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# The dynamics of government debt

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**Abstract**

This paper deals with decision making on government debt in an overlapping-generations model of a small open economy. The government is concerned with the utility of current generations only, but it explicitly takes the effect of current decisions on future government decisions into account. Fiscal policy is constrained by viability conditions. An analytical solution for the time paths of debt and taxes is derived. Decreasing as well as increasing debt levels can be obtained. Conditions are given determining which of these patterns prevails. Finally, the effects of (anticipated and unanticipated) shocks in the exogenous parameters on the time path of government debt are analyzed.

*JEL classification:* D78; H30; H63

*Keywords:* Government debt; Dynamics; Overlapping generations

## 1. Introduction

In many western countries a similar evolution of government debt occurred after World War II. First, debt ratios declined, until, somewhere in the mid seventies, they started to rise. Recently, the EC countries have proclaimed by way of the treaty of Maastricht that stabilization of debt ratios (at a relatively low value) is the policy goal to be targeted. These phenomena raise the question as to the determining factors of the evolution of debt. One obvious candidate for explaining this evolution is the interest rate. But the age structure of the population may also constitute an important factor. The effect of the age structure depends to

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a large extent on the ruling tax regime. In the overlapping-generations model, to be developed in Section 2, all consumers, both young and old, pay a consumption tax at the same rate. For the elderly, tax shifting by debt creation would then be advantageous. For the young, however, tax shifting may imply that they are confronted with part of the burden of the additional debt when they are old. Due to this way of modelling a built-in restriction on debt creation exists. In such a model, aging, i.e. an increase in the number of elderly relative to the number of young, has two effects. The political influence of the elderly, who prefer tax shifting, increases. On the other hand, a decrease in population growth leads to an increase in the interest rate corrected for population growth, which decreases the tax base. This implies that the possibilities for tax shifting decrease in the long run. As a consequence, the current young may be confronted with a higher future tax burden if taxes are shifted now. Therefore, they will be more strongly opposed to tax shifting.

From this perspective, this paper analyzes the choice of the debt policy of a rational government. It is assumed that the government only takes the utility of current generations into account. So, it may be interpreted as a representative government which consists of subsequent generations of politicians with a finite time horizon. There is no way a present politician is able to bind his successors to a planned policy. This does not imply that future policy is exogenously given to a present politician: government debt is the instrument through which he is able to influence future decisions. Therefore, rational expectations of future decisions have to be formed. In particular, generations of politicians are assumed to be able to calculate all possible paths of future taxes (and, hence, debt levels) as a function of the present tax rate and to pick the path that maximizes their welfare, i.e. they are assumed to be Stackelberg leaders towards future generations of politicians.

The assumption of Stackelberg leadership has been introduced before in this context. Persson and Svensson (1989) and Tabellini and Alesina (1990) use Stackelberg behavior in a two-period model. Alesina and Tabellini (1990) do consider an infinite-horizon model but confine themselves to a steady-state analysis. In these papers, debt is used to force future governments with different policy preferences to pursue the policy preferred by the present government. Note that in these models there is no overlapping-generations structure. As a result, there is no inherent tendency to shift taxes to future periods as these future taxes have to be paid by the decision makers themselves.

In the present paper, the possibility of tax shifting is at the heart of the analysis. Like the literature cited above, repudiation [1] is abstracted from. Contrary to this

[1] Tabellini (1991) studies debt as an instrument of intergenerational redistribution in a two-period model that allows for repudiation. Altruism is essential for his results. Without altruism the government budget is always in balance. Other papers dealing with intergenerational redistribution in a political-economic model with altruism include Cukierman and Meltzer (1989) and Hansson and Stuart (1989).

work, successive generations of policymakers are assumed to have identical preferences. Moreover, an infinite-time horizon is considered. The present paper also abstracts from altruism. Assuming that the policy adopted reflects the interests of the different groups present in that period, an explicit solution for the time path of government debt and the tax rate is derived. It appears that government debt can decrease or increase in the course of time. In the latter case, it asymptotically approaches a finite maximum sustainable value.

It can be shown that the time path chosen by representative governments may, but need not, coincide with the one that results in the case of a social-welfare-maximizing government. In the literature cited above, normative and positive solutions always coincide if differences in preferences are absent. This is due to the fact that, as noted above, in these models, tax shifting is irrelevant if preferences are the same over time. [2] It should be noted that, according to the present model, in contrast with the infinite-horizon model in Alesina and Tabellini (1990), a welfare-maximizing government does not necessarily choose a balanced-budget policy. Depending on the social rate of time preference and the interest rate, the government will choose increasing or decreasing debt levels.

The ability to calculate time paths for the debt also makes it possible to analyze how the course of these paths will change due to exogenous changes in the parameters. As shown in Section 4, these changes lead to jumps in the level of debt at the time the change becomes known. Moreover, it may change the maximum sustainable debt level.

## 2. The model

In the sequel, a small open economy will be assumed where debtors are able to borrow from domestic lenders as well as foreign lenders, both against a fixed world interest rate. In this case, the capacity of the domestic capital market does not impose any restrictions on the borrowing behavior of the government. This in contrast with a closed economy, where government borrowing is restricted by the capacity of the domestic capital market.

### *2.1. The consumers*

A standard overlapping-generations model is used where two non-altruistic generations, old and young, are present at the same time. Each individual lives for two periods. In the first period, he is endowed with one unit of income of which

[2] In Tabellini (1991) only the young pay taxes. So, debt creation implies complete shifting of the tax burden. In that case, debt creation is only restricted by the threat of repudiation.

he saves an amount $s$. He has to pay a tax rate $\tau$ over the remaining part. The rest is used for consumption in the first period of his life, $c_t^y$, which therefore equals:

$$c_t^y = (1 - \tau_t)(1 - s_t). \tag{1}$$

When he is old, the savings including the interest revenues net of taxes are consumed:

$$c_{t+1}^o = (1 - \tau_{t+1})(1 + r) s_t \tag{2}$$

where $c_{t+1}^o$ is old-age consumption at time $t + 1$ and $r$ is the given fixed world interest rate. So, at a given time the young and the elderly pay the same tax rate which excludes direct transfers from the current young to the current elderly. As noted in the Introduction, this implies that for the young forbearance of taxation may not be acquittance. Note that the tax system used is actually a consumption tax instead of an income tax. $s_t$ can be seen as savings for future consumption so that these can be considered as pension premiums which are tax deductible in many countries. The value of savings, including accrued interest, then equals the pension benefit which is taxable. The young optimize a lifetime-utility function $U_t = U(c_t^y, c_{t+1}^o)$ subject to Eqs. (1) and (2). The instrument in the optimization is the savings rate. Hence, the first-order condition reads

$$\frac{\partial U_t}{\partial s_t} = -\frac{\partial U_t}{\partial c_t^y}(1 - \tau_t) + \frac{\partial U_t}{\partial c_{t+1}^o}(1 + r)(1 - \tau_{t+1}) = 0. \tag{3}$$

From this follows $s_t^* = s_t^*(\tau_t, \tau_{t+1}, r)$ as the optimal amount of savings.

### 2.2. The government

Government outlays at time $t$ consist of interest payments on the given amount of government debt per consumer of generation $t - 1$, $b_{t-1}$, and given expenditures per consumer of generation $t$, $g_t$. The government generates revenues by levying taxes. Abstracting from the possibility of debt repudiation, directly or indirectly through monetary finance, the government's budget identity reads

$$b_t = \left(\frac{1 + r}{1 + n}\right) b_{t-1} + g_t - T_t \tag{4}$$

where $n$ is the growth rate of the population and $T_t$ is total tax revenue per consumer of generation $t$. From Eqs. (1) and (2) it follows that total tax revenue equals

$$T_t = \tau_t \left[(1 - s_t) + \left(\frac{1 + r}{1 + n}\right) s_{t-1}\right]. \tag{5}$$

Two natural conditions constrain the set of fiscal policies open to the government. Firstly, the government is forced by the foreign and domestic lenders to obey the well-known No-Ponzi-Game condition which simply says that the principal and the service of the debt cannot be financed completely by going into new debt. In other words, it requires that the present value of outstanding debt converges towards 0 asymptotically, i.e.,

$$\lim_{\tau\to\infty}\left(\frac{1+r}{1+n}\right)^{-\tau} b_\tau = 0. \tag{6}$$

Aggregating forward Eq. (4) from period $t$ on to a final period $T$, the government's intertemporal budget constraint results:

$$b_t = \left(\frac{1+r}{1+n}\right)^{t-\tau} b_\tau + \sum_{j=t+1}^{\tau}\left(\frac{1+r}{1+n}\right)^{t-j}(T_j - g_j). \tag{7}$$

The NPG condition then implies that the intertemporal budget constraint for an infinite horizon can be written as

$$b_t = \sum_{j=t+1}^{\infty}\left(\frac{1+r}{1+n}\right)^{t-j}(T_j - g_j) \tag{8}$$

which is the well-known result that the present debt has to be met by future primary surpluses. This condition is tightened by a second natural constraint. When raising taxes, the government faces a maximum tax rate, $\tau^{\max}$, which, obviously, never exceeds 1. This implies that the total of future primary surpluses is finite (assuming that the tax base grows at a rate smaller than $(1+r)/(1+n)$) and thus constitutes a threshold value, $b^{\max}$, above which the (foreign) lenders will no longer be willing to buy government debt since then they know for sure that it will not be repaid.

The threshold value $b^{\max}$ and the maximum tax rate $\tau^{\max}$ constrain the set of fiscal policies open to the government. Which policy will be chosen by the government from this set depends on the decision making process. The government at time $t$ represents currently living generations only and is succeeded by another government every next period. It is assumed to maximize a decision function: [3]

$$W_t = W(U_{t-1}, (1+n)U_t). \tag{9}$$

Every $t$, this decision function is maximized again by a new generation of politicians. In choosing its policy the government explicitly takes account of $s_t^*$, the optimal amount of savings chosen by the consumers. This makes it act as a Stackelberg leader towards the private sector.

Given the fact that current generations are not altruistic towards future generations, the current government does not take the utility of future generations into

[3] Bernheim (1989) calls a decision function as in Eq. (9) "...within an overlapping generations framework, ... the most natural class of welfare functions for a representative government" (p. 124).

account. Therefore, the scope for fiscal policy for an incumbent government is confined to the present period. The chosen policy has, however, implications for the actions to be taken by future governments. It is assumed that the current government takes these actions into account in choosing a policy. This means that it also acts as a Stackelberg leader towards all future governments.

## 3. The evolution of debt

This section focuses on the decision making process when $W_t$ in Eq. (9) is maximized again each period. Only the current young and old generation count in the decision making process. The fiscal policy preferred may depend on an agent's planning horizon. Even when their preferences can be characterized by the same utility function, conflicts of interest between agents may arise if they have different planning horizons. In particular, while the elderly would prefer complete tax shifting, the young may, due to the age-independent tax structure, be interested in policies that smooth taxes over their lifetime. The tax rate set by the government follows from the maximization of Eq. (9). Assuming an interior solution, the first-order condition reads

$$\frac{\partial W_t}{\partial \tau_t} = \frac{\partial W_t}{\partial U_{t-1}} \frac{\partial U_{t-1}}{\partial c_t^{\mathrm{o}}} \left[ -(1+r) s_{t-1}^* \right] + \frac{\partial W_t}{\partial U_t}(1+n)$$
$$\times \left\{ -\frac{\partial U_t}{\partial c_t^{\mathrm{y}}} \left[ (1-s_t^*) + (1-\tau_t)\frac{\partial s_t^*}{\partial \tau_t} \right] \right.$$
$$\left. + \frac{\partial U_t}{\partial c_{t+1}^{\mathrm{o}}} \left[ (1-\tau_{t+1}^*)(1+r)\frac{\partial s_t^*}{\partial \tau_t} - (1+r) s_t^* \frac{\partial \tau_{t+1}^*}{\partial \tau_t} \right] \right\} = 0 \qquad (10)$$

where $\tau_{t+1}^*$ is the tax rate set by the next period government. Note that $\tau_{t+1}^*$ is a function of $\tau_t$ reflecting the fact that an incumbent government acts as a Stackelberg leader towards future governments. [4] In Eq. (10), the terms $\partial W_t / \partial U_{t-1}$ and $\partial W_t / \partial U_t$ can be interpreted as the marginal political power of an old and a young individual respectively. The political preferences of the old and the young generation then follow from Eq. (10) by setting $\partial W_t / \partial U_t$ and $\partial W_t / \partial U_{t-1}$ respectively equal to 0. From $\partial W_t / \partial U_t = 0$, assuming positive savings and $\partial U_{t-1} / \partial c_t^{\mathrm{o}} > 0$ it follows that $\partial W_t / \partial \tau_t < 0$, implying that there is no internal solution. This clearly indicates the preference for low taxes by the present elderly. Since they will have passed away the next period and do not bother about the utility of the present young generation or any other future generation, they will

[4] Since the next government is assumed to be a Stackelberg leader to its successors, $\tau_{t+1}^*$ implicitly takes $\tau_{t+2}^*, \ldots, \tau_\infty^*$ into account.

prefer tax rates to be set at the lowest possible level which is determined by the maximum debt level, $b^{max}$. The political preferences of the young follow by setting $\partial W_t/\partial U_{t-1}$ equal to 0 in Eq. (10). In this case substituting Eq. (3) in Eq. (10) and rewriting gives

$$-\frac{\partial \tau_{t+1}^*}{\partial \tau_t}=\frac{(1-\tau_{t+1}^*)(1-s_t^*)}{(1-\tau_t)s_t^*}. \tag{11}$$

The left-hand side of Eq. (11) is associated with the marginal costs of a one-dollar tax decrease. The right-hand side gives the willingness to substitute present taxes for future taxes. According to Eq. (11), the tax policy preferred by the young is the one where marginal cost equals marginal willingness. [5] Note that since neither negative savings rates nor tax rates or savings rates larger than 1 are allowed, for any $t$ it will hold that $\partial \tau_{t+1}^*/\partial \tau_t \leq 0$. The consequence of this is that if the political power of the young is non-negligible, the government is urged to trade-off current and future tax rates, given the Stackelberg assumption that it takes account of the relation between the current and all future tax rates. A lower current tax rate will lead to a higher debt inherited by the next generation which, as a result, might have to opt for a higher tax rate. [6] For old individuals the trade-off between current and future taxes is of no relevance. As noted before, if the elderly are solely decisive for the tax rate, then the tax rate will be set at its lowest possible value given the viability of the implied policy. The tax rate actually chosen in the political process will result from weighing the preferences of the old and the young generations.

Assume the utility function to be of a logarithmic type, i.e.,$U_t=\ln(c_t^y)+\theta\ln(c_{t+1}^o)$, where $\theta$ is the private discount factor. Furthermore, it is assumed that government expenditures $g$ are constant and smaller than 1 [7] and that $\tau^{max}$ equals 1. Given these assumptions, it immediately follows from Eq. (3) that the young choose $s_t^*=\theta/1+\theta$ for all $t$. Moreover, to derive an explicit solution, the

[5] An alternative interpretation of Eq. (11) is as follows: A young individual has two ways of influencing his second-period consumption possibilities; either directly by changing his savings $s$ or indirectly by choosing a higher (lower) current tax rate in return for a lower (higher) tax rate in the next period. The latter can be seen as 'saving through the government'. Now, Eq. (11) can be interpreted as an arbitrage condition which denotes when the young consumer is indifferent between these two forms of saving. This can be seen by rewriting Eq. (11) as

$$\frac{-[\partial \tau_{t+1}^*/\partial \tau_t](1+r)s_t^*}{1-s_T^*}=\frac{1-\tau_{t+1}^*}{1-\tau_t}(1+r),$$

where the right-hand side is the rate of return of private savings and the left-hand side denotes the rate of return of 'saving through the government'.

[6] This is, however, not necessary. The tax rise may be postponed until period $t+2$ or later, leaving $\tau_{t+1}$ unaffected. In that case, the interests of the young and elderly coincide.

[7] $g<1$ implies that the level of government expenditures per consumer is lower than the initial endowment each consumer receives in the economy.

marginal political power of the individual members of the two generations is assumed to be constant and normalized so that $\partial W_t/\partial U_{t-1} = 1$ and $\partial W_t/\partial U_t = \lambda$ where $0 < \lambda < \infty$. Inserting all this in Eq. (10) results in the following first-order condition for the government at time $t$:

$$-\theta\frac{1-\tau_{t+1}^*}{\theta(1-\tau_t)} - \lambda(1+n)\left\{\frac{1-\tau_{t+1}^*}{\theta(1-\tau_t)} + \frac{\partial\tau_{t+1}^*}{\partial\tau_t}\right\} = 0 \qquad (12)$$

for $t = 1,\ldots,\infty$. Notice in Eq. (12) that, as expected, the relation between the current and future tax rate is of importance only if a young individual has some political power, i.e., $\lambda > 0$. The term between brackets reflects Eq. (11), the marginal trade-off made by the young. However, the government also takes the preferences of the elderly, which are reflected in the first term, into account. Hence, it follows that the tax rate chosen by the government will be lower (or at most equal to) the tax rate preferred by the young. In the extreme case where $\lambda = 0$, Eq. (12) cannot hold with equality implying that there is no internal solution. Given $b^{\max}$, the lowest possible tax rate results.

At any time $t$, the government, as a Stackelberg leader towards future governments, knows that future governments use Eq. (12) to solve for the tax rate. Therefore, it uses this equation to calculate the relation between its own and future government decisions.

To derive a solution for the infinite horizon problem the following procedure is used: first the finite horizon problem is solved and then a solution for the infinite horizon problem is derived as the limit of the finite horizon problem. [8] Assume the time horizon to be finite, $\mathrm{T}$. Obedience of the No-Ponzi-Game condition for a finite $\mathrm{T}$ implies that all government debt has to be repaid, i.e., [9]

$$b_{\mathrm{T}} = 0. \qquad (13)$$

The calculation executed by the forward-looking government at time $t$ goes backward starting in the final period $\mathrm{T}$. In that period, the government has no choice but to obey the terminal condition $b_{\mathrm{T}} = 0$. The tax rate in the final period, then, follows immediately from the government's budget constraint, Eq. (4):

$$\tau_{\mathrm{T}}^* = \frac{1+\theta}{1+(1+\hat{r})\theta}\left[(1+\hat{r})b_{\mathrm{T}-1} + g\right] \qquad (14)$$

where $\hat{r} = (r-n)/(1+n)$, $\hat{r}$ denotes the effective interest rate, and, hence, $1+\hat{r} = (1+r)/(1+n)$. In period $\mathrm{T}-1$, the incumbent government explicitly

[8] Note that standard dynamic programming techniques are not applicable in this case because of the form of the target function.

[9] Strictly speaking, the No-Ponzi-Game condition for a finite horizon reads $\lim_{t\to\mathrm{T}}[(1+r)/(1+n)]^t b_t = 0$. But since $[(1+r)/(1+n)]^t > 0$ for every finite $t$ this implies $b_T = 0$.

takes account of Eq. (14). Inserting it in its own budget constraint and taking the derivative with respect to $\tau_{\tau-1}$ gives

$$\frac{\partial \tau_{\tau}^{*}}{\partial \tau_{\tau-1}} = -(1+\hat{r}). \tag{15}$$

Inserting Eqs. (14) and (15) into Eq. (12) and solving for $\tau_{T-1}$ learns that an interior solution will be obtained if:

$$b_{\tau-2} \le b_{\tau}^{\max} = -\frac{1+(1+\hat{r})}{(1+\hat{r})^2} g + \frac{[1+(1+\hat{r})\theta][(1+\hat{r})+1]}{(1+\theta)(1+\hat{r})^2}.$$

If $b_{\tau-2} = b_{\tau}^{\max}$ the tax rate will be equal to $\tau^{\max}$. Moreover, $b_{\tau-2} > b_{\tau}^{\max}$ is simply not possible. The (foreign) lenders will prevent this level of debt from being issued since it can never be repaid. Therefore, an interior solution is always obtained. Given $\tau_{\tau-1}$ thus obtained, the solution for the tax rate $\tau_{\tau-2}$, $\tau_{\tau-3}$ and so on can be obtained in the same way.

The solution for the infinite-horizon analogue of the model follows by taking the limit of the $\tau$ period model. [10] The solution for the tax rate at time $t$ for the infinite-horizon model then reads:

$$\tau_{\infty,t}^{*} = \begin{cases} \tau_{\infty,t}^{\text{int}} & \text{if} \quad b_{t-1} \le b^{\max} \\ \tau^{\max} & \text{if} \quad b_{t-1} = b^{\max} \end{cases} \tag{16}$$

where

$$\tau_{\infty,t}^{\text{int}} = 1 - \frac{(1+\theta)(1+\hat{r})}{1+(1+\hat{r})\theta} \cdot \frac{\beta}{1+\beta}[b^{\max} - b_{t-1}] \tag{17}$$

and

$$b^{\max} = -\frac{g}{\hat{r}} + \frac{1+(1+\hat{r})\theta}{(1+\theta)\hat{r}} \tag{18}$$

where $\tau_{\infty,t}^{\text{int}}$ is the interior solution for the tax rate at time $t$. $\beta$ is defined as $[\lambda(1+n)+\theta]/[\lambda(1+n)\theta]$ and describes the political compromise between the preferences of the present generations. It reflects the political weight in the decision making process attached to present utility (of the old and young generation) relative to future utility (of the present young). Of course, $b^{\max}$ is independent of the political influence of both generations as measured by $\lambda$. Rewriting $b^{\max}$ as $\frac{\theta}{1+\theta} + \frac{1-g}{\hat{r}}$, it becomes clear that, given the assumption $g < 1$,

[10] That the limit of the solution of the finite-horizon problem is indeed a solution for the infinite-horizon problem is easily seen by writing down the first-order conditions of the infinite-horizon model and checking the candidate solution and, furthermore, noting that the finite-horizon solution behaves as a turnpike (see e.g. Blanchard and Fischer, 1989), hence there are no transversality problems.

government debt exceeds savings which implies that the country is a debtor in the world capital market. [11]

The evolution of debt can be traced by substituting solution (16) into the budget restriction (4). This gives the following linear difference equation:

$$(b^{\max} - b_t) = \frac{1+\hat{r}}{1+\beta}(b^{\max} - b_{t-1}). \tag{19}$$

$b^{\max} - b_{t-1}$ can be interpreted as the scope politicians in period $t$ have for increasing the inherited debt. According to Eq. (19) this scope for policy-making changes from one period to another by the factor $(1+\hat{r})/(1+\beta)$. From this, it can be concluded that debt (and thus the tax rate) increases if $\hat{r} < \beta$, decreases if $\hat{r} > \beta$ or remains unchanged if $\hat{r} = \beta$. An intuition for this result can be given by noticing that Eq. (12) implies equality between the marginal political willingness to substitute taxes and the marginal cost of tax substitution. The willingness in the political process to substitute present for future taxes is given by the first two terms in Eq. (12):

$$MRS_{\tau^*_{\infty,t},\tau^*_{\infty,t+1}} = \frac{\partial W_t}{\partial \tau^*_{\infty,t}} \Big/ \frac{\partial W_t}{\partial \tau^*_{\infty,t+1}} = \frac{1-\tau^*_{\infty,t+1}}{1-\tau^*_{\infty,t}}\beta \tag{20}$$

From the definition of $\beta$ it can be seen immediately that this willingness depends on both the private discount factor $\theta$ and the political power balance $\lambda(1+n)$. Lower values for $\theta$ imply more impatience on the side of the individual (young) consumers. They are more interested in present than in future consumption. This, obviously, lowers the political willingness to give up present consumption (and thus present utility) through higher taxes now in exchange for future consumption. A similar argument holds for a lower $\lambda(1+n)$, i.e. more political weight for the present elderly. Then again, the willingness to give up present consumption in exchange for future consumption is lower. On the other hand, the marginal cost of tax shifting is given by

$$MRT_{\tau^*_{\infty,t},\tau^*_{\infty,t+1}} = -\frac{\partial \tau^*_{\infty,t+1}}{\partial \tau^*_{\infty,t}} = (1+\hat{r})\frac{\beta}{1+\beta}. \tag{21}$$

[11] If, instead of an open economy, a closed economy with a linear production technology is assumed (i.e. $f(k) = rk + w$ where $w$ is normalized: $w \equiv 1$) the capital market imposes an additional constraint: $b \le s = \theta/(1+\theta)$. In that case,

$$b^{\max} = \min\left\{\frac{\theta}{1+\theta} + \frac{1-g}{\hat{r}}, \frac{\theta}{1+\theta}\right\},$$

which implies that the constraint imposed by the tax base is redundant if $g < 1$. This implies a lower upperbound for government debt than in the open economy case, thus altering the set of feasible policies. However, in case of $g > 1$, the results for the closed economy coincide with those of the small open economy.

Note that this marginal cost is lower than the marginal cost of a debt increase when debt has to be redeemed completely in the next period. In that case, the marginal cost of a one-dollar debt increase (or, equivalently, a one-dollar tax decrease) would be $1 + \hat{r}$ (see Eq. (15)). The reason for this is that, when debt is not completely redeemed in the next period, it is possible to shift the burden of the one-dollar debt increase partly over to future generations. This decreases the marginal costs. The degree to which this possibility is used depends on the private weight attached to future consumption ($\theta$) and the political power balance ($\lambda(1+n)$) as can be seen from $\beta$. In the optimum the marginal costs of tax shifting have to equal the marginal benefits. From equalizing these two it follows that the tax rate (and, hence, debt) increases if $\hat{r} < \beta$, decreases if $\hat{r} > \beta$ or remains unchanged if $\hat{r} = \beta$.

The following proposition summarizes the results:

*Proposition 1. If* $b_0 < b^{max}$, $\lambda > 0$ *and* $\theta > 0$, *for every finite t, it holds that:*

(a) *If* $\hat{r} < \beta$ *then*

$$b_{t-1} < b_t,$$
$$\tau_{\infty,t-1} < \tau^*_{\infty,t}.$$

(b) *If* $\hat{r} > \beta$ *then*

$$b_{t-1} > b_t,$$
$$\tau^*_{\infty,t-1} > \tau^*_{\infty,t}.$$

(c) *If* $\hat{r} = \beta$ *then*

$$b_{t-1} = b_t,$$
$$\tau^*_{\infty,t-1} = \tau^*_{\infty,t}.$$

The regimes (a) and (b) in the proposition correspond with the cases (a) and (b) in part (A) of Fig. 1

Regime (c) corresponds with the case where the budget is balanced every period. The adjoining tax rate is then given by

$$\tau = \frac{1+\theta}{1+(1+\hat{r})\theta}[\hat{r}b + g].$$

Part (B) of Fig. 1 shows the evolution of the debt (Eq. (19)). Case (a) is the case $\hat{r} < \beta$ and the system converges towards the steady state $b^{\max}$ through a sequence of increasing levels of debt. Case (b) for which $\hat{r} > \beta$ results in an ever decreasing debt. [12]

[12] Here, an unstable process occurs where the debt and the tax rate tend to minus infinity. In principle, negative debt also poses a problem of viability: in the case of foreign borrowers, the rest of the world must be willing to serve it. It is possible to avoid such a process by imposing e.g. a non-negativity restriction on the tax rate. Then the debt will converge to a finite minimum level. Calculations for this case have appeared in an earlier version of this paper and are available upon request. Since they do not provide additional insights they are left out here for expositional reasons.

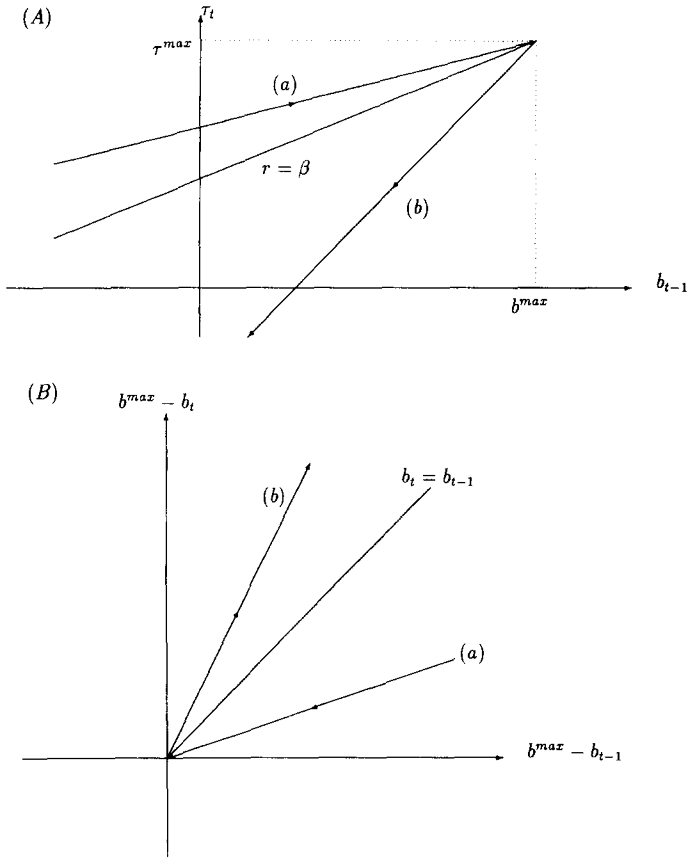


Fig. 1. Possible debt–tax patterns (part A) and the evolution of debt (part B).

It is interesting to compare the evolution of the debt described in Proposition 1 with the evolution that would result from the maximization of a social welfare function. It can easily be shown [13] that, if the government maximizes a Benthamite social welfare function with a social rate of time preference ρ, a proposition analogous to Proposition 1 with β replaced by ρ can be derived. So, any evolution of debt and taxes that can result from decision making by a representative government can also be found by maximizing a social welfare function that explicitly takes account of the utility of future generations. So, a representative democracy, where decision making is based on the utility of the currently living generations only, may implicitly take account of the welfare of all

[13] See Meijdam et al. (1994).

future generations. However, it will do this only to a limited extent: since $\beta > 1$ the implicit social rate of time preference can never be below 1. This result provides a justification for the common practice to mimic a positive government by a normative social welfare maximizing government with a high social rate of time preference compared to the individual rate of time preference (see e.g. van der Ploeg and van de Klundert, 1991).

## 4. The effects of parameter changes

The analysis in the previous section led to a description of the evolution of the debt as a function of the exogenous parameters $\lambda$, $r$, $\theta$, $g$ and $n$. This section analyzes how this evolution is affected by a change in these parameters. [14] Two cases have to be distinguished: one where the change in the parameters is unanticipated, the other where it is anticipated. In case of an unanticipated change in the parameters these effects remain unknown until the time of the change, $t = \tilde{t}$. If, however, the change is anticipated, the effects are taken into account from period 1 onwards as the news of the change in the parametersarrives. [15] FromEq. (19) it follows that, given $b_{t-1}$, a change in $\lambda$, $n$, $\theta$, $r$ or $g$ in period $t$ has two effects: one running through $b^{\max}$, changing the scope for policy making, the other running through $(1+\hat{r})/(1+\beta)$, affecting the factor at which the scope for policy making is changing from one period to another. The following table summarizes the effects on $b^{\max}$ and $(1+\hat{r})/(1+\beta)$:

| | $\partial\lambda$ | $\partial\theta$ | $\partial r$ | $\partial g$ | $\partial n$ |
|---|---|---|---|---|---|
| $\partial b^{\max}$ | 0 | + | ± | − | + |
| $\partial[(1+\hat{r})/(1+\beta)]$ | + | + | + | 0 | − |

In the sequel, changes in the exogenous variables at some time $\tilde{t}$ will be investigated. The reference situation is $\hat{r} < \beta$, i.e. increasing debt levels and tax rates.

*Effect of an increase in political power of an old individual.* This is reflected by a decrease in $\lambda$ at time $\tilde{t}$. Since $b^{\max}$ is independent of political elements, this

[14] Notice the difference with a comparative statics analysis where changes in the stationary state are analyzed. Here, changes in the whole time path of debt due to changes in the exogenous parameters are analyzed.

[15] Note that, if the debt level at the time the news of the change arrives (either at $t = 1$ or at $t = \tilde{t}$, the time of the change) is larger than the new $b^{\max}$ a situation occurs where debt can never be repaid and the government is in fact bankrupt.

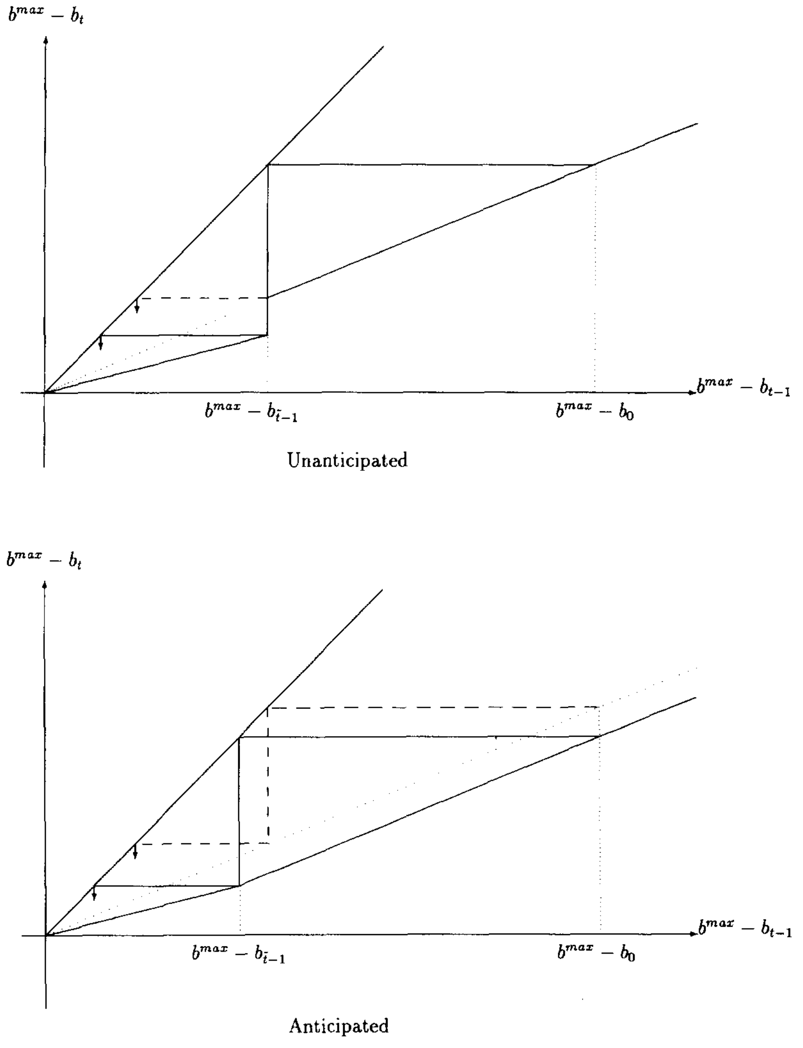


Fig. 2. $\partial\lambda < 0$.

does not affect the maximum level of debt. However, this maximum sustainable level is approached faster. Because the main concern of the elderly is a low tax rate, each period a larger part of the scope for policy-making is consumed. The implications of this are represented in Fig. 2. [16]

[16] In this figure and the figures to come, the dotted lines refer to the situation where there is no change in the respective exogenous parameter. The dashed line denotes the debt path followed in this case. The shift of the axis is due to a change in $b^{max}$ changing the scope for policy-making. Illustrated is the case where $\tilde{t} = 2$.

In the case of an unanticipated decrease in $\lambda$, debt jumps to a higher level at the time of the change, $\tilde{t}$. Due to this jump, the old generation present at that time enjoys large gains from its increased political power. Subsequent generations of elderly can profit from their increased political power to a much lower degree, because they are left with a debt level much closer to the maximum level $b^{max}$. Moreover, each generation reinforces this effect for their successors by consuming a larger part of their scope for policy-making.

In case of an anticipated change in the political power of the elderly, the generations at $t = 1$ will immediately exploit the possibility to jump to a higher debt level which arises due to the change in political power at $\tilde{t}$.[17] Note that, because of this jump to a higher debt level and the chosen debt policy, all generations of elderly with increased political power, including the elderly of generation $\tilde{t}$, are confronted with a smaller scope for policy-making. Hence, the gains from their increased political power are relatively low.

*Effect of an increase in the private discount factor.* If the private discount factor $\theta$ increases, two opposing forces are at work as can be seen from the table above. Firstly, due to higher savings, the tax base enlarges making lower taxes and larger debt levels possible (i.e. $b^{max}$ increases, widening, ceteris paribus, the scope for policy-making). Secondly, the relative political weight of present utility decreases (i.e. $\beta$ decreases) implying, ceteris paribus, a preference for higher taxes now relative to future taxes. As a result, less debt is passed on to the future thus increasing the scope for policy-making for future generations of politicians.

Fig. 3 provides an illustration of an unanticipated and an anticipated change. In case of an unanticipated increase in $\theta$, the two opposing effects are at work at the same time $\tilde{t}$. Therefore, the total effect is ambiguous and depends on the magnitude of the increase.[18] For small alterations in $\theta$ the tax-base effect dominates, for larger alterations in $\theta$ the political weight effect dominates which even might lead to a regime switch from increasing debt levels to decreasing debt levels. The upper part of Fig. 3 illustrates a jump to a lower level $b_{\tilde{t}}$. Savings increase, making more debt creation possible. However, politicians are not inclined to do so, due to the higher political weight attached to future utility. If there is an anticipated change, the generations before time $\tilde{t}$ can and will take advantage of the enlargement of $b^{max}$ due to the increase in $\theta$ at time $\tilde{t}$. Hence, at $t = 1$ there is a jump to a higher level of debt. At time $\tilde{t}$, only the increased political weight of the future is left, implying a slower growth of the debt. Note that the generations

[17] The generations between $t = 1$ and $t = \tilde{t}$ then pursue debt policies that would be unsustainable if they were continued after $\tilde{t}$. This can easily be seen from Fig. 2 by extending the solid line of policies between 0 and $\tilde{t} - 1$. It will not pass through the origin. However, because of the change in political power at $\tilde{t}$, a switch is made towards a sustainable policy.

[18] Note that, if $\hat{r} > \beta$, the political weight effect always dominates and a jump at time $\tilde{t}$ to a lower level of government debt can be observed.

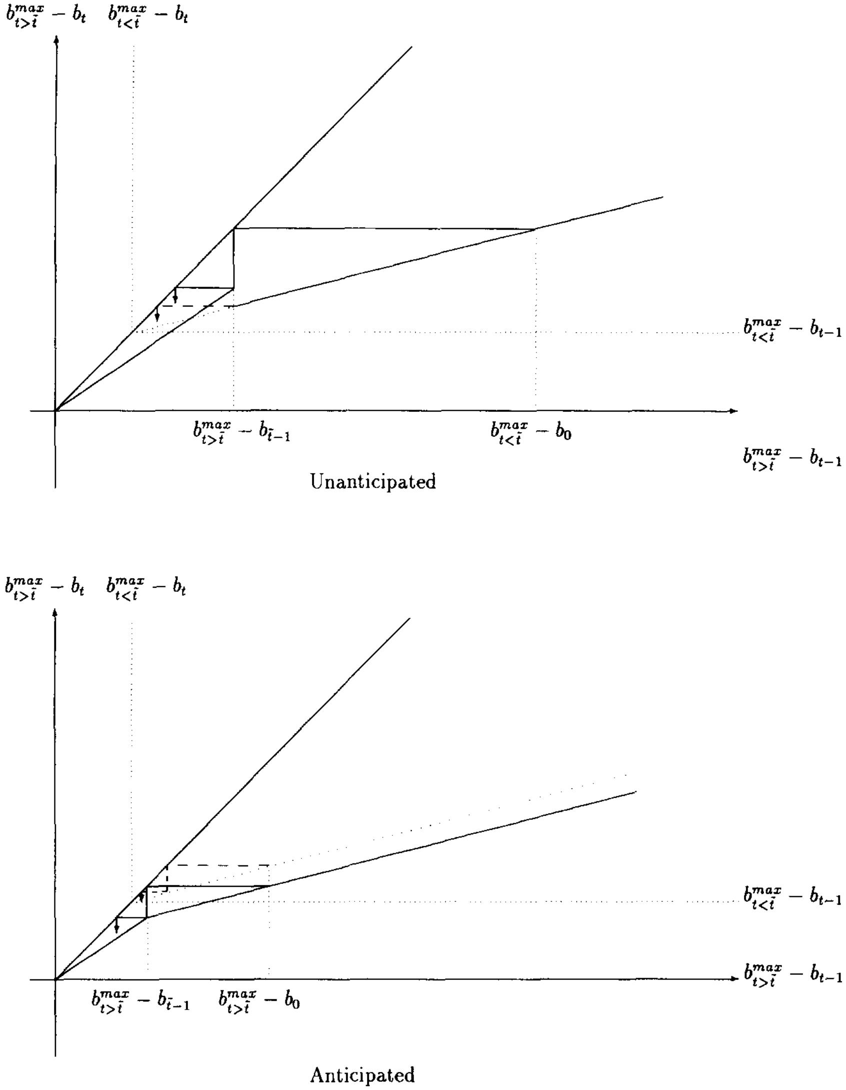


Fig. 3. $\partial\theta > 0$.

at $t = 1$ only experience the enlargement of the tax base and not the change in political weights. Therefore, they partly exploit the increase in the tax base. Due to this anticipation effect, the generations at $\tilde{t}$ profit only partly from the widening of the tax base.

*Effect of an increase in the interest rate.* An increase in the interest rate has two effects. Firstly, it enlarges the interest burden of existing debt and makes debt creation in the future more costly. Secondly, the interest revenues on savings rise which leads to a widening of the tax base. Clearly, these two effects are opposing. The increase in the interest burden necessitates, ceteris paribus, lower debt growth. The effect on the tax base makes, (again) ceteris paribus, larger debt levels possible. These effects can be derived from Eq. (19). The increased interest burden of existing debt necessitates an increase of the factor, $(1+\hat{r})/(1+\beta)$, by which the scope for policy-making changes. The higher costs of future debt issuance and the growth of the tax base are both contained in the effect on $b^{\max}$. From rewriting $b^{\max}$ as $\theta/(1+\theta)+(1-g)/\hat{r}$ it immediately follows that $\partial b^{\max}/\partial r<0$ since $g<1$. So, the positive tax-base effect of an increase in $r$ is dominated by the negative interest-burden effect.

An increase in $r$ leads to a jump to a lower level of debt, either at $t=\tilde{t}$ (unanticipated, the upper part of Fig. 4) or at $t=1$ (anticipated, the lower part of Fig. 4) [19]. After $\tilde{t}$ the erosion of the scope for policy-making is slowed down because of the increased interest burden of inherited debt, implying less debt to be passed on to future generations.

*Effect of an increase in government expenditures.* The effect of a change in the government expenditures on $b^{\max}$ is clearly negative. An increase in $g$ means that a larger share of tax revenues has to be used for financing these expenditures, leaving less room for debt repayments. This naturally implies that less debt can be accumulated, i.e. $b^{\max}$ is lower. The growth rate of debt is unaffected since the political weights and the interest rate remain unchanged. Hence, an increase in $g$ leads to a negative jump in debt. When this jump is observed depends on whether it is anticipated or unanticipated. An anticipated change in $g$ at time $\tilde{t}$ leads to jump at $t=1$, an unanticipated at $t=\tilde{t}$. (Fig. 5).

*Effect of a decrease in population growth.* The effect of a decrease in population growth, i.e. $\partial n<0$, comprises two effects: an increase in the political power of the old generation ($\partial\lambda(1+n)<0$), because of the increase of their relative number, and an increase in the effective interest rate ($\partial\hat{r}>0$). The world interest rate $r$, of

[19] A jump to a lower debt level at $t=1$ in anticipation of a future increase of the interest rate may seem odd, since consumers do not exhibit altruistic behavior towards future generations. So why should they bother? The reason for this is, of course, that, since they are Stackelberg leaders towards future generations, they know the behavior of generation $\tilde{t}$ in response to the change in the interest rate. This causes a reaction of generation $\tilde{t}-1$ to the displayed behavior of generation $\tilde{t}$ which in turn causes a reaction of generation $\tilde{t}-2$, etcetera.

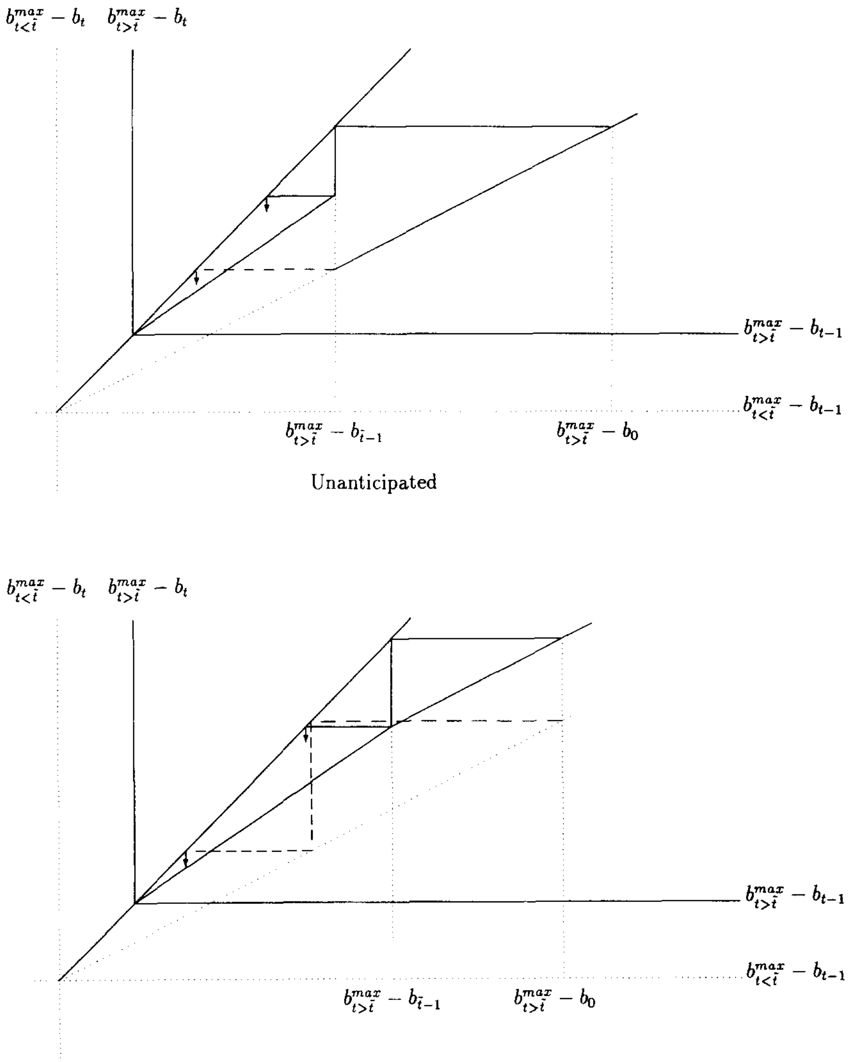


Fig. 4. $\partial r < 0$.

course, remains fixed, just as the political influence of a young individual, $\lambda$. Rewriting Eq. (19) gives

$$b^{\max} - b_t = [1 + \hat{r}] \Big/ \left[ \frac{1+\theta}{\theta}(1+n) + \frac{1}{\lambda} \right] (b^{\max} - b_{t-1}).$$

Therefore, a decrease in $n$ implies that the scope for policy-making is diminished

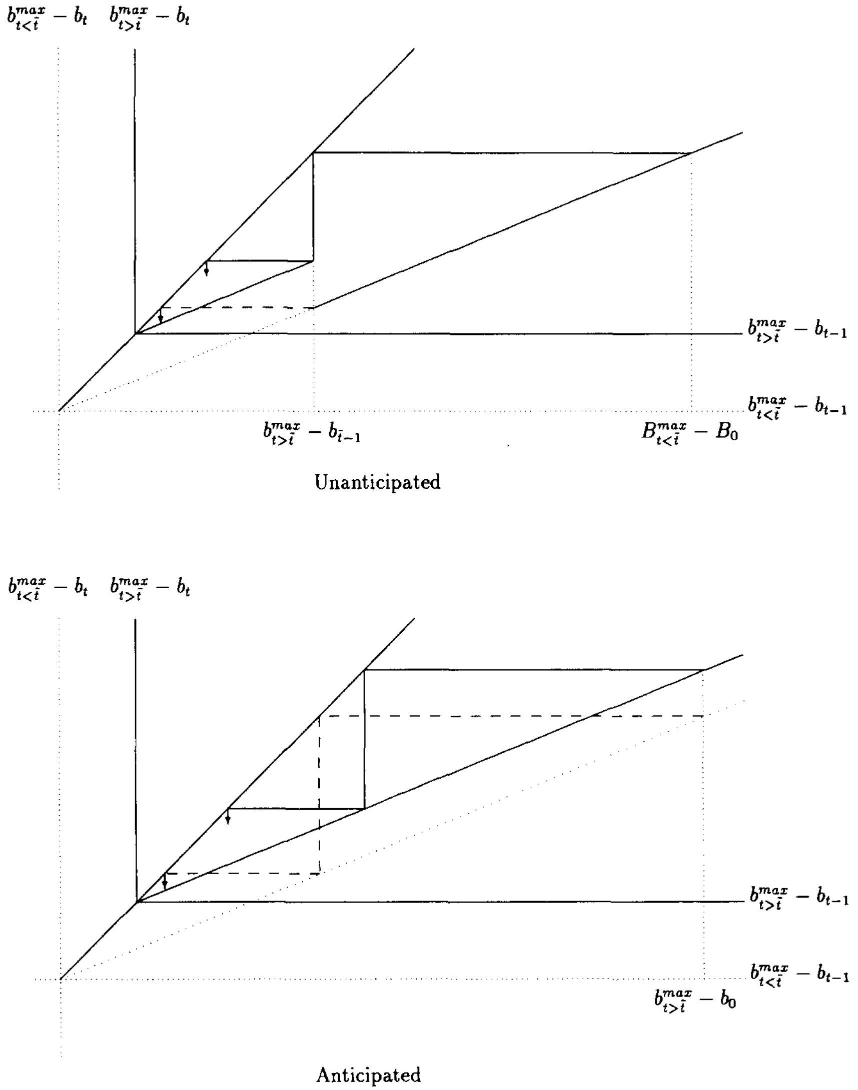


Fig. 5. $\partial g > 0$.

less quickly from one period to another by successive governments. Apparently, the effect of an increase in the political power of the elderly is dominated by the effect of the increase in the effective interest rate. Hence, ceteris paribus, a smaller part of the scope for policy-making is used by the present generation of politicians. Assuming $g < 1$, this effect is reinforced because a decrease in $n$ leads, through the increase in the effective interest rate, to a decrease in $b^{\max}$, implying a

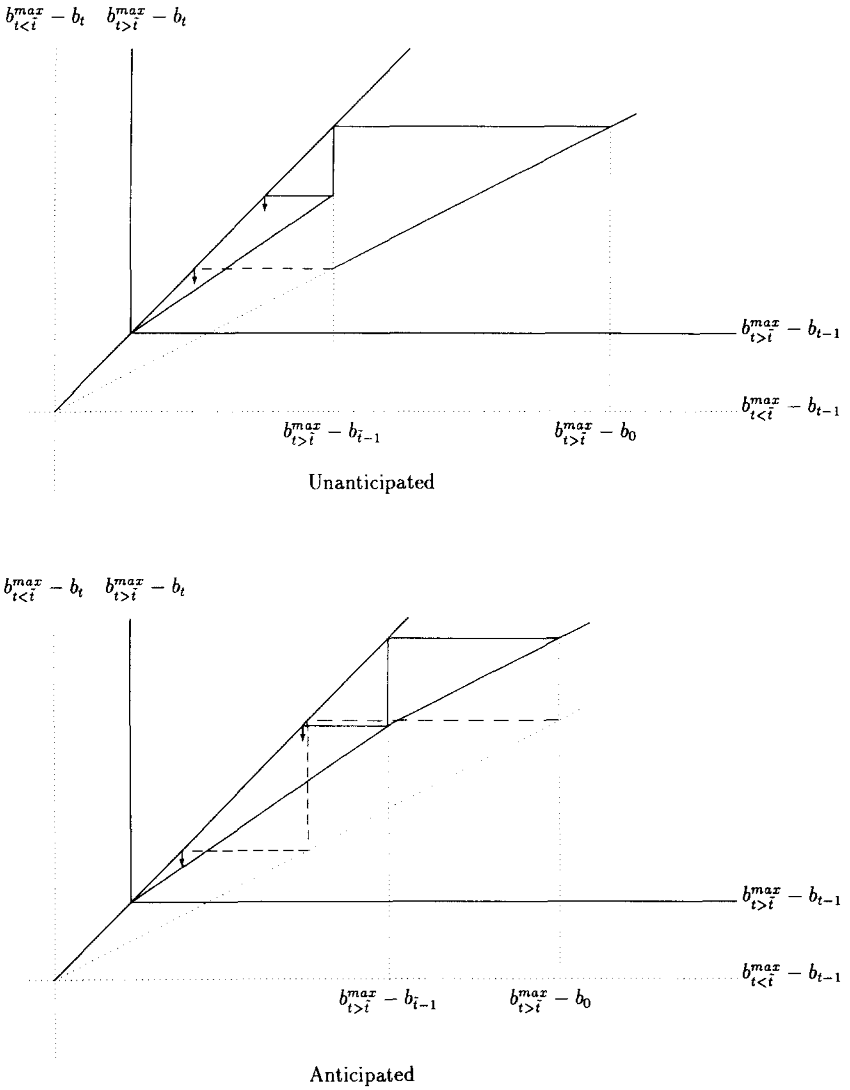


Fig. 6. $\partial n < 0$.

contraction of the scope for policy-making. Lower population growth lowers the tax base and by this $b^{max}$ decreases.

Fig. 6 gives an illustration of an unanticipated (upper part) and an anticipated (lower part) decrease in the population growth rate. The decrease in $b^{max}$ necessitates a jump to a lower level of debt, either at period 1 (anticipated) or at period $\tilde{t}$ (unanticipated).

### 4.1. Parameter changes and actual debt policy

Returning to the actual evolution of debt after World War II described in the Introduction, it is interesting to check whether this evolution can be explained by the model through parameter changes. With respect to the initially declining debt levels, according to Proposition 1, this can be the case if $\beta$ is lower than $\hat{r}$. Although the effective interest rate was quite low in that period, the discount factor may have been so high that $\beta$ was even lower. Indications of this may be found in the fact that savings by consumers were relatively high given the low interest rate. Moreover, in this same period many funded pension plans were initiated, also indicating the relatively high importance the generations of that time attached to the future. Starting in the sixties, the post-war baby-boom generation started to enter the labor market. In terms of this model this implies a (small) increase in $n$. It might be hypothesized that this generation was more consumption minded than the previous one witnessed by the fact that savings did not increase in spite of the increase in the interest rate starting in the early seventies. So, a larger generation with a lower discount factor entered the labor market. According to the model, this might have caused the shift from decreasing to increasing debt levels as observed in practice. This growth of debt continued during the eighties leading to rather high levels of debt. Recently, in several countries policy measures were enacted to mitigate the growth rate of debt. Moreover, as expressed by the EMU-norm regarding the required debt ratio for the member states of the EC, maximum levels for the debt ratio were fixed. These phenomena can be interpreted in the framework of the model. In particular, many EC-countries expect an aging of their population. As noted before, such a decrease in population growth should lead to a lower maximum level of debt. Moreover, this maximum level should be approached at a slower pace. Fig. 7 illustrates this effect of an anticipated decrease in population growth. [20]

In this figure, the generation of politicians at time $t = 2$ expect an aging of the population to occur at time $t = 3$. Moreover, the generation that is young at $t = 2$ is relatively large. In the simulation $n = 0.3$ was taken in the first two periods to indicate an increasing population, followed by a period where the dependency ratio increases ($n = -0.1$ for $t = 3$) after which the population stabilizes ($n = 0$). If the length of one period were equal to 25 years, and period 1 would correspond with the first 25 years after World War II, this stylized demographic development would roughly correspond with the current EC-situation. The population is expected to stabilize at a lower level than the current one in the second or third decade of the next century, after a once-only increase of the dependency ratio due

[20] In the simulation the following parameter values are used: $\lambda = 0.55$, $g = 0.5$, $\theta = 0.5$, $r = 2.1$, $b_0 = 0$.

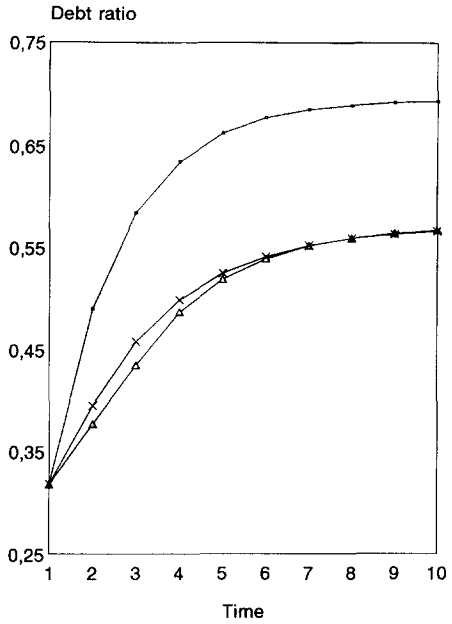


Fig. 7. The evolution of debt.

- • - no change; - × - baby bust; - △ - baby boom and bust.

to the retirement of the post-war baby boom. [21] It appears that, in anticipation of the decrease in population growth, debt jumps to a lower level at time $t = 2$. After this, debt gradually approaches the new maximum level. Notice from Fig. 7 that if the population would stabilize after $t = 2$ instead after $t = 3$, the debt ratio would be slightly higher in these periods. So, the baby boom, i.e. the increase in the dependency ratio at $t = 3$ ($n = -0.1$), hardly has any effect on the evolution of debt. But the baby bust, i.e. the permanent decrease in population growth ($n = 0$) affects the evolution of debt to a large degree.

## 5. Concluding remarks

The literature dealing with strategic decision making on government debt, surveyed in the Introduction, mostly uses a two-period model. If, however,

[21] Notice from Fig. 7 that an increasing debt ratio has been assumed for the whole post-war period. In this simulation the factors that contributed to a declining debt ratio during this period are not taken into account.

infinite-horizon models are developed, only steady-state comparisons are made. In this literature, it is concluded that there is a bias towards non-optimal (either too large or too low) levels of debt if and only if succeeding governments have different preferences. The model of decision making on government debt developed in this paper takes a different view; it focuses on the possibility of tax shifting in the absence of differences in preferences. An overlapping-generations model is used. Consumers, both young and old, pay a consumption tax at the same rate. Although the time horizon is infinite, the analyses are not restricted to steady-state comparisons. Given logarithmic utility functions, explicit solutions for the development of the tax rate and the level of debt are derived. A sequence of representative governments will gradually increase or decrease debt. In the former case, debt converges to a level determined by the maximum taxing capacity. In the case of decreasing debt, it may converge to a level determined by some lower bound on the tax rate. Under a social planner, debt *may* take the same route as under a representative government. However, even though differences in preferences are abstracted, this is by no means necessary. Representative governments may choose another time path of debt than a social planner because of the inherent tendency of consumers to shift taxes to future generations.

In this model the time path of debt can be calculated completely. Therefore, it is possible to derive how this path will change due to some foreseen or unforeseen change in the parameters. Such a change leads to a jump in the level of debt and, thus, forcing it to follow another path. This new path may converge towards a different maximum level, or may even decrease instead of increase. The following comparative-dynamics results are worth pointing out here. Firstly, if savings increase due to a lower rate of time preference, this can be concomitant with a desired decrease in the level of government debt. An increase in savings, giving a higher potential to finance government debt, is at the same time an expression of the increased weight that the current young generation places on its future consumption. By the same reasoning, an increase in the rate of time preference leads to an increase in the level of government debt. Secondly, in this model anticipated aging of the population leads to an immediate curtailing of debt. Basically, aging limits the possibility to shift the tax burden to the future by debt creation as it increases the effective interest rate. A combination of these parameter changes may lead to a time path of debt that is roughly consistent with the time path observed in most western countries after World War II.

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