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Strategic trading of forward contracts in oligopolistic industries with non-storable commodities*

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Abstract

We introduce demand uncertainty in the model of Allaz-Vila (1993). Traders' risk attitude in the forward market crucially determines the degree of competitiveness of the spot market. The higher demand uncertainty the lower the competition-enhancing effect of forward trading.

JEL Classification Codes: D43, L13

1 Introduction

A well known result by Allaz and Vila (1993) shows that introducing forward markets for a non-storable commodity reduces oligopolists' market power, because at equilibrium engaging in forward sales is a commitment to high output. Although the strategic insight of their model is very appealing, it is not clear to which extent it holds in a more realistic setting in which demand is uncertain and speculators are risk-averse. We believe these assumptions are more realistic because of the widely observed normal backwardation (i.e. the forward price is lower than the expected future spot price) which indicates that hedgers pay a risk-premium in such markets.

We analyze the effects of adding a forward market to a model of oligopolistic competition where the spot demand by retail consumers is uncertain. We model a market in which a forward contract on the commodity is traded between producers and speculators. In the special case of risk-neutral speculators, our model reproduces the result of Allaz and Vila (1993) showing that introducing forward markets drives the commodity price to the competitive level. In the

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case of risk-averse speculators, however, we show that this strong result does not hold. Because speculators require risk compensation, the forward price falls below the expected spot price, leading the producers to restrain their forward sales. Our main conclusion is that the competition-enhancing effect of forward markets decreases as spot market volatility increases due to the increase of the risk-premium. This indicates that the effectiveness of introducing forward markets for competition strongly depends on the volatility of demand for the commodity. Hence, in markets with stable demand (e.g. metals) forward markets can increase efficiency substantially while in markets subject to frequent exogenous demand shocks (e.g. energy) the opposite applies.

In general, the literature on the competition effects of forward markets (Powell (1993), Newbery (1995) and (1998), Green (1999)) suggests that long-term contracts reduce the market power of producers. The only exception is Green (2003) who independently derives results similar to ours in the context of electricity markets where forward markets are open only to generators and retailers. Our analysis allows free entry in the market for forward contracts, also to pure financial speculators.

2 The model

There are two periods and two risk-neutral producers, i and j. At time $t = 1$ the producers and risk-averse speculators $k \in M$ buy/sell forward contracts at the forward price p_f. At time $t = 2$ a spot market for the good opens and the actual production takes place: we assume the commodity is non-storable, so that it cannot be produced at $t = 1$. The exchange of the actual commodity occurs between the producers and consumers with an uncertain, price-sensitive demand. At the same date $t = 2$ the forward contracts are executed.

Forward market

For simplicity, we assume there is no time-discount in the model. The agents operating in the forward market can buy or sell forward contracts that call for delivery at time 2. The future realization of the spot market demand is uncertain when the forward trading takes place.

The equilibrium forward price p_f (for one unit of commodity) fixed at $t = 1$ rules out all arbitrage possibilities and clears the forward market.

If the speculators operating in the forward market are risk-averse, they would trade the quantity h_k which maximizes the expected utility of their profits $\Pi_k = (p_f - p)\, h_k$, with $h_k > 0$ indicating the sales of forwards; assuming they have mean-variance utility functions with risk-aversion coefficient equal to λ_k, we obtain:

$$E[U_k(\Pi_k)] = E[p_f - p]h_k - \frac{\lambda_k}{2} Var(\Pi_k) \tag{1}$$

Producers' profits and cost functions

The profit of producer i at $t = 2$ is composed of two elements: the profit from selling the output at the spot price, $pq_i - c(q_i)$, and the profit from the forwards sold at p_f, $p_f f_i - c(f_i)$, with $f_i > 0$ indicating a short position in the forward. We assume that for each producer the cost is $c_i(x_i) = b_i x_i$ where $x_i = q_i + f_i$ is the total amount of production taking place at $t = 2$.

Consumers demand

We assume that consumers' demand at $t = 2$ is known and depends on the spot price: $D = a - p + \theta$, where p is the spot price and θ is the realization of the demand shock $\widetilde{\theta}$. For simplicity, we assume that the random variable $\widetilde{\theta}$ has a binomial distribution $\{\theta_L, \theta_H\}$ with probabilities $\{\pi, 1-\pi\}$. Moreover, we normalize $E[\widetilde{\theta}] = 0$ (so that $\theta_L = -\frac{1-\pi}{\pi}\theta_H$) and $Var[\widetilde{\theta}] = \pi\theta_L^2 + (1-\pi)\theta_H^2 = \frac{1-\pi}{\pi}\theta_H^2$.

2.1 Forward market competition à la Cournot

In this section, we characterize the solution of the model in the case the producers compete in quantities on the forward market. Section 3 will analyze the case in which they compete in price.

Notice that on the spot market we always consider competition "à la Cournot", because with Bertrand competition the commitment effect of forward contracting disappears (see Green 1999).

We assume that the financial speculators who operate on the forward market are risk-averse, i.e. $\lambda_k > 0$ for all k.

2.1.1 Equilibrium in the spot market

Given the position in forward f_i and f_j, the producers i and j choose the optimal quantity to sell on the market at $t = 2$. The two competitors maximize profits:

$$\begin{aligned} \Pi_i &= p(\theta)q_i - bx_i = (a + \theta - x_i - x_j)\, q_i - bx_i \\ \Pi_j &= p(\theta)q_j - bx_j = (a + \theta - x_i - x_j)\, q_j - bx_j \end{aligned}$$

where q_i, q_j are, respectively, the quantity of commodity sold on the spot market by i and j, while $x_i = q_i + f_i$ and $x_j = q_j + f_j$. The reaction functions are:

$$\begin{aligned} x_i(x_j) &= \frac{a + \theta - b + f_i - x_j}{2} \\ x_j(x_i) &= \frac{a + \theta - b + f_j - x_i}{2} \end{aligned}$$

The spot market equilibrium, for any given realization of θ, is:

$$x_i^* = x_i(\theta) = \frac{(a + \theta - b) + 2f_i - f_j}{3} \quad (2)$$

$$x_j^* = x_j(\theta) = \frac{(a + \theta - b) + 2f_j - f_i}{3} \quad (3)$$

$$p^* = p(\theta) = \frac{(a + \theta) - f_i - f_j + 2b}{3} \quad (4)$$

Notice that this equilibrium is the same as in Proposition 2.1 of Allaz-Vila (1993) once we correct for the realized shock.

Moreover, given (4), the ex-ante $Var(p^*) = \frac{1}{9}Var[\widetilde{\theta}] = \frac{1-\pi}{9\pi}\theta_H^2$.

2.1.2 No-arbitrage condition in the forward market

We now compute the forward price p_f that clears the forward market at $t = 1$, given rational expectations of the speculators. Assuming rational expectations consists in imposing that the speculators take into account the correct spot price equilibrium formula (4) to decide their optimal exposure in forward contracts. Given CARA utility functions, the optimal position for speculator k in forward contracts for a given forward price p_f and a given distribution of p^* is

$$h_k = \frac{p_f - E[p^*]}{\lambda_k Var(p^*)}$$

so that the aggregate speculators (short) position in forwards is equal to

$$H = \frac{p_f - E[p^*]}{\Lambda Var(p^*)}$$

where $\Lambda = \left(\sum_k \frac{1}{\lambda_k}\right)^{-1}$. The equilibrium forward price p_f is such that the aggregate position of speculators and producers i, j is zero:

$$f_i + f_j + H = 0$$

that gives:

$$\begin{aligned}
\frac{p_f - E[p^*]}{\Lambda Var(p^*)} + f_i + f_j &= 0 \\
p_f - E[p^*] &= -\Lambda Var(p^*)\,(f_i + f_j) \\
p_f &= E[p^*] - \Lambda Var(p^*)\,(f_i + f_j) \\
&= E[p^*] - \Lambda \left(\tfrac{1-\pi}{9\pi}\right)\theta_H^2\,(f_i + f_j) \qquad (5)
\end{aligned}$$

The forward price p_f in (5) is then the unique non-arbitrage forward price when speculators have rational expectations.

2.1.3 The choice of optimal forward exposure by the duopolists

Given the forward price p_f, the expected profit of producer i at $t = 1$ is:

$$\begin{aligned}
E[\Pi_i] &= E[(p^* - b)\,(x_i^* - f_i) + (p_f - b)f_i] \\
&= E[(p^* - b)\,x_i^*] + (p_f - E[p^*])f_i \\
&= E[(p^* - b)\,x_i^*] - \Lambda Var(p^*)\,(f_i + f_j)\,f_i
\end{aligned}$$

where the expected profit collapses to the Allaz-Vila case if $Var(p^*) = 0:$

$$E[(p^* - b)\,x_i^*] = \tfrac{a-b-(f_i+f_j)}{3}\tfrac{a-b+2f_i-f_j}{3}$$

When the producers compete in quantity on the forward market, i solves for the optimal forward position by choosing

$$\max_{f_i} E[\Pi_i] = \tfrac{a-b-(f_i+f_j)}{3}\tfrac{a-b+2f_i-f_j}{3} - \Lambda Var(p^*)\left(f_i+f_j\right)f_i \qquad (6)$$

with $Var(p^*) = \frac{(1-\pi)}{9\pi}\theta_H^2$.

The necessary and sufficient conditions for a maximum of (6) are:

$$\frac{1}{9}\frac{a\pi-b\pi-4f_i\pi-f_j\pi-2\Lambda\theta_H^2 f_i+2\Lambda\theta_H^2 f_i\pi-\Lambda\theta_H^2 f_j+\Lambda\theta_H^2 f_j\pi}{\pi} = 0$$

$$\Rightarrow f_i = \frac{a-b-f_j-\frac{\Lambda\theta_H^2 f_j}{\pi}+\Lambda\theta_H^2 f_j}{4+\frac{2\Lambda\theta_H^2}{\pi}-2\Lambda\theta_H^2} \qquad (7)$$

$$\frac{\partial\left(\frac{1}{9}\frac{a\pi-b\pi-4f_i\pi-f_j\pi-2\Lambda\theta_H^2 f_i+2\Lambda\theta_H^2 f_i\pi-\Lambda\theta_H^2 f_j+\Lambda\theta_H^2 f_j\pi}{\pi}\right)}{\partial f_i} = \frac{-4\pi-2\Lambda\theta_H^2+2\Lambda\pi\theta_H^2}{9\pi} = \frac{2(\Lambda\theta_H^2(\pi-1)-2\pi)}{9} < 0$$

Using (7) and the symmetric condition for j gives us:

$$f_i = f_j = f = \frac{(a-b)\,\pi}{5\pi+3\Lambda\theta_H^2(1-\pi)} = \frac{(a-b)}{5+3\Lambda\theta_H^2\frac{1-\pi}{\pi}} = \frac{(a-b)}{5+3\Lambda Var(\theta)}$$

which is always lower than the forward position in the solution of Allaz-Vila (1993) (see their Proposition 2.3).

Proposition 1 *If the speculators $k \in M$ operating in the forward market are strictly risk-averse, then the optimal forward position for producers is lower than the one with certain demand schedule.*

The interpretation of this result is simple. Risk-averse speculators buy forward contracts at a price which is lower than the expected spot price. The discount is proportional to the variance of the spot price. This represents the premium that risk-averse speculators ask for bearing the price risk. In equilibrium the producers will optimally reduce their short forward position because the cost of selling forwards increases with the risk premium. The lower the short position of each producer, the lower the degree of competitiveness on the spot market at $t = 2$. When demand shocks are highly unpredictable the cost of selling forwards is so high that the introduction of a forward market has little improvement on the overall efficiency.

2.2 Forward market competition à la Bertrand

Green (1999) argues that a less competitive market in long-term contracts has less impact on the spot market allocation. He proves that adding a forward market in which producers compete "à la" Cournot does not change the spot market allocation. On the other hand, a very competitive forward market, with competition "à la" Bertrand should produce larger effects on the spot market.

We verify this intuition extending the result of Proposition 1 to the case in which the producers compete in price in the forward market.

Proposition 2 *If the speculators $k \in M$ operating in the forward market are strictly risk-averse, and if the producers compete in prices on the forward market, then the optimal forward position for producers is lower than the one with certain demand schedule when*

$$Var(\theta) > \frac{9}{2}\Lambda^{-1}$$

Proof: Rewriting the f.o.c. for the profit maximization for producer i we obtain:

$$\frac{1}{9}\frac{a\pi - b\pi - 4f_i\pi - f_j\pi - 2\Lambda\theta_H^2 f_i + 2\Lambda\theta_H^2 f_i\pi - \Lambda\theta_H^2 f_j + \Lambda\theta_H^2 f_j\pi}{\pi} + \frac{1}{9}\frac{-2a\pi + 2b\pi - f_i\pi + 2f_j\pi - \Lambda\theta_H^2 f_i + \Lambda\theta_H^2 f_i\pi}{\pi}\frac{df_j}{df_i} = 0$$

and with Bertrand competition on the forward market $\frac{df_j}{df_i} = -1$ then

$$\frac{1}{9}\frac{3a\pi - 3b\pi - 3f_i\pi - 3f_j\pi - \Lambda\theta_H^2 f_i + \Lambda\theta_H^2 f_i\pi - \Lambda\theta_H^2 f_j + \Lambda\theta_H^2 f_j\pi}{\pi} = 0$$

The optimal forward position is given by the solution:

$$f_i = -\frac{3a\pi - 3b\pi - \Lambda\theta_H^2 f_j - 3f_j\pi + \Lambda\theta_H^2 f_j\pi}{-\Lambda\theta_H^2 + \Lambda\theta_H^2\pi - 3\pi}$$

and imposing symmetry: $f_i = f_j = f$ and solving for f :

$$f^B = -\frac{3}{2}\pi\frac{a-b}{-\Lambda\theta_H^2 + \Lambda\theta_H^2\pi - 3\pi} = \frac{a-b}{2 + \frac{2}{3}\Lambda Var(\theta)}$$

Comparing the result in presence of Bertrand competition f^B with the result with Cournot competition, $f^C = \frac{a-b}{5+3\Lambda Var(\theta)}$, we can conclude that $f^B > f^C$ and $f^B < f^{Allaz-Vila} \Leftrightarrow 2 + \frac{2}{3}\Lambda Var(\theta) > 5 \Leftrightarrow Var(\theta) > \frac{9}{2}\Lambda^{-1}$. ■

If the shock $\widetilde{\theta}$ has high variance, the forward market does not achieve the perfectly competitive allocation as in Allaz and Vila (1993) even if it is very competitive.

3 Conclusion

In this paper we show that introducing forward markets in a duopoly does not always enhance the perfectly competitive allocation on the spot markets, as in Allaz and Vila (1993). Their result relies crucially on the absence of any element of uncertainty over the spot demand of the commodity and on the risk-neutrality of forward buyers, like financial intermediaries. The efficiency-enhancing effect of forward contracts is lower when the demand of the good is very uncertain.

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