Taxes, growth and welfare in an endogenous growth model with overlapping generations

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Abstract

The effects of fiscal policy are analyzed in a two-sector endogenous growth model with overlapping-generations. Firstly, it is shown that long-run growth, and thus utility of future generations, can be stimulated by a flat tax on capital income as well as by a flat tax on labor income or a subsidy on human capital . Secondly, it is shown that a tax on capital income harms all existing generations, while taxing labor income or subsidizing human capital may be a Pareto improvement. Finally, it is shown that taxes on capital and labor are not used in an optimal policy, but that a subsidy on human capital is a necessary instrument to realize the first-best outcome in a market economy.
JEL codes: D62, D91, E62, H21, H23, H31, I28
Keywords: taxes, growth, welfare, overlapping generations

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*I would like to thank Lans Bovenberg, Casper van Ewijk, Theo Leers and Marijn Verhoeven for helpful comments on an earlier draft.

1 Introduction

There is a vast literature on the effects of fiscal policy in Ramsey-models, i.e. models with an infinitely long-lived dynasty as the decision making unit. The general conclusion from this literature is that distortionary taxes are bad for growth and welfare (see e.g. Stokey and Rebelo (1995), Milesi Ferretti and Roubini (1996)). The number of papers on the consequences of taxes in overlapping-generations (OLG) models is much smaller. However, the scarce literature on this subject suggests that the effects of taxation in this kind of models may be remarkably different from those in a Ramsey-model. For example, Uhlig and Yanagawa (1996) and Bertola (1996) show that in a one-sector growth model with OLG, growth may be stimulated rather than slowed down by a tax on capital income. The argument is that in such a model financial wealth rises with age, so that a tax on capital income that is refunded lump sum implies a (future) transfer from the old to the young. This increases savings and thus long-run growth. This paper is aimed to extend the parsimonious knowledge of the effects of taxation on growth and welfare in OLG-models.

In particular, in this paper we analyze the consequences of a tax on capital income, a tax on labor income and a subsidy on human capital in a two-sector growth model which is a synthesis of the Blanchard-Yaari OLG-model (Blanchard (1985)) and the Lucas-Uzawa model of endogenous growth (Lucas (1988)). The model is presented in Section 2. Individuals allocate time either to working, to learning, or to leisure. Leisure takes the form of 'quality time' (hours combined with human capital, see Becker, (1965)). This implies that a broad interpretation can be given to it. That is, leisure can also be thought of as home production or production in the informal sector. Moreover, this way of modelling allows for elasticities of substitution between consumption and leisure not equal to one. It is assumed that new-born generations start with a level of human capital proportional to the average level of human capital in the economy at the time of their birth and accumulate human capital through learning. As in the Lucas-model, there is no physical capital involved in this accumulation process.

The paper makes a number of contributions to the literature. Firstly, in Section 3 we analyze the effects of fiscal policy on long-run growth. In this section we generalize the result of Bertola (1996) to a two-sector growth model where human capital is the engine of growth and labor supply is endogenous. That is, we show that an increase in the tax on capital income raises long-run growth. Moreover, we demonstrate that a tax on labor income may have the same effect. This result that a tax on labor income may stimulate long-run growth is remarkably different from conventional economic wisdom. In general, in a Ramsey model where both physical capital and human capital are reproducible factors, a tax on labor income reduces long-run growth (see e.g. Pecorino (1993), Stokey and Rebelo (1995), Milesi Ferretti and Roubini (1996)). In the dynastic versions of the Lucas-model of endogenous growth found in the literature (Lucas (1988, 1990), Chamley, (1993)), constant proportional taxes on labor income do not affect long-run growth however. If labor supply is exogenous, this result carries over to the OLG-version of this model presented here. But this is not true anymore if the supply of labor to the market is endogenous. In that case, growth rises if the tax on wage income has strong substitution effects so that leisure rises sharply and consumption of goods decreases, or, in other words, if the wage tax causes a strong shift from the formal to the informal sector. Then there will be a shift in the composition of wealth from assets that are merely productive in the formal sector (financial wealth) to assets that are also productive in the informal sector (human wealth). Given the OLG-structure used, this change in the composition of wealth increases growth. Finally, we show in this section that a subsidy on (investment in) human capital stimulates growth.

Secondly, in Section 4 of the paper we study the welfare effects of changes in taxes. Because of the OLG-structure of the model it is not sufficient for this purpose to look at the effects on long-run growth. The effect of changes in taxes on the welfare of the different generations during the period of transition to the new balanced-growth path should also be taken into account. This is done using numerical simulation experiments. These experiments learn that an increase in the tax on capital income harms most generations. Only generations born in the distant future see their welfare increase. Therefore, in a direct democracy where the currently living generations vote on the tax level, such a tax increase will not be accepted. That is, such a democracy will not maximize growth. In contrast, the outcome of a vote on a proposal to increase the tax on labor income is ambiguous. The reason for this is that old generations gain while young generations may lose. However, a tax on labor income may also increase welfare of all generations, i.e. it may be a Pareto improvement. The same is true for a subsidy on human capital. The reason for this is that future generations' initial level of human capital is determined by the average level of human capital at the time they are born. This introduces an externality in the model: the accumulation of human capital has a social aspect that is not taken into account by individuals when they decide how much time to devote to learning. Therefore, without government intervention, the market outcome is suboptimal as growth is too low.

Thirdly, after this positive analysis, in Section 5 we address the normative question whether it is optimal for the government to use taxes on capital and/or labor income. An optimal policy is derived by maximizing a social welfare function as introduced by Calvo and Obstfeld (1988). That is, the government is assumed to maximize the weighted sum of lifetime expected utility of future generations and the expected utility over the rest of their lifetime of those cohorts currently alive. It is shown that, even though both taxes may increase growth, neither a tax on labor income nor a tax on capital income is used in an optimal policy, provided that the government is able to levy time-dependent and age-dependent lump-sum taxes. Moreover, we proof that a subsidy on (investment in) human capital is a necessary instrument in order to be able to reach the first-best outcome in a market economy. Finally, the paper winds up with some concluding remarks.

2 The model

Firms

Production is described by a standard neo-classical production function F which is assumed to be linearly homogeneous in the factors of production capital k and labor (in efficiency units) l. Define $\ell \equiv \frac{l}{k}$ and $f(\ell) \equiv \frac{F(k,l)}{k}$. Then the representative firm's optimization problem can be summarized by the first-order conditions[1]:

$$f_\ell(\ell(t)) = w(t), \tag{1}$$

$$(1-\tau^k)[f(\ell(t)) - \ell(t)w(t)] = r(t), \tag{2}$$

where r is the interest rate, w stands for the real wage rate per labor efficiency unit and τ^k denotes the tax rate on capital income.[2]

[1] For variables on an aggregate level only an index t is used to denote current time. For variables that refer to a particular generation, a second index s is added to denote the time of birth. A subscript denotes a (partial) derivative. A dot indicates the partial derivative with respect to t.

[2] Notice that we abstract from depreciation. Allowing for depreciation does not change any of the results derived below however.

Consumers

A standard Blanchard-Yaari-model with a constant population size normalized to one is used, i.e. the birth rate equals the probability of death (π).[3] Each consumer maximizes expected lifetime utility U, which equals the flow of instantaneous utilities discounted at the rate of time preference ρ plus the probability of death. He contracts to have all of his financial wealth go to an insurance company contingent on his death. In exchange the insurance company will pay a premium πk per unit of time. At each point in time a consumer has one unit of time available which is used either for working (l^f), for learning (l^h) or for leisure. Labor productivity is measured by the efficiency index h which will be called human capital. Learning leads to an increase in human capital. The learning technology is characterized by constant returns to scale:[4]

$$\dot{h}(t,s) = \phi(l^h)h(t,s) \quad \phi' > 0, \phi'' \leq 0. \tag{3}$$

In order to allow for endogenous growth, the stock of human capital at the time of birth should be positive. Moreover, it should not be constant over time. A balanced growth path only exists if the initial stock of human capital of subsequent generations grows at the same rate as the economy. Here we realize this is by assuming that human capital of a new-born generation is proportional to average human capital, i.e. $h(t,t) = \chi h(t)$. The idea is that a new born generation starts with a higher level of human capital (and is therefore able to accumulate human capital faster) than previous generations at their time of birth because it can build on the knowledge gathered by these generations. This social aspect of human capital is not taken into account by individuals when they decide how much time to devote to learning, however. So, a positive externality is associated to the accumulation of human capital.

It is assumed that utility in period t is a CES-function of consumption (c) and leisure (z) with an elasticity of substitution σ. Leisure takes the form of 'quality time', i.e. it is defined as $z(t,s) = [1 - l^h(t,s) - l^f(t,s)]h(t,s)$ This way of modelling, which was introduced by Becker (1965), is used here in order to be able to allow for elasticities of substitution between consumption

[3] For the derivations of the equations in this section see Appendix A. Allowing for population growth does not change any of the results derived below.

[4] One could also assume that the accumulation of human capital requires physical capital. This would complicate the analysis, but as long as the accumulation of human capital is less capital intensive than the accumulation of physical capital this would not affect the main results.

and leisure not equal to one. When leisure is not scaled by the efficiency index, this is not compatible with balanced growth.[5] Moreover, it implies that a broad interpretation can be given to leisure time. One could also think of it as time spent in home production or in the informal sector. In this interpretation it seems logical to assume that the productivity of leisure time rises when human capital grows.

Human wealth (j) is defined as the value of human capital in terms of consumption goods, i.e. $j(t,s) = q(t,s)h(t,s)$ where q is the average value of human capital. Total wealth (v) is the sum of human wealth, financial wealth (k) and the present value of lump-sum transfers from the government (g) and evolves according to a standard Euler equation:

$$\dot{v}(t,s) = [r(t) - \rho]v(t,s), \tag{4}$$

while total spending on consumption and leisure is a constant fraction of total wealth:

$$c(t,s) + w^n(t)z(t,s) = [\rho + \pi]v(t,s). \tag{5}$$

Total consumption spending is allocated to consumption of goods and leisure according to:

$$w^n(t)z(t,s) = \lambda(t)c(t,s), \tag{6}$$

where w^n is the net wage rate and $\lambda(t) \equiv w^n(t)^{1-\sigma}(\frac{1-\alpha}{\alpha})^{-\sigma}$. Notice that if $\sigma = 1$ the CES-utility function boils down to a Cobb-Douglas function and eqs. (5) and (6) give the well-known result that consumption of goods is a constant fraction of wealth. Notice further that $\alpha = \lambda = 0$ gives the special case of exogenous labor supply.

Using first-order conditions (5) and (6)and the equilibrium condition for the goods market, the following expression for the growth rate of average financial wealth ($\nu^k(t) \equiv \frac{\dot{k}(t)}{k(t)}$) can be derived:

$$\nu^k(t) = f(\ell(t)) - \frac{\rho + \pi}{(1 + \lambda(t))K(t)}, \tag{7}$$

where $K(t) \equiv \frac{k(t)}{v(t)}$ is the average share of financial wealth in total wealth.

As all individuals face the same learning function, they will all learn the same number of hours. Consequently, the growth rate of human capital is

[5] See King, Plosser and Rebelo (1988) or Barro and Sala-i-Martin, (1995, Appendix 9A).

equal for all individuals, namely $\phi(l^h(t))$. However, if $\chi \neq 1$ this individual growth rate differs from the growth rate of average human wealth ($\nu^h(t) \equiv \frac{\dot{h}(t)}{h(t)}$) which is given by:

$$\nu^h(t) = \phi(l^h(t)) - \pi(1-\chi). \tag{8}$$

The reason for this is that if $\chi \neq 1$ new-born generations have a level of human capital that is different from the average level of human capital of those who die.

The growth rate of average total wealth ($\nu^v \equiv \frac{\dot{v}(t)}{v(t)}$) is described by:

$$\nu^v = r(t) - \rho - \pi + \pi\frac{v(t,t)}{v(t)}. \tag{9}$$

As transfers are independent of age[6], this can be rewritten as:

$$\nu^v = r(t) - \rho - \pi K(t) - \pi(1-\chi)J(t), \tag{10}$$

where $J(t) \equiv \frac{j(t)}{v(t)}$ denotes the average share of human wealth in total wealth. Here we see that the growth rate of average total wealth differs from the growth rate of individual wealth ($r(t) - \rho$) and depends on the composition of wealth. One reason for this is that at each point in time a part π of the population dies and is replaced by new generations that start without financial wealth. So, in contrast to human wealth or transfer wealth, financial wealth of those who die is not passed on to new generations. Therefore, as in the Blanchard-Yaari model, the larger the share of financial wealth, the lower the growth rate. In the model presented here there may be an additional source for a difference between average growth and individual growth. If $\chi < 1$ (> 1), new generations start with a level of human capital below (above) average. Therefore in that case, the growth rate of average human wealth is smaller (larger) than the individual growth rate. Consequently, the larger the share of human wealth, the larger the difference between the growth rate of average total wealth and the rate of growth of individual total wealth.

It should be noted that the composition of individual wealth changes during lifetime. Suppose there are no transfers. In that case, an individual's wealth at birth consists of human wealth only. From eqs. (8) and (9)

[6] This implies that $\frac{g(t,t)}{v(t)} = 1 - K(t) - J(t)$

it follows that then the growth rate of total individual wealth exceeds the growth rate of individual human wealth, implying that the share of financial wealth in the total wealth portfolio increases during lifetime. Not only the composition of wealth changes during lifetime, but also the allocation of time to work and to leisure. Eqs. (5) and (6) imply that the number of hours leisure depends on the ratio of total wealth over human wealth. Therefore, the older an individual, the larger his leisure time and the lower the number of working hours. In fact, the number of working hours may even become negative, implying that the old individual effectively creates additional leisure time by buying services from younger individuals. This is in line with the broad interpretation of leisure used, which includes home production.

Balanced growth

Firstly, on a balanced growth path, the average value of human capital (q) is constant. From this we can derive that:

$$r + \pi = \phi(l^h) + (1 - l^h + \tau^h)\phi'(l^h). \tag{11}$$

This condition says that the rate of return on financial wealth ($r+\pi$) equals the rate of return on human wealth, which consists of the 'dividend' on human wealth ($(1-l^h+\tau^h)\phi'(l^h)$) and the increase in value ($\phi(l^h)$). Note that eq. 11 implies that if $\phi''(l^h) < 0$ a lower interest rate goes along with more learning. If the learning technology is linear in l^h (i.e. $\phi''(l^h) = 0 \forall l^h$) the interest rate is independent of learning time and is completely determined by the technology parameter ϕ' and the subsidy on human capital. So, neither taxation of labor income nor taxation of capital income does affect the interest rate in this case.

Secondly, a balanced growth path is characterized by $\nu^v = \nu^h = \nu^k = \nu$, which implies:

$$K = \frac{\rho + \pi}{(1+\lambda)[f(\ell) - \nu]}, \tag{12}$$

$$J = \frac{(1-\tau^l)w\ell K + [\rho + \pi]\frac{\lambda}{1+\lambda}}{(1-l^h)\phi'(l^h)}. \tag{13}$$

Using eqs. (5) and (6), the latter equation can be interpreted as an equilibrium condition for the labor market: aggregate supply of human capital (h) equals the demand for human capital for learning ($l^h h$), doing ($\ell k = l^f h$) or leisure ($z = \frac{(\rho+\pi)v\lambda}{(1+\lambda)w^n}$). From the last two equations also the ratio of human capital to physical capital can be derived:

$$\frac{h}{k} = \frac{\ell + \frac{[\rho+\pi]\lambda}{(1+\lambda)Kw^n}}{1-l^h}. \tag{14}$$

Note that it can be derived from eqs. (8), (9) and (11) that if individuals live infinitely long, i.e. $\pi = 0$, then $(1 - l^h + \tau^h)\phi'(l^h) = \rho$. From this it immediately follows that in that case learning time, and therefore long-run growth, is independent of the tax rate on capital income or labor income. The only way the government is then able to stimulate growth is by subsidizing human capital. This is the standard result for Lucas-Uzawa-type models of endogenous growth (see e.g. Lucas, 1990). This result does not hold anymore if the birth rate is positive, however. In that case, tax rates influence the composition of wealth and thus affect the long-run growth rate. This issue is addressed in the next section.

3 Fiscal policy and long-run growth

In this section we analyze how a subsidy on human capital, a tax on capital income and a tax on labor income affect long-run growth. The analysis consists of two parts. Firstly, the qualitative effects of fiscal policy on long-run growth are derived analytically by comparative statics.[7] Secondly, the quantitative effects are illustrated by numerical simulation experiments. It is assumed that the government budget is balanced by transfers to consumers that are lump sum and independent of age. Furthermore, it is assumed that consumption and leisure as well as capital and labor are close substitutes, i.e. $\epsilon = \frac{-f_{\ell\ell}\ell}{f_\ell} < 1$ and $\sigma > 1 \quad \rightarrow \quad \frac{d\,\lambda}{d\,w^n} < 0$ Finally, the comparative statics analysis is based on the assumption that $\chi = 1$, i.e. individuals start with average human wealth. This assumption is made for analytical convenience. It implies that all generations have equal human wealth so that the growth rate of average total wealth does not depend on the share of human wealth (see eq. (10)). Without this assumption, fiscal policy may lead to two opposing forces on the growth rate. On the one hand, as we will see, an increase in τ^h, τ^k or τ^l decreases the share of financial wealth, implying implicit transfers from present to future generations and an increase in growth. If, on the other hand J increases and $\chi < 1$ (or J decreases and $\chi > 1$) this implies that new generations start with a relatively

[7] For the derivation of the comparative statics results we refer to Appendix B.

low level of wealth and thus decreases growth. However, the simulation experiments learn that in most cases the former force is dominant.

A subsidy on human capital

It can easily be derived that a subsidy on human capital indeed stimulates learning and therefore growth in the long run: $\frac{d\,\nu}{d\,\tau^h} = \phi'(l^h)\frac{d\,l^h}{d\,\tau^h} > 0$. If the learning technology is non-linear an increase in learning time decreases the rate of return on human wealth and therefore has a negative effect on the long-run interest rate. This negative effect is outweighed by the positive direct effect of the subsidy on the rate of return on human wealth and thus on the rate of return on financial wealth (see eq. (11)). Consequently, the interest rate rises: $\frac{d\,r}{d\,\tau^h} > 0$. The higher interest rate is the result of a more labor intensive production process: $\frac{d\,\ell}{d\,\tau^h} > 0$ and $\frac{d\,w}{d\,\tau^h} < 0$. As a consequence, the share of financial wealth in total wealth decreases: $\frac{d\,K}{d\,\tau^h} < 0$. As the share of transfers also decreases due to the costs of the subsidy this implies that the share of human wealth increases: $\frac{d\,J}{d\,\tau^h} > 0$. It also immediately follows that the ratio of human to physical capital rises: $\frac{d\,\frac{h}{k}}{d\,\tau^h} > 0$. Note that the effect of a subsidy on human capital is only quantitatively dependent upon the value of π. That is, the result is not qualitatively different if individuals live infinitely long ($\pi = 0$).

Table 1 presents the calibrated steady states for $\chi = 1.25$ and different levels of the tax rates and the subsidy on human capital.[8] Comparing the last column to the first one gives a flavor of the quantitative effects of the introduction of a 10 percent subsidy on human capital can be. In this case, where new generations start with a level of human capital above average, such a subsidy raises the growth rate as well as the interest rate by 0.5 percentage point. In order to realize this the percentage of time used for learning is increased by 9 points. Notice that this not only decreases leisure, but also working time by 3 percentage points. In spite of this, the labor intensity (measured in efficiency units) of the production process rises as the h/k ratio is increased by almost 11 percentage points.

The effects in case $\chi = 0.75$ are comparable to the ones in case $\chi = 1.25$. The effect on the growth rate is, however, slightly smaller in the former

[8] The following specifications for the production function of goods and human capital respectively are used: $f(\ell) = \ell^\gamma$, $\phi(l^h) = \phi_0(l^h)^{\phi_1}$. The paramater values used are: $\gamma = 0.35$, $\alpha = 0.5$, $\sigma = 1.25$, $\phi_0 = 0.06$, $\phi_1 = 0.9$, $\rho = 0.03$, $\pi = 0.01$, $\chi = 1.25$, $\tau_l = \tau_k = 0.25$, $\tau_h = 0$. The growth rate ν and the interest rate r are expressed in percentage points.

	base	$\tau^l = 0.35$	$\tau^k = 0.35$	$\tau^h = 0.10$
ν	2.348	2.344	2.363	2.886
r	5.278	5.280	5.275	5.772
w	1.542	1.542	1.428	1.469
c	0.178	1.178	0.208	0.191
l^f	0.251	0.223	0.244	0.219
l^h	0.311	0.310	0.312	0.401
J	0.739	0.706	0.740	0.878
K	0.115	0.113	0.097	0.106
h/k	0.338	0.380	0.462	0.445

Table 1: The long-run effects of fiscal policy

case. This is due to the fact that the rise in the share of human wealth (J) moderates the positive effect on the growth rate in this case whereas it reinforces the increase in the growth rate if $\chi > 1$. Still, a subsidy on human capital seems a reasonably effective instrument to stimulate growth.

A tax on capital income

The qualitative effects of a tax on capital income are to a large extent analogous to the effects of a subsidy on human capital. That is, it increases learning and growth: $\frac{d\,l^h}{d\,\tau^k} > 0$ and $\frac{d\,\nu}{d\,\tau^k} > 0$. This has a negative effect on the (after tax) interest rate. In contrast to a subsidy on human capital, there is no positive effect on the interest rate that offsets this negative effect. So, if the learning technology is non-linear the interest rate decreases: $\frac{d\,r}{d\,\tau^k} < 0$. The effect of a change in the tax rate τ^k on the share of financial wealth in total wealth is negative: $\frac{d\,K}{d\,\tau^k} < 0$. Consequently, production becomes more labor intensive ($\frac{d\,\ell}{d\,\tau^k} > 0$) and the marginal product of physical capital rises while the wage rate falls.

Note that the growth rate is independent of the composition of wealth if individuals live infinitely long ($\pi = 0$). In that case we have the standard result for Lucas-type of endogenous growth models that long-run growth is not affected by the capital tax. This illustrates that the result that long-run growth is stimulated by a tax on capital income, which is opposite to the conclusions generally drawn in the literature (see e.g. Pecorino (1993), Saint-Paul (1992), Stokey and Rebelo (1995), Milesi Ferretti and Roubini (1996)), is due to the OLG-structure used here. Uhlig and Yanagawa (1996) derive the same result in a two-OLG model with an AK-technology. The argument is that, as the old possess the capital stock, a tax on capital income that is refunded lump sum[9] implies a transfer from the old to the young. Agents compensate for this by higher savings. On the other hand, the lower net return on savings has a negative effect on savings. If this negative substitution effect is outweighed by the positive income effect, savings increase. Bertola (1996) presents a comparable analysis for a Blanchard-Yaari model with an AK-technology. In such a model a lower the interest rate has, apart from an income and a substitution effect, a negative wealth effect on savings as future wage income is discounted less heavily. Consequently, a tax on capital income raises growth only if the income effect dominates both the substitution and the wealth effect. Bertola shows that this is possible if labor productivity decreases with age and the elasticity of intertemporal substitution is strictly less than one. The analysis here can be viewed as a generalization of the results of Bertola to a model where human capital is the engine of growth. Notice that there are some important differences, however. Firstly, our results are based on a utility function with an elasticity of intertemporal substitution equal to one. Secondly, in the present model labor productivity increases with age due to the accumulation of human capital. That we still find that a tax on capital income stimulates growth is due to the fact that it is not savings (i.e. the accumulation of physical capital) that drives growth here, but learning (i.e. the accumulation of human capital). The tax on capital income reduces the rate of return on savings and makes investment in human capital relatively more attractive. Consequently, the number of hours spent working falls and the number of hours spent learning rises.

The simulation results in Table 1 show that the quantitative effect that a 10 percent higher tax on capital income has on long-run growth and the interest rate is very small (compare the third column to the first one). The initial decrease in the after-tax interest rate is almost completely offset by an increase in the marginal product of capital due to the increased labor intensity of the production process. This is in turn caused by the higher ratio of human to physical capital (h/k) which more than offsets the decrease in the number of hours worked. The results for $\chi = 0.75$ are almost the same

[9] In fact, Uhlig and Yanagawa as well as Bertola study the case where the receipts of the increase in the tax on capital income is rebated through a decrease of the tax on labor income. As labor supply is exogenous in their models, this is effectively a (possibly age dependent) lump-sum transfer, however. The last part of this section studies the effect of such a shift in the tax burden from capital to labor in the present model.

as those for $\chi = 1.25$. So we can conclude that long-run growth can be raised by increasing the tax on capital income, but that this is not a very effective instrument to do so.

A tax on labor income

If $\chi = 1$ the sign the effect of a change in the labor income tax on long-run growth is determined by the elasticity of substitution between leisure and consumption. Given the assumption $\sigma > 1$, an increase in the tax rate on labor increases growth: $\frac{d\,\nu}{d\,\tau^l} > 0$. If $\sigma < 1$ the effect is negative, however, and if $\sigma = 1$ or labor supply is exogenous there is no effect. The latter result is well known in the literature on Lucas-type of endogenous growth models. A higher flat tax rate on labor income ceteris paribus decreases the net wage rate, but this does not affect the incentive to invest time in the accumulation of human capital: marginal cost (wage income forgone by learning) and marginal benefits (additional wage income due to accumulated human capital) change at the same rate.[10] When labor supply is endogenous the lower net wage rate leads to an increase in the demand for leisure (in efficiency units), however. If the elasticity of substitution is larger than 1 this goes along with a decrease in the demand for consumption. These changes in demand lead to a change in the wealth portfolio from goods-producing assets (K) to leisure-producing assets (J), i.e. $\frac{d\,K}{d\,\tau^l} < 0$. If individuals face a positive probability of death $\pi > 0$ this increases the growth.

Once again, the result is remarkably different from conventional economic wisdom. Here this is not only due to the OLG-structure used however. It is this structure in combination with the way leisure was introduced in the utility function that determines the result. The assumption of Becker preferences allows for an elasticity of substitution between consumption and leisure larger than one. As a consequence of this, taxation of labor income causes a large shift from the formal to the informal sector, leading to an increase of the incentive to accumulate human wealth while accumulation of physical capital becomes less attractive. Because of the OLG-structure, this causes an implicit transfer from present to future generations which increases growth.

Of course, a tax on labor income can only cause a higher growth rate if learning time increases. Given the increased demand for leisure, this has to be enabled by a decrease in working time ($\frac{d\,\ell}{d\,\tau^l} \leq 0$ although $\frac{h}{k}$

[10] For an analysis of the effect of progressive taxes on labor income in a small open economy see Bovenberg and Van Ewijk (1997).

rises). So, assuming a non-linear learning technology production becomes less labor intensive and the marginal product of labor (in efficiency units) rises ($\frac{d\,w}{d\,\tau^l} > 0$) while the interest rate falls ($\frac{d\,r}{d\,\tau^l} < 0$). This change in factor payments partly offsets the initial effect of the labor tax on the net wage rate.

If new-born generations start with a level of human wealth below the average level (i.e. $\chi < 1$), the growth stimulating effect due to the decrease in the share of financial wealth is reinforced by the shift of human wealth to transfer wealth that also results from a tax on labor income which is rebated through lump-sum transfers. The reason for this is that transfer wealth is independent of age and thus is fully passed on from those who die to newborn generations, while this is not true for human wealth if $\chi < 1$. If, however, initial human wealth is relatively large ($\chi > 1$) the shift from human wealth to transfer wealth has a negative effect on growth. This negative effect may outweigh the positive effect due to the decrease in the share of financial wealth. This is illustrated in Table 1. When we compare the second column to the first one, we see that the effect that a 10 higher percent tax on labor income has on long-run growth is slightly negative if $\chi = 1.25$. The pre-tax wage rate and interest rate are almost not affected. Consequently, the after-tax wage rate falls so that time is re-allocated from work to leisure and learning, and the ratio of human capital to physical capital rises. This has a positive effect on growth, but this is offset by the negative effect due to the decrease in the share of human capital. We conclude that a tax on labor income increases long-run growth if $\sigma > 1$ and $\chi \leq 1$, but not necessarily if $\chi > 1$. However, in both cases, for reasonable parameter values, the effect on growth is very small.

A tax shift from capital income to labor income

Many authors (see e.g. Pecorino (1993), Uhlig and Yanagawa (1996), Bertola (1996)) analyze the effect of a shift from capital income taxation to taxation of labor income. In this section we study the effect of such a shift of the burden of taxation in the OLG-version of the Lucas model presented in this paper with $\chi = 1$. Note that in this case both these taxes increase growth so the effect of a tax shift is ambiguous. The experiment is performed under the condition that, starting from a given set of tax rates and a given level of government expenditures relative to the capital stock, there is no effect on the government budget in the long run, i.e.[11] $d[\tau^l \ell w + \tau^k (f(\ell) - \ell w)] = 0$. Moreover, it is assumed that learning technology is linear. This implies that the long-run interest rate does not change (see eq. (11)). Therefore, it follows that $d\tau^k = -\frac{\ell w}{(f(\ell)-\ell w)(1+\zeta)} d\tau^\ell$ where ζ denotes the effect of the tax shift on the tax base. Using this equation, it can easily be derived that shifting the tax burden from capital income to labor income decreases growth if

$$-(\tau^l + \zeta)\frac{d\lambda}{dw} < \frac{\ell(1+\lambda)}{\epsilon(f(l) - \nu)}. \tag{15}$$

Note that this is evidently true if labor supply is exogenous. In that case, the tax on labor does not affect growth and only the negative effect of the lower τ^k on growth results.

Another sufficient condition for the effect on growth to be negative is $\tau^k = \tau^l = 0$. In that case, the effect on the tax base can be neglected ($\zeta = 0$). Moreover, as $\tau^l = 0$, there is no substitution effect in that case. That is, the decrease in the net wage rate caused by the tax on labor income is exactly compensated by the increase in the wage rate due the rise in capital intensity that results from the higher tax on capital income. Therefore, the only effect that results is the direct effect of the increased capital intensity on the composition of the wealth portfolio and thus on the growth rate. (A lower value for ℓ increases K (see eq. (12)) which in turn decreases growth (see eq. (10)).

If $\tau^l > 0$, the net wage rate falls and a substitution effect, which ceteris paribus increases growth, results. (A decrease in w^n increases leisure and (assuming $\sigma > 1$) decreases consumption. This causes a shift from K to J and thus increases growth). Moreover, if τ^l and/or τ^k is positive, there is a growth stimulating tax-base effect when the tax burden is shifted from capital to labor. The base for the tax on capital income shrinks due to the decrease in the marginal product of capital and the lower labor income diminishes the base for the labor income tax. Consequently, a given decrease in τ^k ceteris paribus leads to a larger increase in τ^l and thus to a stronger impulse for growth. In order for the total effect on growth to be negative, these two positive effects on growth should not be too strong. This implies that τ^l and especially τ^k should not be too high.

[11] Note that it is assumed that there is no subsidy on human capital. Note further that a balanced growth path exists only if government expenditures are adjusted to long-run growth. In order to keep the analysis as simple as possible, this is modeled by assuming that government expenditures are proportional to the stock of capital (cf. Lucas (1990, p. 303)).

4 The welfare effects of fiscal policy

In this section we analyze the welfare effects of fiscal policy using numerical simulation experiments. The experiments start in the steady state as described in Table 1. Then the effect of a change in fiscal policy at time $t = 0$ on (remaining) lifetime utility of the generations living at that time can be described by:

$$\frac{dU(0,s)}{d\tau} = \frac{dU_r(0)}{d\tau} + \frac{dU_w(0)}{d\tau} + \frac{1}{(\pi+\rho)v(0,s)}\frac{dv(0,s)}{d\tau}, \; s \leq 0, \tag{16}$$

where $U_w(t)$ denotes the effect of the timepath of the wage rate on utility and $U_r(t)$ stands for the effect of the timepath of the interest rate (given a certain level of wealth).[12] As the amount of human capital as well as the amount of physical capital is predetermined, the effect of fiscal policy on wealth at $t = 0$ consists of a capital gain (or loss) on human capital (i.e. a jump in the value of human capital q) and a change in the value of transfers:

$$\frac{dv(0,s)}{d\tau} = \chi h(0) e^{(\nu - \phi(l^h))s}\frac{dq(0)}{d\tau} + \frac{dg(0)}{d\tau}. \tag{17}$$

Notice that it follows from eqs. (8) and (17) that the amount of human capital accumulated by an individual is independent of his age if $\chi = 1$. In case $\chi > 1$ (< 1) human capital of older individuals is lower (higher) than that of younger individuals. Notice further that the growth rate of an individual's financial wealth is larger than the growth rate of his human wealth so that the share of financial wealth in his portfolio increases with age. Consequently, for extremely old generations the effect of changes in individual wealth due to changes in the value of human capital and the value of transfers is negligible. The effect on their utility is thus completely determined by the effect of changes in factor prices.

The effect of changes in fiscal policy on lifetime utility of generations born after the shock is given by:

$$\frac{dU(s,s)}{d\tau} = \frac{dU_r(s)}{d\tau} + \frac{dU_w(s)}{d\tau} + \frac{1}{(\pi+\rho)v(s,s)}\frac{dv(s,s)}{d\tau}, \; s > 0, \tag{18}$$

where $\frac{dv(s,s)}{d\tau} = \chi \frac{dj(s)}{d\tau} + \frac{dg(s)}{d\tau}$.

[12] For the definition of U_w and U_r see Appendix C.

	current generations		future generations	
	old	young	near	distant
τ_h	+	-/+	-/+	+
τ_k	-	-	-	+
τ_l	+	+/-	+/-	+/-

Table 2: The welfare effects of fiscal policy

We now discuss the effects of the introduction of a subsidy on human capital and an increase in the tax on capital income and labor income, respectively. All shocks are unanticipated and permanent. Table 2 summarizes the effects on the welfare of the different generations.[13]

INSERT: FIGURES 1A AND 1B

A subsidy on human capital

Figures 1A and 1B[14] display the main short-run effects of an increase in the subsidy on human capital from 0 to 0.10 at $t = 0$. The subsidy leads to a reallocation of time from working and leisure to learning. As a consequence, in the short run, i.e. when the stocks of human capital and physical capital are not yet adapted, the effects on factor prices are exactly opposite to the long-run effects described above: the wage rate initially rises and the interest rate initially falls. The initial net effect of the change in factor prices on utility $(U_r + U_w)$ is positive. Moreover, the value of human capital (j) jumps upward. Consequently, existing generations realize a 'human capital gain' (i.e. q jumps upward). These positive effects are counteracted by a negative effect on the value of transfers (g). For the oldest generations this negative effect is negligible compared to the positive effects. Therefore, their utility unambiguously rises. For the younger generations this is not true however, and the outcome depends on the value of χ. For the case depicted in figures 1A and 1B ($\chi = 1.25$) utility of younger generations also

[13] A "+" indicates that welfare rises unambiguously and a "-" denotes an unambiguous decrease in utility. In case of an "+/-" or an "-/+" the former symbol refers to the effect for low values of χ, the latter to that for high values of χ.

[14] The parameter values used are the same as the ones used for the calibration of the comparative statics results (see Table 1). j and g indicate the change in average human wealth and transfer wealth respectively, relative to $(\rho + \pi)$ times total wealth on the timepath without a shock (see eqs. (17) and (18))

rises due to the introduction of the subsidy. As the growth rate as well as the effect of factor prices on utility increases in the course of time, the same is true for all generations born after the shock. So in this case, introduction of a subsidy on human capital leads to a Pareto improvement.

The reason that the introduction of a subsidy on human capital may lead to a Pareto improvement is that future generations' initial level of human capital is determined by the average level of human capital at the time they are born. This introduces a market failure in the model: the accumulation of human capital has an external effect that is not taken into account by individuals when they decide how much time to devote to learning. Therefore, without government intervention, growth is too low. This may be corrected through a subsidy on human capital that stimulates long-run growth. This resembles the conclusion of Saint-Paul (1992) who analyzes fiscal policy in a one-sector OLG endogenous growth model with an externality. In his model physical capital is the engine of growth and growth can be raised by a subsidy on interest income. In the model presented here growth is driven by the accumulation of human wealth and can be stimulated by subsidizing that. However, although the markets generate too low growth, introduction of a subsidy on human capital does not always lead to a Pareto improvement. For values of $\chi < 1$ (e.g. $\chi = 0.75$) the initial positive effect on human wealth has a smaller weight (see eqs. (17) and (18)) and generations born shortly before or shortly after the shock lose. As human wealth rises fast after the introduction of the subsidy generations born sufficiently long after the shock always gain.

INSERT: FIGURES 2A AND 2B

A tax on capital income

An increase in the tax on capital income ($\tau^k = 0.35$ instead of 0.25) initially decreases the (after tax) interest rate and increases the wage rate (see figures 2A and 2B). In the course of time, the ratio of human capital to physical capital rises and the burden of taxation is shifted to labor. That is, the after-tax interest rate almost recovers and the wage rate falls. Initially, the net effect of this evolution of factor prices on utility is negative. The value of transfers increases at all points in time due to the tax receipts that are rebated in a lump sum fashion. The initial effect on human wealth is also positive. That is, the price of human capital jumps upward. These effects do not outweigh the negative initial factor-price effect however, and all generations living at the time of the shock lose. Consequently, in a direct democracy where currently living generations vote on the tax level, a proposal to increase the tax on capital income will not be accepted.

The factor-price effect rises over time and eventually becomes positive. However, due to the decrease in wages, the price of human capital strongly falls over time. As a consequence of this, human wealth is far below its original level after a few periods. Therefore, generations born shortly after the shock also lose. Because the growth rate increases in the long, human wealth eventually exceeds its original level again and generations born sufficiently long after the shock gain. Since the effect on growth is very small, this takes a very long time, however. As can be seen from the figure, generations born fifty years after the shock still have lower utility.

INSERT: FIGURES 3A AND 3B

A tax on labor income

Figure 3 presents the effects of an increase of the tax on labor income ($\tau^l = 0.35$ instead of 0.25). The (pre-tax) wage rate initially rises, though the rise is not large enough to prevent a decrease in the after tax wage rate. In the course of time, the wage rate decreases to a level slightly below its original level. The evolution of the interest rate is exactly opposite: it initially falls, but it rises over time. The net effect of the change in factor prices on utility is positive at all points in time. Consequently, utility of the oldest generations increases if the tax on labor income rises. Transfer wealth also increases at all points in time due to this tax increase. Human wealth initially strongly falls, however. For the case depicted in figures 3A and 3B ($\chi = 1.25$) this negative effect dominates the positive effects for generations that are young at the time of the shock so that these generations lose. Consequently, in a direct democracy the result of a vote on a proposal to increase the tax on labor income is ambiguous. Whether such a proposal is accepted or not depends on the effect on the utility of the median voter, which may be positive but may also be negative. As in this case the growth rate falls, all generations born after the shock lose if the proposal is accepted.

For lower values of χ the negative effect of the fall in human wealth gets a smaller weight and may be dominated by the positive effect of factor prices and transfers. If that is the case, all generations currently alive as well those born shortly after the shock gain from the tax increase. Moreover, if $\chi < 1$ the long-run growth rate increases and human wealth eventually rises above its original level. Therefore, in that case also generations born in the distant future will gain from the tax increase. Consequently, for low values of χ an increase in the tax on labor income is a Pareto improvement.

5 Optimal fiscal policy

In this section it is assumed that the government maximizes the following social welfare function (see Calvo and Obstfeld, 1988):

$$W(t) = \int_t^{\infty} U(q,q)e^{\hat{\rho}(t-q)}\,dq + \int_{-\infty}^{t} U(t,q)e^{(\rho+\pi)(q-t)}e^{\hat{\rho}(t-q)}\,dq \qquad (19)$$

So it is assumed that the government's objective is to maximize the weighted sum of the lifetime expected utility of representative agents from each of the generations to be born and the expected utilities over the rest of their lifetime of representative agents of those cohorts currently alive. In order to ensure time consistency, for all generations this is measured from the perspective of their date of birth. Alternatively this social welfare function can be written as:

$$W(t) = \int_t^{\infty} \left\{ \int_{-\infty}^{p} u(c(q,p), z(q,p))e^{(\rho+\pi-\hat{\rho})(p-q)}dq \right\} e^{-\rho(p-t)}dp \qquad (20)$$

That is, the social welfare flow at time p is the integral over all cohorts of instantaneous utilities discounted by the private discount factor minus the government's discount factor. The integral over all future p of these welfare flows discounted at rate ρ equals $W(t)$. So, W can be interpreted as a discounted sum of static Benthamite social welfare functions.

In the sequel of this section we first derive the command optimum. Then the question wether the government can realize this optimal plan in a decentralized economy is analyzed.

The command optimum

The first-order conditions for the command optimum can be derived along the lines set out by Calvo and Obstfeld. That is, a two-step procedure can be applied. In the first step, the government solves the static problem given aggregate consumption of goods and leisure. This leads to the following first-order conditions:

$$\frac{\partial c(q,t)}{\partial q} = c(q,t)(\rho - \hat{\rho}), \qquad (21)$$

$$\frac{\partial z(q,t)}{\partial q} = z(q,t)(\rho - \hat{\rho}). \tag{22}$$

These conditions show that an egalitarian plan under which all individuals have the same consumption at any point in time results if the government discounts utility of generations at a rate equal to the rate of time preference ($\hat{\rho} = \rho$). If the government uses a higher (lower) discount rate, consumption of older cohorts exceeds (falls short of) that of younger ones at any point in time.

In the second step the government chooses the aggregate consumption path $\{c(p), z(p)\}_{p=t}^{\infty}$ that maximizes welfare function (20) subject to the resource constraints and the accumulation functions for human capital and physical capital. The first order-conditions for this problem are derived in Appendix D.

The decentralized economy

Can the government realize the optimal plan in a decentralized economy? The answer to this question is yes, provided that the government can use a sufficiently rich set of taxes and transfers. As shown by Calvo and Obstfeld (1988), the government should be able to levy (pay out) taxes (transfers) that depend on an agent's vintage as well as on calendar time. Moreover, in the model presented here, the government also needs a subsidy on human capital. This can easily be seen by comparing the first-order conditions for the second stage of the planning problem with the first-order conditions for private sector (see Appendix A and Appendix D). These conditions are perfectly analogous if $\tau^l = \tau^k = 0$ and $\tau^h = \frac{\pi\chi}{\phi'(l^h(t))}$. The reason why the government needs this additional instrument is the external effect caused by the OLG-structure: when deciding how many hours to learn individuals do not take into account that the wealth of new generations is determined by average human wealth. The larger this effect (the larger χ) and the larger the birth rate π, the higher the optimal subsidy.

Notice that at any point in time t the distribution of wealth over generations living at that time follows from eqs. (21) and (22):

$$\frac{\partial v(q,t)}{\partial q} = v(q,t)(\rho - \hat{\rho}). \tag{23}$$

Moreover, we know that individual wealth grows over time at a rate $r - \rho$ (see eq. (4)) while average wealth grows at this individual growth rate minus

$\pi(1-\frac{v(t,t)}{v(t)})$ (see eq. (9)). From this we can derive the relation between average wealth at some point in time and the wealth of an individual born at that time:

$$v(t,t)=\frac{\rho-\hat{\rho}+\pi}{\pi}v(t). \tag{24}$$

So, if $\rho=\hat{\rho}$, the new born individual starts with average wealth while initial wealth is lower than (exceeds) average wealth if the government discount factor $\hat{\rho}$ is larger (smaller) than the consumer's rate of time preference ρ. It follows that the value of transfers at the time of birth of an individual is equal to

$$g(t,t)=[\frac{\rho-\hat{\rho}+\pi}{\pi}-\chi J(t)]v(t). \tag{25}$$

Note that this may be positive, i.e. there may be transfers from the old to the young, but that it may also be negative. In the latter case, there are transfers from the young to the old.

It should further be noted that eq. (24) implies that $\nu^{v}=r-\hat{\rho}$. That is, optimal behavior of the economy's averages is described by the standard Euler equation that is well known from the Cass-Koopmans-Ramsey optimal growth analysis. The relevant discount rate is, however, the rate at which the government discounts the utility of future generations instead of the private rate of time preference.[15]

6 Concluding remarks

This paper analyzed the effects of fiscal policy in a model which is a synthesis of the Blanchard-Yaari infinite-horizon OLG-model and the Lucas-Uzawa model of endogenous growth. Labor supply was endogenized using so called Becker-preferences. It was shown that a tax on capital income increases long-run growth and that the same may hold for a tax on labor income. The former result can be viewed as a generalization of Bertola (1996). The fact that a tax on labor income may also increase growth is less familiar. This result hinges on three factors. Firstly, the OLG-structure is essential because that implies that the growth rate depends on the average composition of wealth. Secondly, the assumption of a strong substitution of (relatively labor intensive) informal production ('leisure') for formal production when net wages fall is vital, as this causes a shift in the wealth portfolio from

[15] See Calvo and Obstfeld (1988).

human wealth to financial wealth. This may be a reasonable assumption for many countries, especially in the long run. Finally, the stock of human capital of new-born generations should not be too high compared to the average level of human capital. In particular, a sufficient condition for an increase in the tax on labor income to stimulate growth is that the initial stock of human capital does not exceed the average stock.

Numerical simulations showed that, for reasonable parameter values, the effect of fiscal policy on long-run growth is quite modest. So tax policy is not a very effective instrument to affect long-run growth. This conclusion is in line with the results of Mendoza et al. (1997). A subsidy on human capital is much more effective in stimulating growth.

In order to analyze the welfare effects of fiscal policy, it is not sufficient to look at the effects on long-run growth. The effect of changes in taxes or subsidies on the welfare of the different generations during the period of transition to the new balanced growth path should also be taken into account. This was done using numerical simulation experiments. These experiments learned that an increase in the tax on capital income harms most generations. Only generations born in the distant future see their welfare increase. Therefore, in a direct democracy where currently living generations vote on the tax level, such an tax increase will not be accepted. In contrast, the outcome of a vote on a proposal to increase the tax on labor income is ambiguous. The reason for this is that old generations gain while young generations may lose. However, if the level of human capital of new-born generations is lower than the average level raising the tax in labor income may be Pareto improving. That is, all current as well as future generations may gain from it. The pattern of welfare effects that arise when a subsidy on human capital is introduced is also dependent on the initial level of human capital. Generations born shortly before or after the shock may lose when new generations start with relatively large levels of human capital. Introduction of such a subsidy leads to a Pareto improvement however, if the initial level of human capital is larger than the average level. The later result can be viewed as a generalization of the result of Saint-Paul (1992) to a two-sector growth model.

After this positive analysis a normative analysis was presented. It was shown that when a social welfare function as introduced by Calvo and Obstfeld (1988) is to be maximized, a subsidy on human capital is a necessary instrument in order to be able to replicate the command optimum in a market economy. This is due to the fact that new generations start with a level of human capital proportional to the average, which causes an external effect. Taxes on capital income or labor income are not used in an optimal policy however, provided the government is able to use time and age dependent lump-sum transfers. In reality, the government does not have such a rich set of instruments. In that case, there may be a role for taxes on labor and capital income. It would be interesting to see what the optimal policy with a more limited set of instruments would be, but this may be a rather difficult question to answer.

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Appendix A: The consumer's optimization problem

In this appendix the first-order conditions for the consumer's optimization problem are presented. The optimization problem is:

$$\begin{aligned} \underset{c,l^f,l^h}{max} \quad U(t,s) &= \int_t^{\infty} u\left(c(p,s), z(p,s)\right) e^{(\rho+\pi)(t-p)}\, dp, \\ s.t.\ u\left(c(t,s), z(t,s)\right) &= \ln\{[(1-\alpha)c(t,s)^{\frac{(\sigma-1)}{\sigma}} + \alpha z(t,s)^{\frac{(\sigma-1)}{\sigma}}]^{\frac{\sigma}{(\sigma-1)}}\}, \\ z(t,s) &= [1 - l^h(t,s) - l^f(t,s)]h(t,s), \\ \dot{v}^f(t,s) &= [r(t)+\pi]v^f(t,s) - c(t,s) + w^n(t)l^f(t,s)h(t,s) \\ &\quad + \tau^h w^n(t)h(t,s) + b(t), \\ \dot{h}(t,s) &= \phi(l^h(t,s))h(t,s). \end{aligned}$$

The first-order conditions for this problem are:

$$u_c(t,s) = \theta^1(t,s),$$

$$u_z(t,s) = \theta^2(t,s)\phi'(l^h(t,s)),$$

$$\theta^1(t,s)w^n(t) = \theta^2(t,s)\phi'(l^h(t,s)),$$

$$\dot{\theta}^1(t,s) = \theta^1(t,s)(\rho - r(t)),$$

$$\dot{\theta}^2(t,s) = [\rho + \pi - \phi(l^h(t,s)) - (1 - l^h(t,s) + \tau^h)\phi'(l^h(t,s))]\theta^2(t,s),$$

where $\theta^1$and θ^2 denote the shadow prices of physical capital and human capital, respectively. Define $q(t,s)$ as the marginal value of human capital in terms of consumption goods:

$$q(t,s) \equiv \frac{\theta^2(t,s)}{\theta^1(t,s)} = \frac{w^n(t)}{\phi'(l^h(t,s))},$$

where $w^n \equiv (1-\tau^l)w$ denotes the real after-tax wage rate per efficiency unit and τ^h a subsidy on human capital[16]. From the first-order conditions it follows that:

$$\dot{q}(t,s) = [r(t) + \pi - \phi(l^h(t,s)) - (1 - l^h(t,s) + \tau^h)\phi'(l^h(t,s))]q(t,s).$$

[16]For convenience, this subsidy is defined per unit of income from human capital. So the subsidy for an indiviual is $\tau^h w^n(t)h(t,s)$

It can easily be shown[17] that the marginal value of human capital equals the average value of human wealth, i.e. $q(t,s) = \frac{j(t,s)}{h(t,s)}$.

Let $x(t,s)$ be the value at time t of a variable for a consumer born at time s. The corresponding aggregate or average variable is then defined as $x(t) \equiv \int_{-\infty}^{t} x(t,q)\pi e^{(q-t)\pi}\, dq$.

Appendix B: Comparative statics

In this appendix we present the most important results of the comparative statics analysis.

A subsidy on human capital

By differentiating eqs. (1), (2), (11), (12) and (13), it can easily be derived that:

$$\frac{d\, l^h}{d\, \tau^h} = \frac{1+\Psi}{\Theta} > 0,$$

where $\Psi = \pi K[\frac{1}{\epsilon(f(\ell)-\nu)} - \frac{1-\tau^l}{\ell(1+\lambda)}\frac{d\,\lambda}{w^n}] > 0$ and
$\Theta = 1 - (1+\Psi)\frac{\phi''(l^h)}{\phi'(l^h)}(1 - l^h + \tau^h) + \frac{\pi K}{f(\ell)-\nu} > 0$. Furthermore this gives:

$$\frac{d\,\nu}{d\,\tau^h} = \frac{d\, l^h}{d\,\tau^h}\phi'(l^h) > 0,$$

$$\frac{d\, r}{d\,\tau^h} = \frac{\phi'(l^h)(1 + \frac{\pi K}{f(\ell)-\nu})}{\Theta} > 0,$$

$$\frac{d\, K}{d\,\tau^h} = \frac{\frac{\phi'(l^h)}{\pi}(\frac{\pi K}{f-\nu} - \Psi)}{\Theta} < 0.$$

A tax on capital income

The long-run effects of tax on capital income are given by:

$$\frac{d\,\nu}{d\,\tau^k} = \frac{\Psi r}{(1-\tau^k)\Theta} > 0,$$

[17] This result can be derived analogous to derivation of the well-known result that 'marginal Q' coincides with 'Tobin's (average) Q' in Hayashi (1982).

$$\frac{d\,l^h}{d\,\tau^k} = \frac{\frac{d\,\nu}{d\,\tau^k}}{\phi'(l^h)} > 0,$$

$$\frac{d\,r}{d\,\tau^k} = \frac{d\,\nu}{d\,\tau^k}(1 - l^h + t^h)\frac{\phi''(l^h)}{\phi'(l^h)} \leq 0,$$

$$\frac{d\,K}{d\,\tau^k} = \frac{\frac{d\,r}{d\,\tau^k} - \frac{d\,\nu}{d\,\tau^k}}{\pi} < 0.$$

A tax on labor income

The effect of a change in the tax in labor income on growth is:

$$\frac{d\,\nu}{d\,\tau^l} = \frac{\frac{\pi K w}{1+\lambda}\frac{d\,\lambda}{d\,w^n}}{\Theta}.$$

A tax shift from capital income to labor income

It is assumed that the the government budget is constant:

$$d[\tau^l \ell w + \tau^k(f(\ell) - \ell w)] = 0.$$

Moreover, it is assumed that learning technology is linear. This implies that the long-run interest rate does not change (see eq. (11)). Therefore, it follows that:

$$d\,\tau^k = -\frac{\ell w}{(f(\ell) - \ell w)(1+\zeta)}d\,\tau^\ell,$$

where $\zeta \equiv \frac{\tau^k}{(1-\tau^k)} + \frac{\tau^l(1-\epsilon)}{\epsilon(1-\tau^k)}$ denotes the effect of the tax shift on the tax base. Using this equation, it can easily be derived that:

$$\frac{d\,\nu}{d\,\tau^l} = \frac{-\pi K w}{(1+\lambda)\Theta(1+\zeta)}\left[(\tau^l + \zeta)\frac{d\lambda}{dw} + \frac{\ell(1+\lambda)}{\epsilon(f(l) - \nu)}\right].$$

Appendix C: The welfare effects of fiscal policy

The effect of a change in fiscal policy at time t=0 on remaining lifetime utility of the generations living at that time can be described by:

$$\frac{dU(0,s)}{d\tau} = \frac{dU_r(0)}{d\tau} + \frac{dU_w(0)}{d\tau} + \frac{1}{(\pi+\rho)v(0,s)}\frac{dv(0,s)}{d\tau},\ s \leq 0,$$

where:

$$U_w(t) \equiv \int_t^{\infty} \{\ln[\frac{\rho+\pi}{1+\lambda(s)}] + \frac{\sigma}{\sigma-1}\ln[1-\alpha+\alpha\frac{(\lambda(s)}{w^n(s))}^{\frac{\sigma-1}{\sigma}}]\}e^{-(\rho+\pi)(s-t)}ds,$$

denotes the effect of the timepath of wages on utility for a given level of wealth and:

$$U_r(t) \equiv \int_t^{\infty}\int_t^{s}(r(p)-\rho)dp\, e^{-(\pi+\rho)(s-t)}ds,$$

stands for the effect of the timepath of the interest rate on utility for a given level of wealth.

Appendix D: The command optimum

This appendix derives the first-order conditions for the second stage of the two-step procedure to derive the command optimum (see Section 5). Define the indirect utility function:

$$\hat{u}(c(t), z(t)) = \max_{\{c(q,t),z(q,t)\}_{q=-\infty}^{t}} \int_{-\infty}^{t} u(c(q,t), z(q,t))e^{(\rho+\pi-\hat{\rho})(p-q)}dq$$

subject to:

$$\int_{-\infty}^{t} c(q,t)n(t,q)\ dq \leq c(t),$$

$$\int_{-\infty}^{t} z(q,t)n(t,q)\ dq \leq z(t),$$

Then the second-stage planning problem becomes one of maximizing:

$$W(t) = \int_t^{\infty} \hat{u}(c(t), z(t))e^{-\hat{\rho}(p-t)}\ dp$$

subject to:

$$\begin{aligned}
\dot{k}(t) &= k(t)f(\ell(t)) - c(t), \\
\dot{h}(t) &= \phi(l^h(t))h(t) + \pi\chi - \pi, \\
z(t) + l(t) &= (1 - l^h(t))h(t), \\
\ell(t) &= l(t)/k(t).
\end{aligned}$$

The governmment has the instruments: aggregate consumption c, aggregate labor supply in efficiency units l and (aggregate) learning time[18] l^h. This leads to the first-order conditions:

$$\hat{u}_c(t) = \hat{\theta}^1(t),$$

$$\hat{u}_z(t) = \hat{\theta}^2(t)\phi'(l^h(t)),$$

$$\hat{u}_z(t) = \hat{\theta}^1(t)f_\ell(\ell(t)),$$

$$\hat{\theta}^1(t) = \hat{\theta}^1(t)[\hat{\rho} - f(\ell(t)) - \ell(t)f_\ell(\ell(t))],$$

$$\hat{\theta}^2(t) = \hat{\theta}^2(t)[\hat{\rho} - \phi(l^h(t)) - (1 - l^h(t))\phi'(l^h(t)) + (1-\chi)\pi],$$

where $\hat{\theta}^1$and $\hat{\theta}^2$ are teh shadowprices of physical capital and human capital, respectively. The conditions for the first stage and the second stage of the optimization procedure can be linked by noting that in the optimum:

$$u_c(c(t,t), z(t,t)) = \hat{u}_c(c(t), z(t)),$$

$$u_z(c(t,t), z(t,t)) = \hat{u}_z(c(t), z(t)).$$

[18] As all individuals learn the same number of hours and the population size is normalized to 1, aggregate and individual learning time coincide.

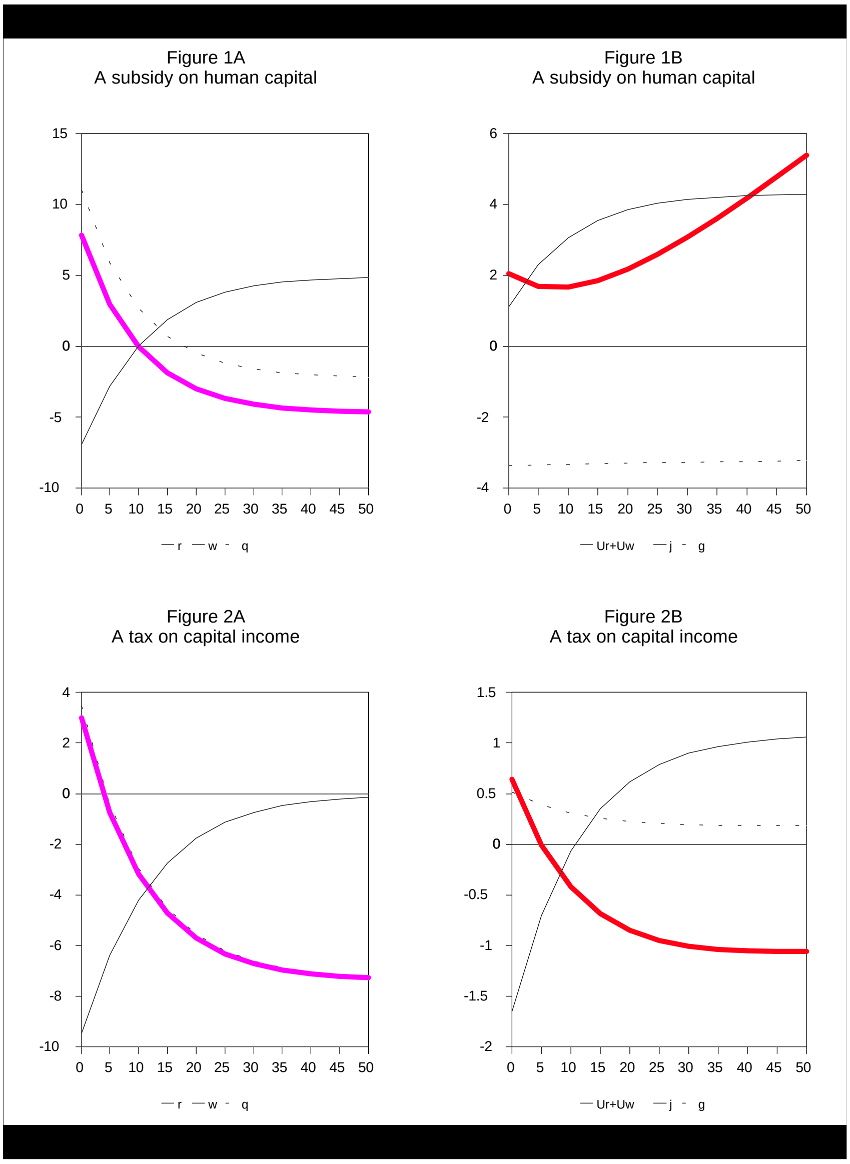

Figure 1A
A subsidy on human capital
r w q
Figure 1B
A subsidy on human capital
Ur+Uw j g
Figure 2A
A tax on capital income
r w q
Figure 2B
A tax on capital income
Ur+Uw j g

Figure 1:

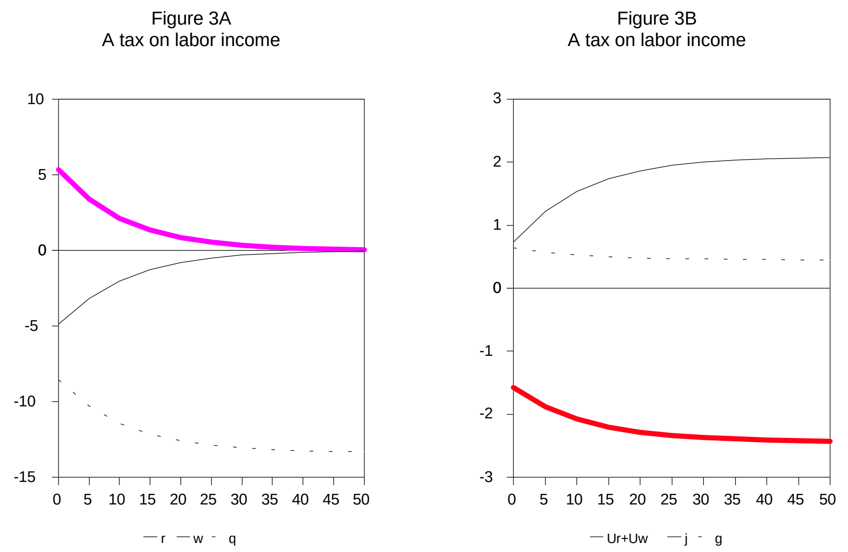

Figure 3A
A tax on labor income
10
5
0
-5
-10
-15
0 5 10 15 20 25 30 35 40 45 50
r w q
Figure 3B
A tax on labor income
3
2
1
0
-1
-2
-3
0 5 10 15 20 25 30 35 40 45 50
Ur+Uw j g

Figure 2: