection of the extended and initial mean-variance frontiers at the point where the investor's optimal portfolio is located. Furthermore, if $\lambda(v) = 0$ holds for all possible v then the extended investment set is spanned by the original K benchmark assets, corresponding to the case where the extended

[2] An excess return is the return in excess of a risk free rate (if available) or some zero beta rate.

and initial mean-variance frontier coincide.

In this paper, we examine whether internationally investing U.S. based mutual funds can extend the efficient investment set of a U.S. investor. We employ a sample of eighteen internationally investing open-end mutual funds over the period 1982 through 1994. The mutual fund data are obtained from the Morningstar Mutual Fund Database. Morningstar reports information about all open-end mutual funds on a monthly basis. The mutual funds in our sample have as investment objective 'foreign' or 'world', and exist over the whole sample period. Our sample is comparable with the sample of Cumby and Glenn [1990], studying the performance of a sample of fifteen U.S. based internationally diversified mutual funds over the period January 1982 through June 1988.

Since performance evaluation depends on the choice of the set of benchmark portfolios, we consider in this paper two sets of benchmark assets. The first set of benchmark assets is equivalent to the one employed by Cumby and Glenn [1990] and consists of the Morgan Stanley World index and an equally weighted portfolio of Eurocurrency Deposits[3] to reflect a currency hedge portfolio. This set of benchmark assets can be interpreted as the initial portfolio of a group of investors that have a widely diversified international portfolio with predetermined country weight allocation corresponding to the market capitalization of the individual countries and who consider to extend their portfolio with an internationally investing mutual fund. The second set of benchmark assets represents the initial portfolio of investors that currently invest efficiently in the US, European and Japanese stock indices as well as the Eurocurrency Deposits. The benchmark assets used are the Morgan Stanley Capital Market Indices (MSCI) for the USA, Europe and Japan, obtained from Datastream. Based on the claim that in international asset pricing the asset returns are better described by multifactor models than by single index models (see, e.g. Korajczyk and Viallet [1989]), the two sets of benchmark assets can alternatively be interpreted as tests whether a two or four factor model prices the mutual funds under consideration.

In Panel A of Table 1 we present some summary statistics for the sample of eighteen mutual funds that we employ. Note the variation in the front loads that the funds charge: five mutual funds can be marked as no-load funds while nine internationally investing funds charge more than 5.75% for

[3] The currencies in this portfolio are the Canadian dollar, the Deutsche mark, The Dutch guilder, the French franc, the Japenese yen, the pound Sterling and the Swiss franc.

Table 1: **Summary Statistics.** Panel A of this table contains some summary statistics for a sample of eighteen mutual funds. The average monthly return (corrected for operating expenses) and standard deviation, are calculated over the period 1982 through 1994. The column 'Net Assets' is the size of the fund in million dollars as reported at the end of 1994. The column 'Expense Ratio' reports the average monthly percentage that a fund took out of its assets for operating expenses over the period 1982 through 1994. The column 'Front Load' shows a one-time deduction the funds charge for an investment made into the fund. The column labelled 'Correl World' shows the correlation between the mutual fund and the MSCI World index. Panel B of the table shows summary statistics for the benchmark indices.

Panel A							
Mutual Fund	Average Return (%)	Stand. Dev. (%)	Net Assets (mln $)	Front Load (%)	Expense Ratio (%)	Correl. World	
Alliance Global Sm.	0.84	6.18	53.80	4.25	0.14	0.69	
Alliance Intl.	1.24	5.31	167.40	4.25	0.13	0.82	
Bailard,Biehl Intl.	0.99	5.21	116.20	0.00	0.09	0.84	
First Invest Global	1.14	5.23	206.80	6.25	0.15	0.78	
Kemper Intl.	1.14	4.60	323.40	5.75	0.11	0.82	
Lexington Wrd. Wide	0.94	5.26	261.80	0.00	0.13	0.69	
New Perspective	1.29	3.94	6560.50	5.75	0.06	0.87	
Oppenheimer Global	1.29	5.28	1809.50	5.75	0.12	0.81	
Phoenix World Opp.	0.82	6.09	122.50	4.75	0.12	0.67	
Pilot Kleinwort Intl.	1.08	5.07	30.00	4.50	0.15	0.85	
Putnam Global Gr.	1.31	4.27	1427.70	5.75	0.11	0.89	
Scudder Intl.	1.25	4.63	2131.80	0.00	0.10	0.86	
T. Rowe Price Intl.	1.35	4.76	5465.60	0.00	0.09	0.87	
Templeton Gr.	1.25	4.10	5727.70	5.75	0.13	0.79	
Templeton Sm. Cmp.	1.27	4.47	1253.00	5.75	0.08	0.73	
Templeton Wrd.	1.24	4.11	5123.40	5.75	0.06	0.79	
United Intl. Gr.	1.27	4.26	603.00	5.75	0.09	0.86	
Vanguard Intl. Gr.	1.41	4.99	2755.80	0.00	0.06	0.85	
Panel B							
Benchmark	Average	Stand.			Correlations		
Indices	Return	Dev.	World	Eurocur	USA	Europe	Japan
World	1.26	4.28	1.00	0.28	0.76	0.82	0.77
Eurocur	0.84	2.82		1.00	-0.07	0.40	0.40
USA	1.25	4.34			1.00	0.59	0.26
Europe	1.41	4.88				1.00	0.51
Japan	1.53	7.57					1.00

a position in the mutual fund. The average returns for the no-load funds do not appear to be different from the funds that charge a load fee. Furthermore, it seems that the expense ratio of a fund is negatively correlated with the size of the fund. This can probably be explained by the fixed costs involved in managing a mutual fund. Moreover, since we defined returns as returns after operating expenses, a high expense ratio influences the average return of the fund. Panel B of Table 1 contains information about the benchmark portfolios. It appears that the Morgan Stanley Japan index realized the highest average return but also involves the highest risk as measured by the standard deviation. Furthermore, the returns on the USA, Europe and Japan stock indices are highly correlated, as expected, with the return on the Morgan Stanley World index.

For both sets of benchmark assets defined above, Table 2 contains the outcomes for the performance measure (3) for the case where the zero beta rate $\frac{1}{v}$ is set equal to the average monthly return on the one-month Tbill over the period 1982-1994, i.e. 0.53%. It appears that in case of returns after operating expenses, ten mutual funds have a positive performance measure in the two benchmark case, while in the four benchmark case only six funds have a positive value in our sample, none of them significant at the 5% level. The outperformance with respect to the two benchmark case of the best performing funds is in the order of magnitude of 0.25% per month, i.e. 3.00% annually. Our outcomes are in accordance with the results of Cumby and Glenn [1990], who find four out of fifteen internationally investing mutual funds with positive Jensen measures. Recall that the Jensen measure corresponds with a stochastic discount factor M_{t+1} that is linear in the benchmark assets. Consequently, the outcome of the Jensen measure can be interpreted in light of (3). This means that investors who already hold an efficient portfolio in the case of four benchmark assets, can only improve the risk-return trade-off by taking a short position in most of the mutual funds under consideration. However, it is important to note that taking a short position in mutual funds is almost impossible for most investors.

As noted by for instance Malkiel [1995], underperformance with respect to a set of benchmark portfolios does not mean that fund managers do not have special abilities in stock selection. Since mutual funds have operating expenses, reported as the expense ratio of the fund, that are deducted from the fund's assets, the performance evaluation of returns before expenses may give some indication for special abilities such as timing or selection ability. However, it has to be noted that the general investing public cannot benefit

Table 2: **Generalized Jensen measure.** The table reports the generalized Jensen measure for two sets of benchmark assets. The first set consists of the Morgan Stanley World index and an equally weighted portfolio of Eurocurrency Deposits, and the second set contains the Morgan Stanley USA, Europe and Japan indices and an equally weighted portfolio of Eurocurrency Deposists. The zero beta rate $1/v$ is set equal to average monthly return on the one-month Tbill over the period 1982-1994, i.e. 0.53. The columns 'Returns after expenses' show the Jensen measure calculated with returns corrected for operating expenses, while the columns 'Returns before expenses' reports the Jensen measure before the fund's expenses are subtracted from its net assets. The results are based on monthly observations for the sample January 1982 through December 1994. Standard errors are reported in parentheses.

Mutual Fund	Generalized Jensen Measure			
	Two Bench. Assets		Four Bench. Assets	
	Returns after expenses	Returns before expenses	Returns after expenses	Returns before expenses
Alliance Global Sm.	-0.29 (0.33)	-0.16 (0.33)	-0.57 (0.24)	-0.44 (0.24)
Alliance Intl.	-0.05 (0.25)	0.08 (0.25)	-0.17 (0.21)	-0.04 (0.21)
Bailard,Biehl Intl.	-0.32 (0.23)	-0.22 (0.23)	-0.36 (0.19)	-0.27 (0.19)
First Invest Global	-0.07 (0.27)	0.08 (0.27)	-0.18 (0.26)	-0.04 (0.26)
Kemper Intl.	-0.05 (0.22)	0.06 (0.22)	-0.14 (0.17)	-0.03 (0.17)
Lexington Wrd. Wide	-0.11 (0.28)	0.02 (0.28)	-0.33 (0.22)	-0.21 (0.22)
New Perspective	0.20 (0.16)	0.27 (0.16)	0.04 (0.12)	0.10 (0.12)
Oppenheimer Global	0.04 (0.26)	0.16 (0.26)	-0.12 (0.23)	-0.00 (0.23)
Phoenix Wrd. Opp.	-0.31 (0.35)	-0.19 (0.35)	-0.57 (0.28)	-0.45 (0.28)
Pilot Kleinwort Intl.	-0.20 (0.22)	-0.05 (0.22)	-0.30 (0.18)	-0.15 (0.18)
Putnam Global Gr.	0.13 (0.16)	0.24 (0.16)	-0.02 (0.12)	0.09 (0.12)
Scudder Intl.	0.02 (0.19)	0.12 (0.19)	-0.09 (0.15)	0.01 (0.15)
T. Rowe Price Intl.	0.08 (0.19)	0.17 (0.19)	-0.02 (0.14)	0.07 (0.14)
Templeton Gr.	0.23 (0.19)	0.36 (0.19)	0.06 (0.14)	0.18 (0.14)
Templeton Sm. Cmp.	0.27 (0.22)	0.35 (0.22)	0.09 (0.18)	0.17 (0.18)
Templeton Wrd.	0.22 (0.18)	0.28 (0.18)	0.03 (0.13)	0.09 (0.13)
United Intl. Gr.	0.11 (0.18)	0.20 (0.18)	0.01 (0.17)	0.10 (0.17)
Vanguard Intl. Gr.	0.12 (0.21)	0.18 (0.21)	0.04 (0.17)	0.10 (0.17)
average	0.00	0.11	-0.14	-0.04

from these superior returns if the costs for obtaining this extra information are too high. Malkiel evaluates the returns from equity mutual funds over the period 1971 through 1991 by assuming the CAPM as the pricing model. In case of returns before expenses, he finds that mutual fund managers outperform the market portfolio with +0.18% on a monthly basis, i.e. about 2.00% annually, whereas underperformance dominates after expenses. In case of returns before expenses, we find in our sample of mutual funds that the performance measures for nine mutual funds are positive with respect to the four benchmark case (average: -0.04% monthly), while in the two benchmark case even fourteen out of eighteen funds have a positive performance measure (average: 0.11% monthly). Results similar to those of Malkiel are obtained by Carhart [1997], Daniel, Grinblatt, Titman and Wermers [1997]. Note that all these papers ignore market frictions. The techniques that we employ to measure the performance of internationally investing mutual funds can be extended to the case of domestic equity portfolios e.g. by imposing short sell restrictions on small as well as large, and high book to market as well as low book to market stocks.

3 Performance Analysis in case of Market Frictions

Thusfar we assumed a frictionless market in evaluating mutual fund performance. It appears that the results are rather sensitive to the assumed initial set of benchmark assets. An investor who already owns a portfolio with efficient country weight allocation, and considers to extend this portfolio with an internationally investing mutual fund, can improve the risk-return tradeoff by taking a short position in most of the international mutual funds. However, an investor is typically confronted with short sales constraints on certain assets. Furthermore, when international investing is taken into account, the question whether an investor should directly invest in international assets or buy an internationally diversified mutual fund depends on the size of the transaction costs involved in buying the assets under consideration. Most of the mutual funds charge a load-fee for an investment into the fund. Moreover, some of the funds also have a back-end sales charge, although this percentage declines the longer the shares are held, and usually disappears entirely over time.

In considering the question whether a mean-variance optimizing investor can extend the efficient set by including additional assets into his portfolio, it is appropriate to incorporate market frictions. In case of short sales constraints, it is shown, for instance by Markowitz [1991] that the mean-variance frontier subject to short sales constraints consists of a finite number of segments of unrestricted frontiers. We denote the total number of segments by P. The assets in the efficient portfolios on the different P unrestricted frontiers coincide with the assets for which the short sales constraints are not binding in the optimization problem subject to short sales constraints (see, e.g. DeRoon, Nijman and Werker [1998]). Let $R_{t+1}^{(v)}$ denote such a L-dimensional subvector of R_{t+1} for which the short sales constraints in the restricted optimization problem are not binding. As shown by Luttmer [1996], if we include short sales constraints, the Law of One Price implies the following generalization of (1):

$$E[m_R(v)_{t+1} r_{t+1}] \leq \iota_N, \tag{4}$$

where the inequality sign reflects the short sales constraints on the additional assets. Since (4) holds, and a mean-variance stochastic discount factor is a linear function of the subset of L assets, the stochastic discount factor corresponding to mean-variance optimizing behavior that prices the assets on segment p of the restricted mean-variance frontier correctly is

$$m_R(v)_{t+1} = v + \alpha^{(v)\prime}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]), \tag{5}$$

with

$$\alpha^{(v)} = Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]).$$

Similar to the case without market frictions, a mean-variance investor cannot extend his efficient set of assets by including a mutual fund with gross return r_{t+1} if the stochastic discount factor given in (5) also prices r_{t+1} correctly, i.e. if it satisfies (4).

Since, in case of short sales restrictions, we can distinguish P different subsets of assets with corresponding mean-variance stochastic discount factors linear in those assets returns, the relationship between the Jensen measure and the performance measure defined in (3) now generalizes to

$$\lambda(v) = E[m_R(v)_{t+1} r_{t+1}] - 1 = v\alpha_J(v), \tag{6}$$

where $\alpha_J(v)$ is the generalized Jensen measure obtained as the intercept from a regression of a mutual fund's return in excess of the zero beta rate corresponding to the benchmark portfolios on a constant and the returns on the benchmark assets in subset p in excess of the same zero beta rate. The interpretation of performance measure (6) is that an investor with stochastic discount factor $m_R(v)_{t+1}$ can extend the efficient set by buying the mutual fund under consideration if and only if $\lambda(v) > 0$. In contrast, if $\lambda(v) \leq 0$ holds for one v, then the restricted mean-variance frontiers intersect, while if $\lambda(v) \leq 0$ holds for all v on all P subsets this corresponds with mean-variance spanning.

In order to illustrate that incorporating short sales restrictions seriously affects the diversification benefits of including mutual funds into the investor's portfolio, we show in Figure 1 the estimated unrestricted optimal portfolio weights for the initial set of four benchmark assets and the Templeton World mutual fund for a range of expected returns, assuming that all parameters coincide with their sample analogue as reported in Table 1.

The global minimum variance portfolio corresponding to this set of initial assets is located at a monthly expected return of 0.87%. Note that the unrestricted efficient portfolio with an expected return of more than 1.31% (15.7% annually) contains a short position in the mutual fund. Moreover, a long position in the mutual fund coincides sometimes with short positions in a number of benchmark assets. Although this is not unrealistic for the currency factor, for the stock market indices a short position is not very realistic. In Figure 2 we show the corresponding optimal weights for the optimization problem under short sales restrictions for the Morgan Stanley USA, Europe and Japan indices, where we also imposed that an investor is no longer required to invest all his wealth in the available assets (see, e.g. Luttmer [1996]). This implies that an investor is allowed to take a long position in a riskless asset with zero return. The vertical line in the figure corresponds to the location of the portfolio where the investor does no longer take a position in the asset with zero return. It is straightforward to show that this portfolio corresponds to the tangency point of the line starting in the origin, i.e. expected return as well as variance equal to zero, to the restricted mean-variance frontier of the risky assets only. We will denote this portfolio as the zero-tangency portfolio.

It appears that only investors whose optimal portfolio expected return lies below 1.31% will take a long position in the mutual fund. Note that since we did not impose short sales restrictions on the Eurocurrency portfolio, investors can still construct portfolios with very high expected returns. This would not be possible if we imposed short sales restrictions on all the assets.

Figure 1: **Optimal Weights.** The figure shows the optimal weights for four benchmark assets and the Templeton World mutual fund if short selling is not excluded. The horizontal line represents the zero weight line, while the vertical line represents the location of the Global Minimum Variance portfolio.

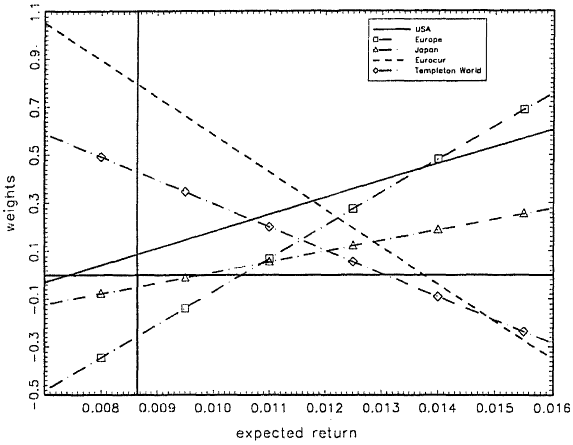
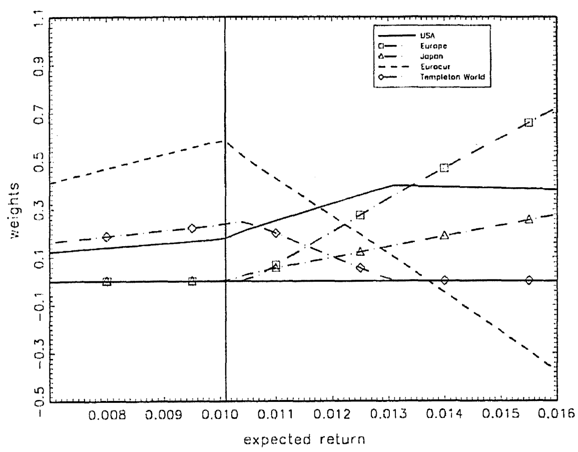


Figure 2: **Optimal Weights under short sales constraints.** The figure shows the optimal weights for the four benchmark assets and the Templeton World fund under short sales constraints for the Morgan Stanley USA, Europe and Japan indices as well as Templeton World fund. The vertical line represents the location of the zero tangency portfolio, i.e. at the right hand side of this line investors invest all their wealth in the risky assets.



In contrast to the mean-variance optimization problem without restrictions, the optimal weights are piecewise linear in the expected portfolio return. As soon as a binding restriction for one of the assets is reached, a transition between two segments of the restricted mean-variance frontier occurs. In the sequel we will denote such a point as a transition point. Note that between two transition points the optimal weights for the assets behave linear in the expected portfolio return.

In order to further illustrate the relevance of taking short sales restrictions into account, we also compute efficient frontiers for adding the other seventeen mutual funds to the benchmark assets, still assuming that all parameters coincide with the values reported in Table 1. In the case of two benchmark assets, a long position is taken for some values of the zero beta rate in thirteen out of eighteen mutual funds, while in the four benchmark case only eight mutual funds give a potential diversification benefit. However, for a restricted mean-variance efficient portfolio with an expected return of more than 1.70% the mutual funds will have zero weight in the four benchmark case. Recall that a zero optimal weight in a mutual fund can be caused by the short sales restriction on the benchmark assets as well as the short sales restriction on the mutual fund.

The short sales constraints assumed so far can be interpreted as an extreme transaction cost that investors have to pay for holding a short position. However, actual transaction costs may not completely prevent investors from taking short positions. Furthermore, there are also transaction costs associated with buying securities. When we incorporate transaction costs in analyzing performance of mutual funds, we have to take into account the investment horizon of the investor. By doing so, we can determine for what investment horizon a mutual fund gives a potential diversification benefit by including it in the investor's portfolio that outweighs transaction costs. Transaction costs can be handled by distinguishing between the return on a short and the return on a long position in the asset, as suggested in Luttmer [1996]. Under the assumption that returns are independently and identically distributed (i.i.d.), we construct a $2K$-dimensional vector $\tilde{R}_{t+1}$. The first K-elements contain the net returns on a long position in the benchmark assets, where a net return[4] is defined as $\tilde{R}^l_{i,t+1} = \tau^l_i R_{i,t+1}$, with $\tau^l_i = \frac{1}{1+a_i}$ where $a_i > 0$ are the transaction costs involved in taking a long position

[4]Note that we define net returns as gross returns after transaction costs, while for instance Bekeart and Urias [1996] denote net returns as returns after operating expenses.

adjusted for the length of the investment horizon, i.e. $a_i = (1+\tilde{a}_i)^{\frac{1}{H}} - 1$ with H is the investment horizon in months and $\tilde{a}_i$ is the cost per transaction. The second K-elements contain the net returns on a short position in the benchmark assets, defined as $\tilde{R}^s_{i,t+1} = \tau^s_i R_{i,t+1}$, with $\tau^s_i = \frac{1}{1-b_i}$ where $b_i > 0$ are the transaction costs involved in taking a short position adjusted for the length of the investment horizon[5]. In a similar way we construct a vector $\tilde{r}_{t+1}$ containing the long, $\tau^l r_{t+1}$, as well as short returns, $\tau^s r_{t+1,}$ on the mutual fund under consideration, i.e. after correcting for the load-fees that the fund charges.

Under the restriction that it is not possible to take a short position in the first K assets and a long position in the second K assets, we can, analogous to the case of short sales restrictions only, denote $\tilde{R}^{(v)}_{t+1}$ as a L-dimensional subvector of $\tilde{R}_{t+1}$ for which the constraints on the long and short position are not binding. Moreover, the stochastic discount factor linear in the subset of L assets that prices the subset of assets correctly is

$$\tilde{m}_R(v)_{t+1} = v + \tilde{\alpha}^{(v)\prime}(\tilde{R}^{(v)}_{t+1} - E[\tilde{R}^{(v)}_{t+1}]), \tag{7}$$

with

$$\tilde{\alpha}^{(v)} = Var[\tilde{R}^{(v)}_{t+1}]^{-1}(\iota_L - vE[\tilde{R}^{(v)}_{t+1}]).$$

It is now straightforward to show that, substituting the expressions of net long and short returns, condition (4) can be generalized to

$$\frac{1}{\tau^s_i} \leq E[\tilde{m}_R(v)_{t+1} r_{i,t+1}] \leq \frac{1}{\tau^l_i}, \;\; i = 1..N, \tag{8}$$

where the inequality signs reflect the short and long sales constraints on the additional assets. In order to check whether an investor can extend the efficient set by including a mutual fund into his portfolio, we can determine

$$\tilde{\lambda}^{(p)}(v) = E[\tilde{m}_R(v)_{t+1} r_{t+1}] - 1 = v\tilde{\alpha}_J(v), \tag{9}$$

where $\tilde{\alpha}_J(v)$ is the generalized Jensen measure obtained as the intercept from a regression of a mutual fund's return in excess of the zero beta rate corresponding to the benchmark portfolios on a constant and the net returns on the benchmark assets in subset p in excess of the same zero beta rate.

[5]The transaction costs $a_i > 0$ and $b_i > 0$ can be interpreted as the ask and bid spread as a percentage of the price $P_{i,t}$ when buying or selling assets.

Now, an investor with stochastic discount factor $\tilde{m}_R(v)_{t+1}$ can extend the efficient set by buying the mutual fund under consideration if and only if $\tilde{\lambda}^{(p)}(v) > a_i$ or sell the mutual fund, if possible, if and only if $\tilde{\lambda}^{(p)}(v) < -b_i$. As may be clear, it is straightforward to adjust (8) and (9) when we impose short sales restrictions on a number of the benchmark assets as well as on the mutual funds under consideration. The empirical implications of incorporating transaction costs will become clear in the next section where we test for diversification benefits by including mutual funds into the initial portfolio.

4 Testing for Spanning and Intersection in case of Market Frictions

The optimal weights reported in Figures 1 and 2 for the benchmark assets as well as the mutual fund are based on the point estimates of Table 1. Consequently, estimation errors in these parameters affect the possible diversification benefits by including the mutual funds into the portfolio. In order to test whether the diversification benefit that can be obtained from including a mutual fund[6] with return vector r_{t+1} to the initial set of benchmark assets with return vector R_{t+1} is significant, we want to test the hypothesis

$$E[m(v)_{t+1}r_{t+1}] = 1. \tag{10}$$

Recall that this corresponds to testing for a shift in the mean-variance efficient frontier. As is well known by now, this test can be based on the regression equation

$$r_{t+1} = a + BR_{t+1} + \varepsilon_{t+1}, \tag{11}$$

with $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon_{t+1}R_{t+1}] = 0$. Ignoring market frictions, spanning implies that $a = 0$ and $B\iota_K - 1 = 0$, and intersection of the extended mean-variance frontier and the mean-variance frontier of the original K assets implies that $av + (B\iota_K - 1) = 0$ for a given value v (Huberman and Kandel [1987], Bekaert and Urias [1996]). This test for mean-variance spanning can easily be extended for investors with other utility functions as shown by DeRoon, Nijman and Werker [1996]. Alternatively, GMM-tests can be used to test for intersection and spanning (see DeSantis [1994], Hansen, Heaton

[6] The tests can easily be extended to the inclusion of a set of N mutual funds.

and Luttmer [1995] and Chen and Knez [1996]). Interpreting the outcome of this test in light of (3), spanning implies that no investors can extend their efficient set by taking a position in the mutual fund, while intersection means that only a particular group, i.e. the group with a stochastic discount factor with expectation v, cannot extend the efficient set of assets.

As shown by DeRoon, Nijman and Werker [1997], a test for a shift in the mean-variance frontier subject to short sales constraints can be implemented in a regression framework as well. Recall that the restricted mean-variance frontier consists of P segments of unrestricted mean-variance frontiers. This means that intersection between the initial and extended mean-variance frontiers can occur at P different unrestricted frontiers. Since the assets in the mean-variance portfolios on such a segment of the frontier coincide with the subset of L assets for which the short sales constraints are not binding in the restricted problem, and a stochastic discount factor $m_R(v)_{t+1}$ is a linear function of the corresponding subvector $R_{t+1}^{(p)}$ only, implies that we can estimate the following P regressions

$$r_{t+1} = a^{(p)} + B^{(p)} R_{t+1}^{(p)} + \varepsilon_{t+1}^{(p)}, \tag{12}$$

and test whether

$$a^{(p)} v + (B^{(p)} \iota_L^{(p)} - 1) \leq 0 \tag{13}$$

holds for one value of v. If (13) holds for all v then the initial set of assets spans the extended set of assets. The inequality sign in (13) has to be replaced by an equality sign when there are no short sales restrictions on the additional assets.

Recall that a segment p of the restricted mean-variance frontier is bounded by two transition points. Since intersection at these two transition points implies spanning at segment p of the restricted mean-variance frontier, a test for mean-variance spanning is equivalent to testing whether (13) holds for two choices of v corresponding to these two transition points. We will denote the two values of v as: $v_{min}^{(p)}$ and $v_{max}^{(p)}$, the minimum and maximum expectation of the set of stochastic discount factors that price the subset of L assets correctly. The value of $v_{min}^{(p)}$ and $v_{max}^{(p)}$ can be determined as the inverse of the zero beta rates corresponding to the transition points bounding segment p of the restricted mean-variance frontier. Testing for spanning is therefore equivalent to testing whether the following two inequality restrictions

$$\begin{aligned} a^{(p)} v_{min}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) &\leq 0 \\ a^{(p)} v_{max}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) &\leq 0 \end{aligned} \tag{14}$$

hold jointly for $p = 1..P$. The joint one-sided inequality constraints can be tested with the Wald test under inequality constraints.

It is straightforward to show that a comparable regression framework as under short sales constraints only can be used for testing for a shift in the mean-variance frontier with transaction costs incorporated. Recall that $\tilde{R}^{(p)}_{t+1}$ denotes a L-dimensional subvector of the net return vector $\tilde{R}_{t+1}$ for which the constraints on the long and short position are not binding, then a test for mean-variance spanning when also transaction costs are incorporated, can be based upon whether in the P regressions

$$r_{t+1} = a^{(p)} + B^{(p)} \tilde{R}^{(p)}_{t+1} + \varepsilon^{(p)}_{t+1}, \tag{15}$$

the following restrictions hold jointly:

$$\begin{aligned} -b_i &\leq a^{(p)} v^{(p)}_{min} + (B^{(p)} \iota^{(p)}_L - 1) \leq a_i \\ -b_i &\leq a^{(p)} v^{(p)}_{max} + (B^{(p)} \iota^{(p)}_L - 1) \leq a_i \end{aligned} \tag{16}$$

for $p = 1...P$, where a_i, b_i are the transaction costs for a long respectively short position in the fund under consideration, adjusted for the length of the investment horizon.

The joint two-sided constraints involved in mean-variance spanning with transaction costs can be tested using a Wald test. In case of a frictionless market, the Wald test statistic simply has a χ^2_{2N} distribution. However, in case of market frictions such as transaction costs and short selling constraints, we have to deal with inequality constraints. Analogous to the derivation of one-sided inequality constraints, as shown by, for instance, Kodde and Palm [1986], the Wald test statistic under two-sided inequality constraints:

$$\xi(v) = min_{-b_i \leq \alpha_J \leq a_i} (\hat{\alpha}_J(v) - \alpha_J(v))' Var(\hat{\alpha}_J(v))^{-1} (\hat{\alpha}_J(v) - \alpha_J(v)) \tag{17}$$

is asymptotically distributed as a mixture of χ^2 distributions, where the $2PN$ dimensional vector $\hat{\alpha}_J(v)$ corresponds to the left hand side of (14) and the $2PN \times 2PN$ covariance matrix $Var(\hat{\alpha}_J(v))$ can be obtained from the restricted covariance matrix of the OLS-estimates of (12).

In Tables 3 and 4 we present the outcomes of the test for mean-variance spanning in a frictionless market for the two respectively four benchmark case. Note that we no longer impose that an investor should invest all his wealth, implying that we assume that a risk free asset with zero return is available[7]. This assumption implies that the upper bound for v is 1. A lower bound for v can be obtained as the inverse of the intercept of the asymptote of the mean-variance frontier. It is straightforward to show that this intercept is equal to the expected return on the global minimum variance portfolio of the benchmark assets, which appear to be 0.940% and 0.886% for the two respectively four benchmark case.

In the two benchmark case it appears that the hypothesis of mean-variance spanning is rejected for eight mutual funds. This means that these funds can significantly extend a widely diversified international portfolio. When we extend the initial set of two assets to four benchmark assets, it appears that thirteen mutual funds give a significant extension of the investment set. Note that a potential diversification benefit can also mean that an investor has to take a short position in the mutual fund, i.e. a rejection of the spanning hypothesis can be caused by out as well as underperformance of the fund. To illustrate whether investors have to take short or long positions in the mutual funds for optimal diversification benefits, we computed for two values of v, one corresponding with a portfolio located near the zero tangency portfolio and one corresponding with a portfolio with an extreme high expected return, the performance measure as given in (3). The (+)'s in the columns labelled 'z-t' in Tables 3 and 4 indicate that for investors whose portfolio is located near the zero tangency portfolio the fund shows outperformance and investors can extend the efficient set by taking a long position in the fund under consideration. Moreover, the (+)'s in the columns labelled 'asy' in Table 3 mean that also for very high expected returns, the funds still show outperformance and investors take a long position in the fund for optimal diversification benefits, while in the four benchmark case (Table 4) for very high expected returns only underperformance of the funds remains and investors take short positions in the mutual funds for diversification benefits.

The second column in the Tables 3 and 4 contain the intervals for the range of expected return values of the initial portfolio for which the hypothesis of intersection will be rejected in case there are no market frictions. For instance, in the four benchmark case, investors that initially hold a portfolio with an expected monthly return between 0.786% and 0.948% (i.e. between 9.4% and 11.4% annually) have a diversification benefit by including the Templeton World mutual fund. The rejection of the full spanning hypothesis is of course caused by the fact that the two frontiers differ substantially for

[7]It is not allowed to take a short position in a risk free asset with zero return.

Table 3: **Spanning and Intersection Tests in the case of two benchmark assets.** The table reports the interval of expected returns for which the hypothesis of intersection can be rejected as well as the p-values associated with the Wald tests for mean-variance spanning in a frictionless market. The initial set of benchmark assets consists of the Morgan Stanley World index and an equally weighted portfolio of Eurocurrency Deposits. Note that we do not impose that an investor should invest all his wealth. The (+) in the column labelled as 'z-t' indicates that an investor located near the zero tangency portfolio takes a long position in the fund for optimal diversification benefits, while a (+) in the column labelled 'asy' means that also for very high expected returns a long position in the fund is taken.

Two Benchmark Assets				
Mutual Fund	Rejection Interval Intersection in Expected Returns (%)	Spanning Test (frictionless)		
		p-value	z-t	asy
Alliance Global Small.	$\langle 0.526, 0.969\rangle$	0.000	(+)	(-)
Alliance Intl.	–	0.744	(-)	(-)
Bailard,Biehl Intl.	$\langle 0.933, 0.991\rangle$	0.039	(-)	(-)
First Invest Global	–	0.305	(+)	(-)
Kemper Intl.	–	0.765	(-)	(-)
Lexington World Wide	$\langle 0.692, 0.983\rangle$	0.000	(+)	(-)
New Perspective	$\langle 0.891, 1.025\rangle$	0.000	(+)	(-)
Oppenheimer Global	–	0.810	(+)	(-)
Phoenix World Opport.	$\langle 0.719, 0.963\rangle$	0.000	(-)	(-)
Pilot Kleinwort Intl.	–	0.566	(-)	(-)
Putnam Global Growth	$\langle 0.938, 0.956\rangle$	0.121	(+)	(+)
Scudder Intl.	–	0.958	(+)	(+)
T. Rowe Price Intl.	–	0.283	(+)	(+)
Templeton Growth	$\langle 0.843, 1.040\rangle$	0.000	(+)	(-)
Templeton Small Cmp.	$\langle 0.837, 1.040\rangle$	0.000	(+)	(-)
Templeton World	$\langle 0.837, 1.037\rangle$	0.000	(+)	(-)
United Intl. Growth	$\langle 0.930, 0.959\rangle$	0.078	(+)	(+)
Vanguard Intl. Growth	$[0, 0.916\rangle$ and $\langle 0.939, +\infty\rangle$	0.116	(+)	(+)

Table 4: **Spanning and Intersection Tests in the case of four benchmark assets.** The table reports the interval of expected returns for which the hypothesis of intersection can be rejected as well as the p-values associated with the Wald test for mean-variance spanning in a frictionless market. The initial set of benchmark assets consists of the Morgan Stanley USA, Europe and Japan indices and an equally weighted portfolio of Eurocurrency Deposits. Note that we do not impose that an investor should invest all his wealth. The (+) in the column labelled 'z-t' indicates that an investor located near the zero tangency portfolio takes a long position in the fund for optimal diversification benefits, while a (+) in the column labelled 'asy' means that also for very high expected returns a long position in the fund is taken.

Four Benchmark Assets				
Mutual Fund	Rejection Interval Intersection in Expected Returns (%)	Spanning Test (frictionless) p-value	z-t	asy
Alliance Global Small.	$[0,0.811\rangle$ and $\langle 0.932,+\infty\rangle$	0.067	(–)	(–)
Alliance Intl.	$[0,0.829\rangle$ and $\langle 0.879,+\infty\rangle$	0.117	(–)	(–)
Bailard,Biehl Intl.	$\langle 0.895, 1.168\rangle$	0.003	(–)	(–)
First Invest Global	$\langle 0.795, 0.897\rangle$	0.039	(–)	(–)
Kemper Intl.	$\langle 0.552, 0.923\rangle$	0.000	(+)	(–)
Lexington World Wide	$[0,0.447\rangle$ and $\langle 0.886,+\infty\rangle$	0.035	(–)	(–)
New Perspective	$\langle 0.843, 0.924\rangle$	0.024	(+)	(–)
Oppenheimer Global	–	0.819	(–)	(–)
Phoenix World Opport.	$[0,0.961\rangle$ and $\langle 1.531,+\infty\rangle$	0.146	(–)	(–)
Pilot Kleinwort Intl.	$\langle 0.902, 1.632\rangle$	0.002	(–)	(–)
Putnam Global Growth	$\langle 0.857, 0.982\rangle$	0.117	(+)	(–)
Scudder Intl.	$\langle 0.761, 0.911\rangle$	0.007	(+)	(–)
T. Rowe Price Intl.	$\langle 0.827, 0.911\rangle$	0.029	(+)	(–)
Templeton Growth	$\langle 0.786.0.957\rangle$	0.000	(+)	(–)
Templeton Small Cmp.	$\langle 0.779, 0.966\rangle$	0.000	(+)	(–)
Templeton World	$\langle 0.786, 0.948\rangle$	0.000	(+)	(–)
United Intl. Growth	$\langle 0.800, 0.932\rangle$	0.002	(+)	(–)
Vanguard Intl. Growth	$\langle 0.843, 0.918\rangle$	0.032	(+)	(–)

Table 5: **Spanning Tests imposing transaction costs.** The table shows the transaction costs a fund may charge such that the hypothesis of mean-variance spanning is just rejected at the 5% level for various investment horizons. The initial benchmark assets are the Morgan Stanley World index and an equally weighted portfolio of Eurocurrency Deposits.

Mutual	Holding Period (in months)						
Fund	1	6	12	18	24	30	36
Alliance Global Small.	0.40	0.95	2.00	3.10	4.15	5.30	>6.00
Alliance Intl.	0.40	-	-	-	-	-	-
Bailard,Biehl Intl.	0.15	-	-	-	-	-	-
First Invest Global	0.35	-	-	-	-	-	-
Kemper Intl.	0.30	-	-	-	-	-	-
Lexington World Wide	0.55	1.15	2.45	3.75	5.00	>6.00	>6.00
New Perspective	0.80	0.90	1.30	1.90	2.45	3.10	3.75
Oppenheimer Global	0.45	-	-	-	-	-	-
Phoenix World Opport.	0.20	0.65	1.45	2.25	3.05	3.90	4.70
Pilot Kleinwort Intl.	0.30	-	-	-	-	-	-
Putnam Global Growth	0.70	-	-	-	-	-	-
Scudder Intl.	0.45	-	-	-	-	-	-
T. Rowe Price Intl.	0.60	-	-	-	-	-	-
Templeton Growth	0.90	1.55	2.65	3.90	5.20	>6.00	>6.00
Templeton Small Cmp.	1.00	1.70	3.15	4.70	>6.00	>6.00	>6.00
Templeton World	0.95	1.45	2.60	3.90	5.20	>6.00	>6.00
United Intl. Growth	0.60	-	-	-	-	-	-
Vanguard Intl. Growth	0.60	-	-	-	-	-	-

this region of expected returns. If the corresponding entry in the column is empty, intersection cannot be rejected for any choice of the expected return.

In order to analyze the impact of frictions we first of all present the mean-variance spanning tests imposing transaction costs. In order to test the hypothesis whether it is efficient to invest directly in international assets or to buy an internationally diversified mutual fund, we also assume that there are transaction costs involved in taking a position in the benchmark assets. Following Luttmer [1996] we impose transaction costs for the benchmark assets that equal 0.5% for buying as well as selling. In Table 5 we present for the case of two benchmark assets, the transaction costs a mutual fund may charge such that the hypothesis of mean-variance spanning is just rejected at the 5% level for different investment horizons. It appears that for an investment horizon of only one month all the mutual funds in the sample give a diversification benefit in the case of two benchmark assets when we incorporate transaction costs. This can probably be explained by the fact that we fixed the transaction costs for the benchmark assets at 0.5% for buying as well as selling, and what makes these assets relatively expensive compared to the mutual funds. For holding periods of more than six months only seven mutual funds give a significant improvement in the risk-return trade-off.. An empty entry in a column indicates that the fund does not provide any diversification benefits, even in case of zero transaction costs. Comparing the outcomes of Table 5 to the actual load-fees that the funds charge, it appears that an investment horizon (holding period) of about two years is required to have a significant improvement in the risk-return trade-off.. An exception is Lexington World Wide, which can be marked as a no-load fund, that gives diversification benefits for all the holding periods by including it in the investor's portfolio.

In Table 6 we present the outcomes of the test for mean-variance spanning imposing short sales restrictions on the Morgan Stanley World, USA, Europe and Japan indices as well as the mutual fund under consideration. We do not impose a short sales restriction on the Eurocurrency Deposits portfolio.

It appears that in the case of two benchmark assets, the hypothesis of mean-variance spanning under short sales restrictions is rejected for four mutual funds, indicating that these funds still show outperformance. Consequently, investors that own a portfolio with predetermined country weight allocation can improve their portfolio's risk-return trade-off by including one of these mutual funds. However, in the case of four benchmark assets the hypothesis of mean-variance spanning is not rejected anymore. Apparently, the combination of short sales restrictions on the benchmark assets as well as on the mutual funds makes the diversification benefit that appeared to be present in the frictionless market disappear completely. Moreover, the outperformance present in the two benchmark case is for portfolios with efficient country weight allocation not present anymore. So, although Figure 2 suggested that a mean-variance optimizing investor takes a long position in the Templeton World fund, it appears that there is not a significant diversification benefit by taking a long position in the mutual fund after imposing short sales constraints.

Table 6: **Spanning Tests under short sales constraints.** The table reports the Wald test statistic under inequality constraints and the corresponding p-value for mean-variance spanning. Note that we do not impose short sales restrictions on the currency hedge portfolio in both sets of benchmark assets.

Mutual Fund	Two Benchmark Assets p-value	Four Benchmark Assets p-value
Alliance Global Small.	0.522	0.747
Alliance Intl.	0.912	0.817
Bailard,Biehl Intl.	0.713	0.802
First Invest Global	0.552	0.845
Kemper Intl.	1.000	0.734
Lexington World Wide	0.228	0.792
New Perspective	0.017	0.487
Oppenheimer Global	0.435	0.923
Phoenix World Opport.	0.567	0.744
Pilot Kleinwort Intl.	0.773	0.725
Putnam Global Growth	0.164	0.743
Scudder Intl.	0.505	0.806
T. Rowe Price Intl.	0.299	0.671
Templeton Growth	0.002	0.291
Templeton Small Cmp.	0.006	0.189
Templeton World	0.002	0.368
United Intl. Growth	0.187	0.500
Vanguard Intl. Growth	0.228	0.495

5 Conditional Performance Evaluation

Recent studies show that returns on stocks and bonds are predictable over time (see e.g., Ferson and Harvey [1993], Keim and Stambaugh [1986]). Time-varying expected returns and (co)variances imply time-varying mean-variance frontiers. Consequently, mean-variance optimizing investors will dynamically adjust their portfolios because of the changing economic conditions. Therefore it can be the case that under certain economic conditions there are diversification benefits for a mean-variance optimizing investor by including additional assets into his portfolio while under different circumstances these benefits are absent. The implication for performance evaluation of mutual funds is, as advocated in recent papers of Ferson and Schadt [1996] and Chen and Knez [1996], that the evaluation should be conditional upon the state of the economy. The aim of conditional performance evaluation is to distinguish mutual funds with real timing or selection ability of the fund manager from managed portfolio strategies that can be replicated using publicly available information.

In previous sections we assumed that the expected returns on the assets and the corresponding (co)variances are constant over time. We will now relax this assumption. Let us first of all consider the case of a frictionless market in conditional performance evaluation of mutual funds. Using a set of information variables z_t, supposed to reflect the state of the economy, a test for conditional mean-variance spanning can be based on the following regression:

$$r_{t+1} = a + \gamma' z_t + BR_{t+1} + \varepsilon_{t+1}. \tag{18}$$

One can easily test for mean-variance spanning for arbitrary values of the information variables z_t as well as for mean-variance spanning for specific values of z_t (see appendix A for further details). The first case coincides with $a = \gamma = 0$ and $B\iota_K - 1 = 0$, while the second case holds for $a = -\gamma' z_t$ and $B\iota_K - 1 = 0$. Alternatively, one can incorporate conditional information by adding so-called scaled returns to the regression equation (18) (see, e.g. Cochrane [1997], Bekeart and Urias [1996]). However, the disadvantage of this method of conditional performance evaluation is the dimensionality problem that arises in estimating and testing when the set of information variables or the set of initial benchmark assets is large.

Following previous studies on conditional performance evaluation (see, Ferson and Schadt [1996] and Chen and Knez [1996]) we use the following set of information variables: 1) the lagged level of the one-month Tbill yield, 2) the lagged dividend yield on the Morgan Stanley World index (in the two benchmark case), 3) the lagged term spread measured as the difference between a constant maturity 10-year bond yield and a constant maturity 1-year bond yield and 4) a dummy for the month of January.

Since rejection of the hypothesis of mean-variance spanning for arbitrary values of the information variables z_t does not imply that mean-variance spanning is absent under specific economic circumstances, we consider only tests for the hypothesis of mean-variance spanning under a number of sets of specific values for the information variables lagged Tbill yield, lagged dividend yield and lagged term spread. Moreover, we examine whether it affects the diversification benefits when these specific economic circumstances occur in the month January or in the other months of the year. In Table (7) we report outcomes for tests of the hypothesis of mean-variance spanning in a frictionless market, conditional upon three different sets of information variables, when the initial set of assets consist of the Morgan Stanley World and the currency hedge portfolio, i.e. the two benchmark case.

Table (7) indicates that under specific economic circumstances some mutual funds give diversification benefits while these are absent in other circumstances. Moreover, some mutual funds only provide investors with an improved risk-return trade-off in January while in the rest of the year the fund does not give any diversification benefits. For instance, First Invest Global gives diversification benefits in January conditional upon information set 1 and 2, i.e. the term spread equal to or lower than the average value and the Tbill yield and Dividend yield equal to the average value over the period 1982-1994. However, when the Tbill yield is below the average over the sample period (set 3) then the fund does not give an improvement in the risk-return trade-off in any of the months. If we compare the outcomes of the conditional mean-variance spanning test with the unconditional mean-variance spanning test (Table 3) then it appears that, not surprisingly, outperformance is found roughly for the same funds but that for some funds (First Invest Global, Oppenheimer Global) distinction can be made when the fund under- or outperforms.

Similar to the unconditional mean-variance spanning tests, conditional mean-variance spanning tests with market frictions incorporated can be based upon a regression framework (see appendix A for further details). In case of transaction costs, a test for conditional mean-variance spanning can now be

Table 7: **Conditional Spanning Tests in a frictionless market.** The table reports outcomes of a test for conditional mean-variance spanning ignoring market frictions, in the case of two benchmark assets, for specific values of the information variables. Panel A of the table reports a number of sets of specific values for the information variables and their average value (between parentheses) over the sample period 1982-1994. Panel B reports the p-values associated with the Wald test for mean-variance spanning in a frictionless market conditonal upon the values for the information variables.

Panel A: specific values information variables (annualized)						
Variable (average)	set 1		set 2		set 3	
Term spread (1.6%)	0.4%	0.4%	1.6%	1.6%	1.6%	1.6%
Month	January	Other	January	Other	January	Other
Tbill yield (6.0%)	6.0%	6.0%	6.0%	6.0%	2.4%	2.4%
Div yield (3.1%)	3.1%	3.1%	3.1%	3.1%	3.1%	3.1%
Panel B: p-values mean-variance spanning tests in frictionless market						
Mutual Fund						
Alliance Global Small.	0.000	0.000	0.000	0.000	0.000	0.000
Alliance Intl.	0.431	0.813	0.367	0.830	0.429	0.767
Bailard,Biehl Intl.	0.009	0.030	0.014	0.038	0.007	0.021
First Invest Global	0.010	0.208	0.034	0.277	0.070	0.370
Kemper Intl.	0.274	0.648	0.131	0.651	0.376	0.652
Lexington World Wide	0.000	0.000	0.000	0.000	0.000	0.000
New Perspective	0.000	0.000	0.000	0.000	0.000	0.000
Oppenheimer Global	0.132	0.015	0.670	0.105	0.105	0.031
Phoenix World Opport.	0.000	0.000	0.000	0.000	0.000	0.000
Pilot Kleinwort Intl.	0.821	0.787	0.814	0.590	0.815	0.794
Putnam Global Growth	0.107	0.119	0.101	0.098	0.115	0.118
Scudder Intl.	0.957	0.942	0.867	0.960	0.954	0.981
T. Rowe Price Intl.	0.383	0.246	0.186	0.224	0.426	0.317
Templeton Growth	0.000	0.000	0.000	0.000	0.000	0.000
Templeton Small Cmp.	0.000	0.000	0.000	0.000	0.000	0.000
Templeton World	0.000	0.000	0.000	0.000	0.000	0.000
United Intl. Growth	0.051	0.015	0.063	0.022	0.023	0.004
Vanguard Intl. Growth	0.160	0.186	0.119	0.112	0.186	0.194

based on testing whether in the P regressions

$$r_{t+1} = a^{(p)} + \gamma^{(p)\prime} z_t + B^{(p)} \tilde{R}_{t+1}^{(p)} + \varepsilon_{t+1}^{(p)} \tag{19}$$

the following restrictions hold jointly:

$$\begin{aligned} -b_i &\leq a^{(p)} v_{min}^{(p)} + \gamma^{(p)\prime} \tilde{z} v_{min}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) \leq a_i \\ -b_i &\leq a^{(p)} v_{max}^{(p)} + \gamma^{(p)\prime} \tilde{z} v_{max}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) \leq a_i \end{aligned} \tag{20}$$

for $p = 1...P$, where $\tilde{z}$ denotes a specific choice for the information variables, $\tilde{R}_{t+1}^{(p)}$ is, as before, the L-dimensional net return vector of the initial assets for which the constraints on the long and short position are not binding, $v_{min}^{(p)}$ and $v_{max}^{(p)}$ are the inverses of the zero beta rates corresponding to transition points bounding segment p of the conditional mean-variance frontier and a_i, b_i are the transaction costs involved in taking a long respectively short position. The joint constraints in (20) can be tested with the Wald test under two-sided inequality constraints given in (17).

Comparable with the unconditional case, we impose transaction costs for the benchmark assets that equal 0.5% for buying as well as selling. Taking a position in the currency hedge portfolio is assumed to be free of charge. In contrast to the unconditional case, we now fix the total transaction costs involved in taking a long or short position in the mutual fund at 0.5%, and we test whether under specific economic circumstances, $\tilde{z}$, a mutual fund provides diversification benefits in the case of two benchmark assets. Note that we consider only an investment horizon (holding period) of one month. Table (8) presents the outcomes for the conditional mean-variance spanning tests with transaction costs incorporated.

It appears that in case of transaction costs the potential diversification benefits are rather sensitive for the specific values of the information variables $\tilde{z}$. For instance, when the term spread, Tbill yield and dividend yield are almost equal to their average value over the sample period (set 2), only two mutual funds give an improvement in the risk-return trade-off in January. However, in the other months of the year, ten mutual funds show a significant shift in the conditional mean-variance frontier. A similar pattern is observed for other values of the information variables $\tilde{z}$, suggesting that a dynamic trading strategy of taking a position in the mutual funds under consideration in eleven months of the year, and not having a position in the mutual funds in January, gives optimal diversification benefits.

Table 8: **Conditional Spanning Tests imposing transaction costs.** The table reports outcomes of a test for conditional mean-variance spanning where transaction costs of 0.5 are taken into account, in the case of two benchmark assets, for specific values of the information variables. Panel A of the table reports a number of sets of specific values for the information variables and their average value (between parentheses) over the sample period 1982-1994. Panel B reports the p-values associated with the Wald test under two-sided inequality constraints for mean-variance spanning conditional upon the corresponding set of the information variables.

Panel A: specific values information variables (annualized)						
Variable (average)	set 1		set 2		set 3	
Term spread (1.6%)	0.4%	0.4%	1.6%	1.6%	1.6%	1.6%
Month	January	Other	January	Other	January	Other
Tbill yield (6.0%)	6.0%	6.0%	6.0%	6.0%	2.4%	2.4%
Div yield (3.1%)	3.1%	3.1%	3.1%	3.1%	3.1%	3.1%
Panel B: p-values mean-variance spanning tests with transaction costs						
Mutual Fund						
Alliance Global Small.	0.257	0.154	0.367	0.055	0.193	0.155
Alliance Intl.	1.000	0.724	1.000	0.142	0.990	0.943
Bailard,Biehl Intl.	0.632	0.997	0.943	0.748	0.394	0.857
First Invest Global	0.002	0.106	0.011	0.373	0.049	0.527
Kemper Intl.	1.000	0.461	1.000	0.088	1.000	0.587
Lexington World Wide	0.284	0.018	0.612	0.002	0.105	0.014
New Perspective	0.153	0.006	0.426	0.000	0.219	0.041
Oppenheimer Global	0.047	0.000	0.425	0.006	0.043	0.003
Phoenix World Opport.	0.417	0.423	0.377	0.114	0.147	0.119
Pilot Kleinwort Intl.	0.887	0.790	0.841	0.372	0.941	0.906
Putnam Global Growth	0.969	0.278	0.948	0.002	0.923	0.396
Scudder Intl.	0.908	0.416	0.968	0.069	0.953	0.624
T. Rowe Price Intl.	1.000	0.089	1.000	0.002	0.999	0.218
Templeton Growth	0.276	0.037	0.313	0.000	0.214	0.054
Templeton Small Cmp.	0.009	0.002	0.034	0.000	0.003	0.002
Templeton World	0.315	0.042	0.335	0.000	0.198	0.046
United Intl. Growth	0.374	0.012	0.715	0.001	0.110	0.005
Vanguard Intl. Growth	1.000	0.301	1.000	0.008	0.999	0.423

The final step is to incorporate short sales constraints on certain assets. Recall that short sales constraints can be interpreted as extreme transaction costs that investors have to pay for taking a short position in the assets under consideration. Therefore we can, similar to the unconditional mean-variance spanning tests with short sales restrictions, estimate the following P regressions

$$r_{t+1} = a^{(p)} + \gamma^{(p)\prime} z_t + B^{(p)} R_{t+1}^{(p)} + \varepsilon_{t+1}^{(p)} \tag{21}$$

and test whether the following two inequality restrictions

$$\begin{aligned} a^{(p)} v_{min}^{(p)} + \gamma^{(p)\prime} \bar{z} v_{min}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) &\leq 0 \\ a^{(p)} v_{max}^{(p)} + \gamma^{(p)\prime} \bar{z} v_{max}^{(p)} + (B^{(p)} \iota_L^{(p)} - 1) &\leq 0 \end{aligned} \tag{22}$$

hold jointly for $p = 1...P$. The interpretation of $v_{min}^{(p)}$ and $v_{max}^{(p)}$ is similar to the one in (20). Note that the inequality sign in (22) has to be replaced by an equality sign when there are no short sales constraints on the additional assets. The joint constraints (22) can be tested using the Wald test under one-sided inequality constraints (see, for instance, DeRoon, Nijman and Werker [1997]). Table (9) presents the outcomes for the conditional mean-variance spanning tests when we impose short sales constraints on the Morgan Stanley World index and the fund under consideration.

It appears that, similar to the case of conditional mean-variance spanning with transaction costs incorporated, mutual funds provide less diversification benefits in January compared to the rest of the year. Recall that the diversification benefits present in the conditional mean-variance spanning with transaction costs incorporated can be due to out as well as underperformance of the fund. Moreover, to have an improvement in the risk-return trade-off of his initial portfolio it can also mean that the investor has to take a short position in some of the benchmark assets. The combination of short sales restrictions on the mutual funds as well as on some of the benchmark assets therefore leads to less diversification benefits of the mutual funds compared to the case of transaction costs only.

6 Concluding Remarks

In this paper we examined whether internationally investing U.S. based mutual funds can extend an investor's efficient investment set. It appears that

Table 9: **Conditional Spanning Tests with short sales restrictions.** The table reports outcomes of a test for restricted conditional mean-variance spanning, in the case of two benchmark assets, for specific values of the information variables. Note that we do not impose short sales restrictions on the currency hedge portfolio. Panel A of the table reports a number of sets of specific values for the information variables and their average value (between parentheses) over the sample period 1982-1994. Panel B reports the p-values associated with the Wald test under one-sided inequality constraints on mean-variance spanning conditional upon the corresponding set of the information variables.

Panel A: specific values information variables (annualized)						
Variable (average)	set 1		set 2		set 3	
Term spread (1.6%)	0.4%	0.4%	1.6%	1.6%	1.6%	1.6%
Month	January	Other	January	Other	January	Other
Tbill yield (6.0%)	6.0%	6.0%	6.0%	6.0%	2.4%	2.4%
Div yield (3.1%)	3.1%	3.1%	3.1%	3.1%	3.1%	3.1%
Panel B: p-values mean-variance spanning tests with short sales constraints						
Mutual Fund						
Alliance Global Small.	0.165	0.146	0.259	0.312	0.108	0.119
Alliance Intl.	0.580	0.995	0.637	0.903	0.623	0.615
Bailard,Biehl Intl.	0.516	0.572	0.542	0.576	0.510	0.515
First Invest Global	0.002	0.124	0.016	0.773	0.033	0.357
Kemper Intl.	0.618	0.386	0.553	0.358	0.653	0.380
Lexington World Wide	0.153	0.024	0.403	0.041	0.052	0.007
New Perspective	0.120	0.010	0.320	0.006	0.142	0.037
Oppenheimer Global	0.039	0.002	0.317	0.095	0.023	0.004
Phoenix World Opport.	0.245	0.320	0.255	0.320	0.065	0.077
Pilot Kleinwort Intl.	0.880	0.646	0.873	0.769	0.882	0.953
Putnam Global Growth	0.828	0.344	0.744	0.135	0.851	0.312
Scudder Intl.	0.899	0.378	0.805	0.444	0.893	0.439
T. Rowe Price Intl.	0.812	0.132	0.596	0.120	0.773	0.200
Templeton Growth	0.164	0.040	0.224	0.001	0.122	0.040
Templeton Small Cmp.	0.008	0.003	0.024	0.001	0.004	0.001
Templeton World	0.188	0.058	0.230	0.003	0.114	0.038
United Intl. Growth	0.238	0.024	0.970	0.021	0.062	0.006
Vanguard Intl. Growth	0.696	0.272	0.596	0.155	0.762	0.318

the answer to this question depends first of all on, the assumed set of benchmark assets supposed to reflect the current portfolio choice, and secondly, on the assumption of a frictionless market. Using simple linear regressions we tested for mean-variance spanning in a frictionless as well as a market with transaction costs and short sales restrictions incorporated. Furthermore, it has been shown that these tests for mean-variance spanning are closely related to the issue of performance evaluation of mutual funds.

A risk-averse mean-variance optimizing investor that initially holds a widely diversified international portfolio with predetermined country weight allocation and considers to extend his portfolio with an internationally diversified mutual fund can improve his portfolio risk-return trade-off by taking long or short positions in the mutual funds. Although transaction costs and short sales constraints reduce the set of mutual funds that give potential diversification benefits, an extension of the portfolio with an internationally investing mutual fund is still worthwhile.. Alternatively, this can be interpreted that even after imposing transaction costs and short sales constraints some mutual funds still show outperformance for investors with an investment strategy with predetermined country weight allocation. However, most mean-variance optimizing investors that already own an internationally diversified portfolio with efficient country weight allocation can only have an improvement in the risk-return trade-off by taking short positions in the mutual funds. Consequently, incorporating short sales restrictions seriously affects these potential diversification benefits for this group of the investing public. Moreover, it means that internationally diversified mutual funds do not show outperformance for investors that already own a diversified portfolio of international stocks with efficient country weight allocation.

A Testing for Conditional Mean-Variance Spanning

In this appendix we show how to test for mean-variance spanning in a frictionless as well as a market with short sales constraints and transaction costs when we incorporate conditional information. Recall that in a frictionless market there exists a stochastic discount factor M_{t+1} such that

$$E[M_{t+1}R_{t+1}|\, I_t] = \iota_K, \tag{23}$$

where ι_K is a K-vector of ones and I_t is the public information set available at time t. Denote z_t as a set of information variables, including a constant, supposed to reflect the state of the economy, and assume that

$$\begin{aligned} E[R_{t+1}|z_t] &= \gamma_R' z_t, \\ E[r_{t+1}|z_t] &= \gamma_r' z_t. \end{aligned}$$

The mean-variance stochastic discount factor $m(v)_{t+1}$ given by

$$m(v)_{t+1} = v + \alpha(v)'(R_{t+1} - E[R_{t+1}|z_t]), \tag{24}$$

with

$$\alpha(v) = Var[R_{t+1}|z_t]^{-1}(\iota_K - vE[R_{t+1}|z_t])$$

has the lowest variance of all stochastic discount factors with expectation v that price R_{t+1}correctly. Denote $Var[R_{t+1}|z_t]$ as Σ_{RR} and $Cov[r_{t+1}, R_{t+1}|z_t]$ as Σ_{rR}. Now, a mean-variance optimizing investor will not have a diversification benefit if the stochastic discount factor given in (24) also prices the mutual fund's return r_{t+1} correctly. This implies that

$$\begin{aligned} E_t[m(v)_{t+1}r_{t+1}] &= 1 \Leftrightarrow \\ v\gamma_r' z_t + \Sigma_{rR}\Sigma_{RR}^{-1}(\iota_K - v\gamma_R' z_t) &= 1 \Leftrightarrow \\ (\gamma_r' - \Sigma_{rR}\Sigma_{RR}^{-1}\gamma_R')vz_t + (\Sigma_{rR}\Sigma_{RR}^{-1}\iota_K - 1) &= 0. \end{aligned} \tag{25}$$

If this equality holds for one value of v then there is intersection, while if (25) holds for all v then there is spanning. It is now straightforward to show that $(\gamma_r' - \Sigma_{rR}\Sigma_{RR}^{-1}\gamma_R')$ and $\Sigma_{rR}\Sigma_{RR}^{-1}$ can consistently be estimated by the OLS estimates for γ and B in the following regression equation

$$r_{t+1} = \gamma' z_t + BR_{t+1} + \varepsilon_{t+1}, \tag{26}$$

with $E[\varepsilon_{t+1}z_t] = E[\varepsilon_{t+1}R_{t+1}] = 0$. Consequently, the hypothesis that there is intersection for a given value of v and z_t can be tested by testing

$$\gamma'\tilde{z}v + (B\iota_K - 1) = 0, \tag{27}$$

and the hypothesis of spanning for arbitrary values of z_t can be tested by testing

$$\gamma = 0 \text{ and } (B\iota_K - 1) = 0, \tag{28}$$

while spanning for specific values of z_t occurs for

$$\gamma'\tilde{z} = 0 \text{ and } (B\iota_K - 1) = 0, \tag{29}$$

where $\tilde{z}$ denotes a specific choice for the information variables.

When we incorporate short sales constraints (23) generalizes to

$$E_t[m_R(v)_{t+1}r_{t+1}] \leq 1, \tag{30}$$

where $m_R(v)_{t+1}$ is the stochastic discount factor that prices the subset of L assets on segment p of the restricted mean-variance frontier correctly. This implies that (25) generalizes to

$$(\gamma_r' - \Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}\gamma_R')vz_t + (\Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}\iota_K - 1) \leq 0. \tag{31}$$

As before, it is straightforward to show that $(\gamma_r' - \Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}\gamma_R')$ and $\Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}$ can consistently be estimated by the OLS estimates for $\gamma^{(p)}$ and $B^{(p)}$ in the regression equation

$$r_{t+1} = \gamma^{(p)\prime}z_t + B^{(p)}R_{t+1}^{(p)} + \varepsilon_{t+1}. \tag{32}$$

Recall that the restricted mean-variance frontier consists of P segments. Denote $v_{min}^{(p)}$ and $v_{max}^{(p)}$ as the minimum and maximum expectation of the set of stochastic discount factors that price the subset of L assets correctly, then testing for spanning for specific values of z_t is therefore equivalent to testing whether the following system of inequality restrictions

$$\begin{aligned} \gamma^{(p)\prime}\tilde{z}v_{min}^{(p)} + (B^{(p)}\iota_L^{(p)} - 1) &\leq 0 \\ \gamma^{(p)\prime}\tilde{z}v_{max}^{(p)} + (B^{(p)}\iota_L^{(p)} - 1) &\leq 0 \end{aligned} \tag{33}$$

hold jointly for $p = 1..P$.

Comparable with the unconditional case, transaction costs can be handled by distinguishing between the return on a short and a long position in the assets. Denote $\tau_i^l = \frac{1}{1+a_i}$ and $\tau_i^s = \frac{1}{1-b_i}$ as the transaction costs involved in taking a long respectively short position in the assets, and construct a $2K$-dimensional vector $\tilde{R}_{t+1}$ where the first K elements contain the net returns on a long position and the second K elements the net returns on a short position in the benchmark assets. Now it is straightforward to show that condition (30) can be generalized to

$$\frac{1}{\tau^s} \leq E_t[\tilde{m}_R(v)_{t+1}r_{t+1}] \leq \frac{1}{\tau^l}, \tag{34}$$

where $\tilde{m}_R(v)_{t+1}$ is the stochastic discount factor that prices the subset of L assets correctly, and the inequality signs reflect the short and long sales constraints on the additional asset. Substituting the expression for $\tilde{m}_R(v)_{t+1}$ in (34) gives

$$-b_i \leq (\gamma_r' - \Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}\gamma_R')vz_t + (\Sigma_{rR}^{(p)}\Sigma_{RR}^{-1(p)}\iota_K - 1) \leq a_i, \tag{35}$$

then a test for conditional mean-variance spanning for specific values of z_t when also transaction costs are incorporated can be based upon whether in the P regressions

$$r_{t+1} = \gamma^{(p)\prime}z_t + B^{(p)}\tilde{R}_{t+1}^{(p)} + \varepsilon_{t+1}, \tag{36}$$

the following restrictions hold jointly

$$\begin{aligned} -b_i &\leq \gamma^{(p)\prime}\tilde{z}v_{min}^{(p)} + (B^{(p)}\iota_L^{(p)} - 1) \leq a_i \\ -b_i &\leq \gamma^{(p)\prime}\tilde{z}v_{max}^{(p)} + (B^{(p)}\iota_L^{(p)} - 1) \leq a_i. \end{aligned} \tag{37}$$

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