

REASONING ABOUT THE TRINITY: A MODERN FORMALIZATION OF A MEDIEVAL SYSTEM OF TRINITARIAN LOGIC

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In Christian discourse, paralogisms arise when one blindly applies classical logic to the traditional definition of the trinity. Some people conclude, on the basis of these paralogisms, that logic cannot be coherently applied to the trinity. Others conclude that it can, but that the right logic is not classical logic, but an extension of it. An anonymous treatise from the late Middle Ages addresses the problems of paralogisms by introducing a semi-formal theory of predication and syllogistic reasoning which can be applied to both the trinity and to other subject matter. In this paper we formalize the theory presented in the medieval text, providing the contemporary philosopher and logician with a sound logic for reasoning about the trinity.

1. Introduction

For many people, the phrase “logic in religious discourse” will bring to mind the paralogisms of the trinity, the various syllogistic arguments which have apparently true premises and valid structure, but which result in an apparently false conclusion. For example:

*The Father is God.
God is the Son.*

Therefore, the Father is the Son.

For both believers and non-believers, paralogisms like this pose a significant problem for the rational interpretation of Christian doctrine of the trinity. Many people draw the conclusion that the trinity is one part of religious discourse where there is no logic, where ordinary rules of inference simply do not apply. Others disagree and believe that the paralogisms are simply that — paralogisms — and that once the correct logic underlying trinitarian reasoning is isolated, the paralogisms will no longer appear to be valid. People in the latter camp can be divided into two types, those that believe that logic can be applied to the nature of God, but think that the appropriate logic is not the same logic as that used to reason about non-divine things, and those that believe that one and the same logic can be used for both reasoning about God and about non-divine things.

An example of someone in the first camp is the 14th-century French logician Jean Buridan, who says in Book III, Part I, ch. 4 of his book on consequences:

But it should be carefully noted that these rules do not hold in the case of God, [the terms for Whom] supposit for a simple thing one and triune at the same time. Whence although the Father is the same as the simple God and the Son is the same as the simple God, the Father is nevertheless not the Son; and although the same Father is God and not the Son, it is false nevertheless that the Son is not the same as God [3, 3.4.8, p. 265].¹

More interesting are those in the other camp, and believe that not only can logic be applied to the trinity, but that it is the *very same* logic that we use in ordinary reasoning. This is the view of the anonymous author of a logical treatise *De modo predicandi ac syllogizandi (DMPS)* contained in MS. Munich, Bayerische Staatsbibliothek, lat. 17290, ff. 136r – 145v and edited in [7]², which discusses

¹*Sed diligenter aduertendum est quod hae regulae non tenent in terminis diuinis, qui supponunt pro re una simplicissima simul et trina. Vnde licet deo simplici sit idem pater et eidem deo sit idem filius, tamen filius non est pater; et licet idem pater sit deus et non filius, tamen falsum est quod filius deo non sit idem* [2, p. 85].

² Few details about the authorship or localization of the text are known. Because Thomas Aquinas is referred to as a saint, the text was almost certainly written after his canonization

modes of predication and syllogistic reasoning in the trinity. In this text, the author argues that the same logic can be used to reason about both divine and created things by making a distinction between different modes of being and modes of predication, distinctions which collapse when we talk about created things. He supports this conclusion by presenting a syllogistic logic which is adequate for reasoning about the trinity and from which ordinary, Aristotelian syllogistics can be recovered. (Interestingly, this author is often classed with people who fall in the former category, that is, the category of people who believe that logic can be applied to God but don't accept a single logic (see, e.g., [4, p. 86]).)

In this paper we give a formal reconstruction of the trinitarian syllogistic theory presented in the anonymous text, and show how it can be used to explain why traditional paralogsms appear to be valid but are in fact invalid.

Chapter 7 of [10] is an expanded version of this paper. The technical details omitted from this paper can be found there.

2. The text

The text can be divided into three main parts, each of which builds upon the previous one:

1. A discussion of modes of being.
2. A discussion of modes of predication.
3. A discussion of syllogistic reasoning.

According to the author, the first of these is properly within the scope of philosophy (or, when it concerns the trinity, theology); the latter two make up the scope of logic.

in 1323. On the basis of other textual and conceptual references, a composition date of the late 14th or early 15th century can be postulated, possibly in a Germanic setting. For further discussion of this, see Maierù's introduction [7, pp. 251, 255 – 257]. This manuscript is the only known manuscript containing this text. The text is, unfortunately, incomplete; in *DMPS*, par. 106, p. 286 an objection is introduced, and the text breaks off directly after, leaving the objection unaddressed. A translation of this text into English is given in Appendix C of [10]. Chapter 7 of [10] is an expanded version of present paper, and the technical details omitted from this paper can be found there. All Latin references are taken from [7], and the English translations from [10].

The modes of being which can be found in the trinity are discussed in *DMPS*, paragraphs 6 – 24, p. 266 – 269; in *DMPS*, par. 25, p. 269, the author notes that, modes of being having been spoken of, we can now move to a discussion of modes of predication and syllogistic reasoning, for, as he says, “In truth logic, in so far as it suffices for the present purpose, consists in modes of predication and syllogistic reasoning.”³ Because predications are predications in some mode of being, before logic proper is discussed it is first required that the philosophical issues of modes of being be covered. Speaking anachronistically, we can say that the first 24 paragraphs were setting up the semantics of our system, explaining the underlying factors which will make certain predications true or false, and that starting in *DMPS*, par. 25 we are now being given syntax. Facts about generating modes of predication from the modes of being are discussed in *DMPS*, par. 25 – 32, pp. 269 – 270, and the discussion of syllogisms, which makes up the rest of the text, begins in *DMPS*, par. 33, p. 271. In presenting his syllogistic system, our author uses of two typically medieval developments in logic: supposition theory and expository syllogisms. From standardly accepted facts about the supposition of terms and the reduction of certain classes of general syllogisms to expository syllogisms, the author is able to isolate a class of divine syllogisms which are valid, and to justify their validity. Rules governing the validity of categorical syllogisms with mixed premises are given in *DMPS*, par. 51, p. 275 (for affirmative syllogisms) and *DMPS*, par. 57 – 60, p. 277 (for negative syllogisms). After a discussion of how these rules relate to expository syllogisms, the author summarizes the class of valid syllogisms which have two positive premises in *DMPS*, par. 93 – 96, pp. 284 – 295, and the class of valid syllogisms which have a negative premise in *DMPS*, par. 98 – 105, pp. 285 – 286. Unfortunately, *DMPS*, par. 106, p. 286 provides a counterexample to the system which has just been outlined, and as the text breaks off we are left with no indication as to how the author would have resolved this problem.

The central argument of the text is that in order to properly reason about the nature of the trinity, we must distinguish three different modes of being and predication. When the author discusses the different modes of being of an object (divine or created), it should be understood that what he is speaking of is more properly called modes of identity, that is, different ways that two objects can be

³[I]ogica vero, quantum ad propositum sufficit, in modis predicandi ac sylogizandi consistit.

identical. He never speaks of an object simply *existing* in one of these modes of being, but rather of one object *being the same as* another object in one of these modes of being. The author distinguishes three modes of being, that is, three ways in which two things can be identical with each other:

- Essentially (*Essencialiter*)
- Personally or Identically (*Personaliter/Ydemptice*)
- Formally or Properly (*Formaliter/Proprie*)

Roughly speaking, two things are essentially the same if they share the same essence; but things which are essentially the same may still yet differ in the accidental properties that they share or in the definitions which define them. This distinction of types of identity can be found as early as Abelard. (For further discussion of Abelard's views, see [5], especially p. 242.) In his *Theologica 'scholarium'* II, 95 – 99, Abelard distinguishes three ways that things can be the same [1, pp. 454 – 456]:

- Essentially or in number (*Essentialiter siue numero*)
- Properly or by definition (*Proprietate seu diffinitione*)
- In likeness (*Similitudine*)

Abelard's three ways of being the same correspond to the three modes of being in the anonymous text we're considering.⁴ Abelard's essential identity is also called *idem quod* sameness, and Knuuttila glosses it as “[t]he sameness pertaining to the subject and predicate of a singular proposition in the sense that there is a third of which both are said.” This is distinguished from *idem qui* sameness, glossed as “the sameness between the meanings of terms.” This *idem qui* sameness covers both personal and formal (or proper) identity [6, p. 193]. Basically, if two things are essentially identical, then they share the same essence. If they are personally identical, then they share the same properties and definitions. Finally, if two things are formally identical, then they share sufficient similarity that they can be placed under the same genus, or form. (In *DMPS*, par. 32, p. 270 the author says that there “is a certain mode of being in which some things are formally the same, on the condition that in whatever way one

⁴As Knuuttila notes, “The originally Abelardian distinction between intensional (personal) and extensional (essential) identity was widely employed in later medieval Trinitarian theology and influenced late medieval logic” [6, p. 195].

is the other is also in the same way” (*est quidam modus essendi quo aliqua sunt formaliter idem, ita quod in quocumque est unum in eodem est et alterum*). It is not clear whether this condition is a sufficient or necessary condition for two things being formally identical.)

The author’s system of divine syllogistics is based on distinguishing these three modes of being. In created beings, these distinctions collapse, which explains why ordinary, Aristotelian syllogistics works as well as it does, and why for so long no one realized that there was more to the story than that. (At the beginning of the text, in *DMPS*, par. 1, p. 265, the author apologizes for Aristotle, noting that because Aristotle’s focus was on the mode of being as it is found in created things, and hence his syllogistic system, which is based on predications expressing that mode of being, does not accommodate reasoning about non-created, i.e., divine things, we cannot fault him for not recognizing that his system could be extended to accommodate reasoning about the divine nature.) So what exactly do we mean when we speak of the trinity, or the divine nature, in the context of discussing this anonymous text? The author makes as few controversial assumptions about the nature of the trinity as possible. In *DMPS*, par. 4, pp. 265 – 266, the author says that:

The mode of being in divinity is that three persons are one most simple essence and likewise the most simple essence [is] three persons and each of them.⁵

This view is essentially a compressed version of the Athanasian Creed, adopted in the 6th century:

We worship one God in Trinity, and Trinity in Unity; neither confounding the Persons: nor dividing the Substance. . . . But the Godhead of the Father, of the Son, and of the Holy Ghost, is all one: the Glory equal, the Majesty coeternal. . . . The Father eternal: the Son eternal: and the Holy Ghost eternal. And yet they are not three eternals: but one eternal.⁶

⁵*Modus essendi in divinis est quod tres persone sunt una essentia simplicissima et eadem simplicissima essentia tres persone et quolibet earum* (*DMPS*, par. 4, pp. 265 – 266).

⁶*Unum Deum in Trinitate, et Trinitatem in Unitate veneremur; neque confundentes personas: neque substantiam separantes. . . . Sed Patris et Filii et Spiritus Sancti una est divinitas: aequalis gloria, coaeterna majestas. . . . Aeternus Pater: aeternus Filius: aeternus [et] Spiritus Sanctus. Et tamen non tres aeterni: sed unus aeternus.*

The examples that the author uses when discussing the trinity mention eight different persons or properties of the trinity, for which we introduce notation now:

E := Essence	Su := Substance
F := Father	P := Fatherhood/Paternity
S := Son	Wi := Wisdom
HS := Holy spirit	C := Charity/Love

P, Wi, and C are called by the author “personal properties” (*DMPS*, par. 23 – 24, pp. 268 – 269), following Peter Lombard. These properties are the distinguishing properties of the persons of the Father, the Son, and the Holy Spirit, respectively. (The essence also has essential attributes, namely *sapientia* *<et>* *essencialiter dicte, iusticia, bonitas, etc.* (*DMPS*, par. 19, p. 268). But we need not introduce new terms for these essential attributes, since they are all formally identical with the essence (*DMPS*, par. 19, 32, pp. 268, 270 – 271), and the author makes no further mention of them.)

The comparison of the three modes of being used in this text with Abelard’s three modes of identity gives us some idea of what is meant when it is said that two objects are personally the same, or that they are formally distinct, but it does not give us information about the nature of the relationships ‘being essentially the same as,’ ‘being personally the same as,’ and ‘being formally the same as.’ No clear statement of the properties of these relations is given in the text, but we can extract some of them by looking at the examples of identities and distinctions that the author makes in *DMPS*, par. 5 – 24, pp. 266 – 269. A summary of these examples is given in Table 1 (note that some of the cells are not wholly filled in because the text is underspecific), where we let $=_e$, $=_p$, and $=_f$ be the relations of essentially identity, personal identity, and formal identity, respectively. Since essential identity is an equivalence class of which all parts of the trinity are members, we omit it from the table since it would appear in every cell.

3. The formal system

The formal system we present here was developed in order to be able to model reasoning within a particular natural language, namely medieval Latin as it was used by logicians, specifically by the logician who is the author of the text under

	E	F	S	HS	P	Wi	C	Su
E	$=_f$	$\neq_f, =_p$	$\neq_f, =_p$	$\neq_f, =_p$	\neq_f	\neq_f	\neq_f	\neq_f
F	$\neq_f, =_p$	$=_p, =_f$	\neq_p, \neq_f	\neq_p, \neq_f	$=_p, =_f$	\neq_f	\neq_f	\neq_f
S	$\neq_f, =_p$	\neq_p, \neq_f	$=_p, =_f$	\neq_p, \neq_f	\neq_f	$=_p, =_f$	\neq_f	\neq_f
HS	$\neq_f, =_p$	\neq_p, \neq_f	\neq_p, \neq_f	$=_p, =_f$	\neq_f	\neq_f	$=_p, =_f$	\neq_f
P	\neq_f	$=_p, =_f$	\neq_f	\neq_f	$=_p, =_f$	\neq_p, \neq_f	\neq_p, \neq_f	\neq_f
Wi	\neq_f	\neq_f	$=_p, =_f$	\neq_f	\neq_p, \neq_f	$=_p, =_f$	\neq_p, \neq_f	\neq_f
C	\neq_f	\neq_f	\neq_f	$=_p, =_f$	\neq_p, \neq_f	\neq_p, \neq_f	$=_p, =_f$	\neq_f
Su	\neq_f	\neq_f	\neq_f	\neq_f	\neq_f	\neq_f	\neq_f	$=_f$

Table 1: Formal and personal identity in the trinity

consideration. This is the cause of certain otherwise non-standard modeling choices that we make. In particular, we have designed our system to deal with ambiguous natural language statements such as

Homo est animal.

Because Latin does not have an indefinite or definite article, this sentence is ambiguous between the reading *omnis homo est animal* and *quidam homo est animal*. When the sentence is literally translated into English, this ambiguity manifests itself in questionable grammar: “Man is animal.” A more natural translation would add definite or indefinite articles or quantifiers, e.g., ‘the essence is the father’ for *essencia est pater*, which adds two definite articles which are not present in the Latin. Another way that features of our formal model will be determined by features of Latin is in the use of context-dependent indexicals like *hoc* (‘this’). When we say things such as *haec tabula est viridis*, we are saying something more than ‘some table is green’ but something less than ‘all tables are green.’ We will introduce specific operators into our language to be able to deal with issues surrounding the use of indexicals in this manner.

3.1. Language and models

We use the language $\mathcal{L}_{\text{trin}}$ consisting of a set of terms \mathfrak{T} ; the relations $=_e, =_p, =_f$ and their negations \neq_e, \neq_p, \neq_f ; the functions $_{es}$ and $_{fs}$; the quantifiers $A, E,$ and $!$; and two punctuation symbols, $[$ and $]$. \mathfrak{T} contains all of $E, Su, F, S, HS, P, Wi,$ and $C,$ and potentially other terms, e.g., ‘man,’ ‘cat,’ ‘Socrates.’ We use t as a variable ranging over \mathfrak{T} , and we use $=_*$ as a meta-variable over $=_e, =_p,$

$=_f$ when we need to make statements about all three relations. $!t$ is to be read ‘this t ’ (English) or *hoc t* (Latin). This operator will be used in formalizing ambiguous natural language sentences such as the ones just discussed.

Traditional Aristotelian syllogistic logic is a term logic, not a predicate or propositional logic. This means that the formal system we develop will be neither a predicate nor a propositional logic, though, as we’ll see below, we will use predicate logic as a meta-logic when giving the truth conditions for formulas in models. Instead we will develop a logic whose basic constituent is the categorical proposition, though we will go a step beyond traditional medieval syllogistics by allowing boolean combinations of these categorical propositions. We begin by giving a definition of the set of *basic terms* and the set of *quantified terms* in our language:

Definition 1 (Terms). *The set $\mathbb{T}_{\text{trin}} = \mathbb{T}_{\text{trin}}^{\text{basic}} \cup \mathbb{T}_{\text{trin}}^{\text{quant}}$ is the set of terms of $\mathcal{L}_{\text{trin}}$ where*

- $\mathbb{T}_{\text{trin}}^{\text{basic}}$ is the set of basic terms of $\mathcal{L}_{\text{trin}}$, defined recursively as follows:
 - If $t \in \mathfrak{T}$, then $t, t_{\text{es}}, t_{\text{fs}} \in \mathbb{T}_{\text{trin}}^{\text{basic}}$.
 - If $t \in \mathbb{T}_{\text{trin}}^{\text{basic}}$, then $[t]_{=*} \in \mathbb{T}_{\text{trin}}^{\text{basic}}$.
 - Nothing else is in $\mathbb{T}_{\text{trin}}^{\text{basic}}$.

We call terms of the form $[t]_{=*}$ equivalence terms.

- $\mathbb{T}_{\text{trin}}^{\text{quant}}$ is the set of quantified terms of $\mathcal{L}_{\text{trin}}$ defined as follows:

$$\{\text{At} : t \in \mathbb{T}_{\text{trin}}^{\text{basic}}\} \cup \{\text{Et} : t \in \mathbb{T}_{\text{trin}}^{\text{basic}}\} \cup \{!t : t \in \mathbb{T}_{\text{trin}}^{\text{basic}}\}$$

If $t \in \mathbb{T}_{\text{trin}}^{\text{quant}}$ then we let t^- be the result of removing the quantifiers from the front of t .

Definition 2 (Categorical Propositions). *The set CAT_{trin} of categorical propositions of $\mathcal{L}_{\text{trin}}$ is defined as follows:*

- If $t, t' \in \mathbb{T}_{\text{trin}}$, then $t =_e t'$, $t =_p t'$, $t =_f t' \in \text{CAT}_{\text{trin}}$. We call categorical propositions of this type affirmative.
- If $t, t' \in \mathbb{T}_{\text{trin}}$, then $t \neq_e t'$, $t \neq_p t'$, $t \neq_f t' \in \text{CAT}_{\text{trin}}$. We call categorical propositions of this type negative.
- Nothing else is in CAT_{trin} .

Note that all categorical propositions are of the form $Qt =_* Q't'$ for terms t, t' and quantifiers (possibly null) Q, Q' . If ϕ is a categorical proposition, then we indicate the type of identity in ϕ by ϕ_* , and we call the term on the left-hand side of the identity sign the ‘subject’ and the term on the right-hand side the ‘predicate.’

Definition 3 (WFFs). *The set WFF_{trin} of well-formed formulas of $\mathcal{L}_{\text{trin}}$ is defined recursively:*

- If $\phi \in \text{CAT}_{\text{trin}}$, then $\phi \in \text{WFF}_{\text{trin}}$.
- If $\phi \in \text{WFF}_{\text{trin}}$, then $\neg\phi \in \text{WFF}_{\text{trin}}$.
- If $\phi, \psi \in \text{WFF}_{\text{trin}}$, then $\phi \wedge \psi, \phi \vee \psi, \phi \rightarrow \psi \in \text{WFF}_{\text{trin}}$.
- Nothing else is in WFF_{trin} .

In order to prove some of the theorems in section 3.2, we need to isolate a special class of terms called *divine terms*; we’ll use the distinction between divine and created (non-divine) terms in our proof.

Definition 4 (Divine Terms). *The set $\text{T}_{\text{div}} \subseteq \text{T}_{\text{trin}}$ of divine terms of $\mathcal{L}_{\text{trin}}$ is the set of all terms $t \in \text{T}_{\text{trin}}$ such that t only contains E, Su, F, S, HS, P, Wi, C and nothing else.*

We define the sets CAT_{div} and WFF_{div} from Definitions 4, 2, and 3 by replacing trin with div throughout.

Formulas gain meaning when they are interpreted in models.

Definition 5 (Trinitarian Models). *A structure*

$$\mathfrak{M}_{\text{trin}} = \langle O, I, \{t : t \in \mathfrak{T}\}, \dot{=}_e, \dot{=}_p, \dot{=}_f, \dot{=}_{\text{es}}, \dot{=}_{\text{fs}} \rangle$$

is a trinitarian model iff:

1. O is a set of objects such that $\dot{E}, \dot{\text{Su}}, \dot{F}, \dot{S}, \dot{\text{HS}}, \dot{P}, \dot{\text{Wi}}, \dot{C} \in O$. We use o, x, y, z , etc., as meta-variables ranging over O .
2. $I : \mathfrak{T} \rightarrow 2^O$ associates a set of objects with each term of \mathfrak{T} , such that $I(E) = \{\dot{E}\}$, $I(\text{Su}) = \{\dot{\text{Su}}\}$, $I(F) = \{\dot{F}\}$, $I(S) = \{\dot{S}\}$, $I(\text{HS}) = \{\dot{\text{HS}}\}$, $I(P) = \{\dot{P}\}$, $I(\text{Wi}) = \{\dot{\text{Wi}}\}$, and $I(C) = \{\dot{C}\}$. I can be extended to I' which covers equivalence terms: $I'([t]_{=_*}) = \{x \in O : \text{there is a } y \in I(t) \text{ and } x =_* y\}$.

3. $\dot{=}_e$ is a partial equivalence relation on O such that if $o \notin I(t)$ for all $t \in \mathbb{T}_{div}$, then for all $o' \in O$, $\langle o, o' \rangle \notin \dot{=}_e$ (that is, it is an equivalence relation on the interpretation of divine terms).
4. $\dot{=}_p$ is a partial symmetric relation on O satisfying the conditions in Table 1 such that if $o \notin I(t)$ for all $t \in \mathbb{T}_{div}$, then for all $o' \in O$, $\langle o, o' \rangle \notin \dot{=}_p$.
5. $\dot{=}_f$ is a equivalence relation on O satisfying the conditions in Table 1.
6. \dot{e}_s, \dot{f}_s are partial unary functions such that if $o \in I'([\dot{E}]_{=e})$ then $o_{\dot{e}_s} = \dot{E}$; if $o \in [\dot{F}]_{=f}$ then $o_{\dot{f}_s} = \dot{F}$; if $o \in [\dot{S}]_{=f}$ then $o_{\dot{f}_s} = \dot{S}$; if $o \in [\dot{HS}]_{=f}$ then $o_{\dot{f}_s} = \dot{HS}$; and undefined otherwise.

Conditions 3, 4, and 5 of Definition 5 capture the fact that when we are reasoning about non-divine things, we can only make formal predications. When explaining why essential and identical predications do not show up in Aristotelian syllogistics, our author notes that though the terminists and the realists may disagree about whether there are only formal identities between created objects, or whether there are also personal identities, nevertheless they agree that all predications are predications of formal identity:

And because in creation all predications are formal, because according to common opinion of the terminists all the things which are the same in creation are formally the same, therefore the mode of syllogizing through propositions concerning identical predications is not necessary in creation.

However, according to the mode of the realists, according to which not all things in creation which are the same are formally [the same], still all predications are formal, which is clear because what is not formally the same according to the realists, according to they themselves must necessarily be denied of each other if indeed they are identically the same.⁷

The author does not specify whether, in the case of created objects, we are able to state non-identities of the essential and personal type (that is, whether we can

⁷*Et quia in creaturis omnes predicaciones sunt formales, quia iuxta opinionem communem terministarum omnia que sunt idem in creaturis sunt formaliter idem, ideo non fuit necesse in creaturis modus sylogizandi per proposiciones de predicacione ydemptica.*

Secundum modum autem realistarum, secundum quem non omnia in creaturis que sunt idem sunt formaliter (idem), adhuc omnes predicaciones sunt formales, quod patet, quia que non sunt formaliter idem secundum realistas, secundum ipsos necessario negantur de semetipsis si etiam ydemptice sint idem (DMPS, par. 39 – 40, pp. 272 – 273).

say of two created objects o and o' that $o \neq_e o'$, and so forth). Because he does not say that it is possible to make statements of non-identities of these types when dealing with created objects (only that we cannot make predications of identities of these types), we have opted to not build into the system the ability to express these negated identities.

We now give the truth conditions of the members of WFF_{trin} in a trinitarian model. Boolean combinations of categorical propositions are as expected:

Definition 6 (Truth Conditions of Boolean Formulas).

$$\begin{aligned} \mathfrak{M} \models \neg\phi & \quad \text{iff } \mathfrak{M} \not\models \phi \\ \mathfrak{M} \models \phi \wedge \psi & \quad \text{iff } \mathfrak{M} \models \phi \text{ and } \mathfrak{M} \models \psi \\ \mathfrak{M} \models \phi \vee \psi & \quad \text{iff } \mathfrak{M} \models \phi \text{ or } \mathfrak{M} \models \psi \\ \mathfrak{M} \models \phi \rightarrow \psi & \quad \text{iff } \mathfrak{M} \models \neg\phi \text{ or } \mathfrak{M} \models \psi \end{aligned}$$

For the categorical statements, we correlate the quantifiers of $\mathcal{L}_{\text{trin}}$ with quantifiers in ordinary mathematical logic via an interpretation function int . Two of the quantifiers are standard — $\text{int}(A) = \forall$ and $\text{int}(E) = \exists$. As we noted earlier, indexical pronouns like ‘*hoc*,’ which we formalize with $!$, indicate something more than existence but something less than universality. Pronouns like ‘*hoc*’ are essentially context-dependent choice functions that, given a term, will pick out an appropriate witness for that term, given the context. We capture these two facts by interpreting $!$ with a generalized quantifier (cf. [8, 11]). For a term t , we indicate such a context-dependent choice function as $\chi_!(t)$, which means we can define $\text{int}(!)$ as $\{\{\chi_!(t)\}\}$ for appropriate t .

This leaves us with the empty quantifier, which shows up in formalizations of Latin sentences such as *essencia est pater* and *homo est animal*, which, as we noted above, are essentially ambiguous. Our author does not say how these sentences should be interpreted, but, given how his discussion of modes of being mirrors Abelard’s three ways of being identical, it’s reasonable that he would also subscribe to Abelard’s view of predication. Knuuttila summarizes Abelard’s view thus:

In his *Logica Ingredientibus* Abelard argues that the simple affirmative statement ‘A human being is white’ [*homo est albus*] should be analysed as claiming that that which is a human being is the same as that which is white (*idem quod est homo esse id quod album est*) [6, p. 192].

It is natural to read ‘that which is a human being’ universally, and ‘that which is white’ particularly. Thus, for statements of identity, involving $=_*$, we stipulate that the int of the empty quantifier of a subject is \forall , and the int of the empty quantifier of a predicate is \exists . For statements of non-identity, involving \neq_* , we stipulate that the int of the empty quantifier on either side of \neq_* is \forall . The difference in how the empty quantifier is treated when it appears in a predicate is a result of the distributive force of negation; see Definition 8 below.

Given these preliminaries, we can now give a uniform truth condition for categorical sentences:

Definition 7 (Truth Conditions of Categorical Formulas). *Let Q, Q' be (perhaps empty) quantifiers, and $t, t' \in \mathfrak{T}$. Then,*

$$\mathfrak{M} \models Qt =_* Q't' \text{ iff } \text{int}(Q)x \in I(t) (\text{int}(Q')y \in I(t') (\langle x, y \rangle \in \dot{=}_*))$$

We will see examples of these conditions in the next section when we discuss the formalization of natural language sentences concerning the trinity. Note that defining the truth conditions for the empty quantifiers in this way automatically deals with the issue of existential import, by allowing the inference, regularly accepted in the Middle Ages, from *omnis homo est mortalis* to *quidam homo est mortalis*, but not automatically allowing the inference, which is not so readily accepted by the medieval logicians (cf. [9, §1.2]), from *nullus homo est immortalis* to *quidam homo non est immortalis*, because $\mathfrak{M} \models At \neq_f t'$ when both $I(t) = \emptyset$ and $I(t') = \emptyset$.

3.2. Properties of the system

In this section we look at how the model presented in the previous section can be used to model the syllogistic theory presented in the anonymous text.

First, note that it doesn't really make sense to talk of axioms in the context of a syllogistic logic. This is because what is valid in a syllogistic logic is not sentences, but arguments, which means that the ‘axioms’ are simply rules for moving from two premises to a conclusion. In ordinary, non-divine, syllogistics, these rules are the perfect syllogisms, Barbara, Celarent, Darii, and Ferio. That is, for $t, t', t'' \notin \mathfrak{T}_{\text{div}}$:

Rule 1.

Barbara: If $\mathfrak{M} \models At' =_f t$ and $\mathfrak{M} \models At'' =_f t'$, then $\mathfrak{M} \models At'' =_f t$

Celarent: If $\mathfrak{M} \models At' \neq_f t$ and $\mathfrak{M} \models At'' =_f t'$, then $\mathfrak{M} \models At'' \neq_f t$

Darii: If $\mathfrak{M} \models At' =_f t$ and $\mathfrak{M} \models Et'' =_f t'$, then $\mathfrak{M} \models Et'' =_f t$

Ferio: If $\mathfrak{M} \models At' \neq_f t$ and $\mathfrak{M} \models Et'' =_f t'$, then $\mathfrak{M} \models Et'' \neq_f t$

The validity of the affirmative syllogisms, Barbara and Darii, are governed by the rule called by the *dici de omni* by the medieval logicians, and the validity of the negative syllogisms, Celarent and Ferio, by the rule *dici de nullo*:

Rule 2 (*Dici de omni*). Whenever some predicate is said of some distributed subject, then of whatever is said to be of that distributed subject, of the same thing indeed it is said to be of that predicate.⁸

Rule 3 (*Dici de nullo*). Whenever some predicate is denied of some distributed subject, then of whatever is said to be of that distributed subject, of the same thing indeed it is denied to be of that predicate.⁹

The admissibility of the *dici de omni et de nullo*, and consequently of the four perfect syllogisms, follows straightforwardly from the fact that $=_f$ is an equivalence relation:

Proof.

Barbara Assume $\mathfrak{M} \models At' =_f t$ and $\mathfrak{M} \models At'' =_f t'$. Then by Definition 7, the following two formulas hold:

$$\forall x \in I(t')(\exists y \in I(t)(\langle x, y \rangle \in \dot{=} _f)) \quad (1)$$

$$\forall z \in I(t'')(\exists w \in I(t')(\langle z, w \rangle \in \dot{=} _f)) \quad (2)$$

Take arbitrary $x \in I(t'')$. From (2) it follows that there is a $y \in I(t')$ such that $\langle x, y \rangle \in \dot{=} _f$. From (1), we know that there is some $z \in I(t)$ such that $\langle y, z \rangle \in \dot{=} _f$. Since $=_f$ is transitive, we can conclude that $\langle x, z \rangle \in \dot{=} _f$. Since x was arbitrary, we have shown that the following holds:

$$\forall x \in I(t'')(\exists z \in I(t)(\langle x, z \rangle \in \dot{=} _f)) \quad (3)$$

⁸*Quandocumque aliquod predicatum dicitur de aliquo subiecto distributo, tunc de quocumque dicitur tale subiectum distributum de eodem eciam dicitur tale predicatum* (DMPS, par. 36, p. 272).

⁹This rule is never explicitly stated by the author, but it would have been well-known to his audience.

and hence that $\mathfrak{M} \models At'' =_f t$.

Celarent Assume $\mathfrak{M} \models At' \neq_f t$ and $\mathfrak{M} \models At'' =_f t'$. Then by Definition 7, (1) for every $x \in I(t')$ and $y \in I(t)$, $\langle x, y \rangle \notin \dot{=}_f$, and (2) for every $z \in I(t'')$ there is a $w \in I(t')$ such that $\langle z, w \rangle \in \dot{=}_f$. Take arbitrary $x \in I(t'')$. By (2) there is some $y \in I(t')$ such that $\langle x, y \rangle \in \dot{=}_f$. By (1), for all $z \in I(t)$, $\langle y, z \rangle \notin \dot{=}_f$. Now, suppose that there is a $w \in I(t)$ such that $\langle x, w \rangle \in \dot{=}_f$. Since $\langle x, w \rangle \in \dot{=}_f$ and $\langle x, y \rangle \in \dot{=}_f$, by transitivity and symmetry of $=_f$, this means that $\langle y, w \rangle \in \dot{=}_f$, which is a contradiction. Since $x \in I(t'')$ was arbitrary, we can conclude that the following holds:

$$\forall x \in I(t'') (\forall y \in I(t) (\langle x, y \rangle \notin \dot{=}_f)) \quad (4)$$

and hence $\mathfrak{M} \models At'' \neq_f t$.

Darii Assume $\mathfrak{M} \models At' =_f t$ and $\mathfrak{M} \models Et'' =_f t'$. Then by Definition 7, (1) for every $x \in I(t')$ there is a $y \in I(t)$ such that $\langle x, y \rangle \in \dot{=}_f$, and (2) there is a $\hat{z} \in I(t'')$ and $w \in I(t')$ such that $\langle \hat{z}, w \rangle \in \dot{=}_f$. (1) and (2) together give immediately that there is a $y \in I(t)$ such that $\langle \hat{z}, y \rangle \in \dot{=}_f$, and hence there exists a $z \in I(t'')$ and a $y \in I(t)$ such that $\langle z, y \rangle \in \dot{=}_f$, which is the same as saying that $\mathfrak{M} \models Et'' =_f t$.

Ferio Assume $\mathfrak{M} \models At' \neq_f t$ and $\mathfrak{M} \models Et'' =_f t'$. Then by Definition 7, (1) for every $x \in I(t')$ and $y \in I(t)$, $\langle x, y \rangle \notin \dot{=}_f$, and (2) there exists $z \in I(t'')$ and $w \in I(t')$ such that $\langle z, w \rangle \in \dot{=}_f$. Suppose that there is a $y \in I(t)$ such that $\langle z, y \rangle \in \dot{=}_f$. Then by symmetry and transitivity, we would have $\langle w, z \rangle \in \dot{=}_f$ and hence $\langle w, y \rangle \in \dot{=}_f$, which violates (1), and hence $\mathfrak{M} \models Et'' \neq_f t$.

□

A corollary of this is that Rules 2 and 3 are both sound.

We are left with the cases where the terms do fall in T_{div} . The admissibility of the essential analog of Rule 1 follows immediately from the proof of the admissibility of that same rule, by substitution of $=_e$ for all occurrences of $=_f$. For the other cases, the standard rule of validity for affirmative syllogisms only holds when the propositions in the premises and the conclusion are all of the same type. When they are not, we must use the following rules:

Rule 4 (*Dici de omni* for mixed affirmative syllogisms).

- Whenever some predicate is said formally of some distributed subject, then of whatever that subject is predicated identically, of the same that predicate is predicated identically.¹⁰
- Whenever some predicate is predicated identically of some distributed subject, then of whatever that subject is predicated formally, of the same that predicate is predicated identically.¹¹

For mixed negative syllogisms — that is, ones with at least one negative premise — our rule is split into four parts:

Rule 5 (*Dici de nullo* for mixed negative syllogisms).

- When some predicate is formally denied of some distributed subject, then it is not necessary that of whatever that subject is predicated identically that of the same thing that predicate is denied identically or formally.¹²
- Whenever some predicate is denied identically of some distributed subject, then it is not necessary, if that subject is predicated identically of some term, that of the same that predicate is denied identically.¹³
- If some predicate is denied formally, that is in formal predication, of a distributed subject, of whatever that distributed subject is formally predicated, of the same that predicate is denied in formal predication.¹⁴
- Whenever some predicate is denied identically of some distributed subject, then of whatever that subject is said formally, of the same that

¹⁰*Quandocumque aliquod predicatum dicitur formaliter de aliquo subiecto distributo, tunc de quocumque predicabitur tale subiectum ydemptice, de eodem predicabitur et tale predicatum ydemptice* (DMPS, par. 51a, p. 275).

¹¹*Quandocumque aliquod predicatum predicatur ydemptice de aliquo subiecto distributo, tunc de quocumque predicabitur tale subiectum formaliter, de eodem predicabitur tale predicatum ydemptice* (DMPS, par. 51b, p. 275).

¹²*Quando aliquod predicatum negatur formaliter de aliquo subiecto distributo, tunc non oportet quod de quocumque predicatur ydemptice tale subiectum, quod de eodem negatur ydemptice vel formaliter tale predicatum* (DMPS, par. 57, p. 277).

¹³*Quando aliquod predicatum negatur ydemptice de aliquo subiecto distributo, tunc non oportet, si tale subiectum predicatur ydemptice de aliquo termino, quod de eodem negatur ydemptice tale predicatum* (DMPS, par. 58, p. 277).

¹⁴*Si aliquod predicatum negatur formaliter, idest in predicacione formali, de subiecto distributo, de quocumque predicatur formaliter tale subiectum distributum, de eodem negatur in predicacione formali tale predicatum* (DMPS, par. 59, p. 277).

*predicate is denied identically.*¹⁵

To formalize these, we define the notion of the distribution of a term within a formula:

Definition 8 (Distribution). *A term t is in the scope of \forall iff one of the following holds:*

1. $t \in \mathbb{T}_{\text{trin}}^{\text{quant}}$ and is of the form At' .
2. $t \notin \mathbb{T}_{\text{trin}}^{\text{quant}}$ and is a subject.
3. $t \notin \mathbb{T}_{\text{trin}}^{\text{quant}}$ and is a predicate of a negative categorical.

If t is in the scope of \forall in a categorical proposition ϕ , then we say that t is distributed in ϕ .

With this definition, we can give the following formal statements of Rules 4 and 5.

Rule 6 (*Dici de omni* for mixed affirmative syllogisms). *If t and t' are the terms of ϕ and t is distributed in ϕ , and Q is any quantifier, then*

- *If $\phi = \phi_f$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_p t$, then $\mathfrak{M} \models Qt'' =_p t'$.*
- *If $\phi = \phi_p$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_f t$, then $\mathfrak{M} \models Qt'' =_p t'$.*

Proving the admissibility of this rule is straightforward:

Proof. Assume $\phi = \phi_f$, $\mathfrak{M} \models \phi$, and $\mathfrak{M} \models Qt'' =_p t$. Since t is distributed in ϕ and ϕ is affirmative, we know that ϕ is either of the form $At =_f Q't'$ or $t =_f Q't'$, for some possibly empty quantifier Q' . Looking at Table 1, the only formal identities (other than those which fall out of the reflexivity of $=_f$) are between the persons and their personal properties, and since the persons are personally identical with both themselves and their personal properties, it follows that $\mathfrak{M} \models Qt'' =_p t'$. The other case follows similarly. \square

¹⁵*Quandocumque aliquod predicatum negatur ydemptice de aliquo subiecto distributo, tunc de quocumque dicitur tale subiectum formaliter, de eodem negatur tale predicatum ydemptice* (DMPS, par. 60, p. 277).

In Rule 1, there are only two syllogistic forms which have only affirmative premises, Barbara and Darii. For both of these, there are four possible ways to form a divine syllogism: either both premises are formal, both are personal, the major is personal and the minor formal, or the major is formal and the minor personal (*DMPS*, par. 54, p. 276). In the first case, the syllogism is valid, because:

Secondly I say that if some predicate is said formally of a distributed subject, then of whatever thing that distributed subject is said formally, of the same indeed that predicate is said formally.¹⁶

Which is to say that the traditional *dici de omni* remains valid when considering categorical propositions with divine terms, not just ones containing only created terms.

In the second case, the syllogism is not valid, because:

When some predicate is predicated identically of a distributed subject, and if then that [subject] is said identically of some third term, then it is not necessary that that predicate indeed may be said of the same third term.¹⁷

The third and fourth cases are covered by Rule 6.

Now for the negative syllogisms, Celarent and Ferio. Again we have four cases — the major premise is formal and the minor personal, the major premise is personal and the minor formal, both are personal, or both are formal. All four are expressed explicitly in the rule:

Rule 7 (*Dici de nullo* for mixed negative syllogisms). *If t is a distributed subject in ϕ and Q is any quantifier, then*

1. *If $\phi = \phi_f$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_p t$, then neither $\mathfrak{M} \models Qt'' \neq_p t'$ nor $\mathfrak{M} \models Qt'' \neq_f t'$ follows necessarily.*

¹⁶*Secundo dico quod si aliquod predicatum dicitur formaliter de subiecto distributo, tunc de quocumque dicitur formaliter tale subiectum distributum de eodem eciam dicitur formaliter tale predicatum* (*DMPS*, par. 53, p. 276).

¹⁷*Dico igitur primo... quod quando aliquod predicatum predicatur ydemptice de subiecto distributo, et si tunc tale dicitur ydemptice de aliquo tercio termino, tunc non oportet quod tale predicatum eciam dicatur ydemptice de eodem tercia termino* (*DMPS*, par. 52, p. 275 – 276).

2. If $\phi = \phi_p$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_p t$, then $\mathfrak{M} \models Qt'' \neq_f t'$.
3. If $\phi = \phi_f$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_f t$, then $\mathfrak{M} \models Qt'' \neq_f t'$.
4. If $\phi = \phi_p$ and $\mathfrak{M} \models \phi$, then if $\mathfrak{M} \models Qt'' =_f t$, then $\mathfrak{M} \models Qt'' \neq_p t'$.

Again, proving the admissibility of these rules is straightforward:

Proof.

1. We can prove this case by noting that $\mathfrak{M} \models E \neq_f F$ and $\mathfrak{M} \models F =_p E$, but $\mathfrak{M} \models F =_p F$ and $\mathfrak{M} \models F =_f F$.
2. This follows from the fact that, per Table 1, personal identities and non-identities only occur between the persons and their personal properties or between the persons and the essence, and that each person is formally distinct from both the essence and the personal properties which are not his characteristic property.
3. This valid case is identical with Celarent or Ferio (*DMPS*, par. 59, p. 277).
4. This case follows from (2) by contraposition.

□

With these tools to hand, it is possible to show that the four rules characterizing valid mixed affirmative syllogisms given in *DMPS*, par. 93 – 96, pp. 284 – 285 and the eight rules for mixed negative syllogisms given in *DMPS*, par. 98 – 105, pp. 285 – 286 are correct. These proofs are straightforward, and are left as exercises to the reader. More interesting is to see how this formal system can be applied to resolve the apparent paralogisms; we look to this in the next section.

4. Resolving the paradoxes

In the previous section we introduced the ! quantifier but didn't say much about its usage. The ! quantifier is used when we formalize natural language sentences about the trinity in order to make their import explicit.

A paralogism arises when a syllogism appears to be sound but where the conclusion is intuitively false. These paralogisms can be blocked by recognizing the various ways that categorical propositions containing divine terms can be

ambiguous. There are two main ways that categorical predications like this can be ambiguous. First, the type of identity being expressed by *est* is not made explicit. Paralogisms that arise from this type of ambiguity make up a large percentage of the fallacious arguments concerning the trinity:

[M]any [fallacies] which are made in divinity, are made from identical conjunction of extremes with a middle, and because of this they are believed to be able to be connected with each other identically; or from identical and formal conjunctions, because of which conjunctions they are believed to be able to be connected with each other formally.¹⁸

As a result, to avoid paralogisms of this type we need to make explicit the type of identity (cf. *DMPS*, par. 81, p. 281). If we make explicit which type identity is being expressed by *est* (for purposes of examples we will take it to be $=_f$), then we still have a potential ambiguity, because there are two ways that we can interpret the sentence *essencia est formaliter pater*. By the default interpretation of the empty quantifiers that we introduced in the previous section, this sentence should be interpreted as *omnis essencia est pater*. But since in *omnis essencia est pater*, *essencia* stands for just one object, (namely \dot{E}), we could also interpret the sentence as *hoc essencia est pater* without changing the truth conditions of the sentence (cf. *DMPS*, par. 34, p. 271).

However, there is a second way that we could interpret *omnis essencia est pater*, namely by generalizing the subject term, e.g., *omnis res que est essencia est pater* (cf. *DMPS*, par. 56, 74, pp. 276, 280). The two interpretations are not equivalent, and they do not have the same signification:

Briefly I say that these two propositions: every essence is the father, and: every thing which is the essence is the father, by the mode of signification and imposition do not have the same mentals (*mentales*) unless you want to abuse the term; and the subject of this: everything which is the essence is the father, taking the first ‘is’ identically, supposits formally for many things, namely for the three persons; however the subject of this: every essence is the father, supposits formally for one thing alone, namely for the essence, and only indistinctly and identically for

¹⁸[M]ulte (fallacie) que fiunt in divinis, fiunt ex coniunctione ydemptica extremorum cum medio, et propter hoc creduntur inter se posse coniungi ydemptice; vel ex coniunctionibus ydemptica et formali, propter quas coniunctiones creduntur inter se posse formaliter coniungi (*DMPS*, par. 75, p. 280).

the three persons.¹⁹

The truth conditions for both versions are intuitive. *Hoc essentia est essentialiter pater* is a singular proposition, whose truth conditions are governed by Definition 7, that is, it is true if and only if the particular, singular thing which is the essence stands in the essential identity relation with [something that is] the father. *Omnis res que est essentia est essentialiter pater* is true if and only if everything which is the essence stands in the essential identity relation with [something that is] the father. Formally, the distinction is between:

$$!E =_f F / =_f$$

and

$$E / =_* =_f F / =_f$$

Notice the introduction of $=_*$ into the first term; as the author notes in *DMPS*, par. 82, p. 281 – 282, if we want to expound *essentia* as *omnis res que est essentia*, we need to ask which type of identity is being expressed by this *est*. In *DMPS*, par. 84 – 88, pp. 282 – 283, the author argues in favor of interpreting *omnis essentia est pater* as only *hec essentia est pater*, and not as *omnis res que est essentia est pater*. While if we interpret it as *omnis res que est essentia*, then we can reason according to Rules 4 and 5, if we do so, then *non salvabis omnes modos Aristotelis, ut patet de disamis* (*DMPS*, par. 84, p. 282). Instead, if we singularize the subject terms and pay attention to the modification of the copulae introduced by *essentialiter*, *personaliter*, and *formaliter*, then “you will solve all paralogisms; you will even save all the modes of Aristotle.²⁰ Taking this route, we will see that “many apparent distortions in the infidels themselves follow according to the mode of complete [distribution], of which nothing follows from the aforementioned modification of the copulae.”²¹ And thus we are

¹⁹*Sed breviter dico quod iste due propociones: omnis essentia est pater, et: omnis res que est essentia est pater, ex modo significacionis et impositionis non habent easdem mentales, nisi velis abuti terminis; et subiectum istius: omnis res que est essentia est pater, summendo primum ‘est’ ydemptice, supponit pro pluribus formaliter, scilicet pro tribus personis; subiectum autem illius: omnis essentia est pater, supponit pro uno solo formaliter, scilicet pro essentia, et indistincte vel ydemptice pro tribus personis* (*DMPS*, par. 83, p. 282).

²⁰*solves omnes paralogismos; salvabis eciam omnes modos Aristotelis* (*DMPS*, par. 84, p. 282).

²¹*multa apparencia distorta ipsis infidelibus sequuntur ad modum de completa (distribucione), quorum nullum sequitur ad modificacione copularum predictam* (*DMPS*, par. 89, p. 283).

able to resolve the paralogisms. Furthermore, we can extract Aristotelian syllogistics from within the framework that we have provided. This allows us to say that reasoning about the trinity is not a “special case” which cannot be handled by regular syllogistic logic. Instead the situation is almost the other way around: Reasoning about creation is just a special case or a reduction of trinitarian syllogizing. We can do all of our logical reasoning within one formal system that handles propositions about divine and created things equally well.

The fact that the predications used in syllogisms about the trinity can be formal, identical, or essential explains why we have paralogisms. The expository syllogism

Hoc essentia divina est pater.
Filius est essentia divina.
Igitur, pater est filius.

is valid and sound if the statements are all taken to be essential predications. The paralogism arises when we interpret the conclusion as making a personal or formal predication. Once this misinterpretation is cleared up, by making the type of predication explicit via our formal system and reasoning with expository syllogisms, then the paralogisms disappear.

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