TILBURG UNIVERSITY

TILEC

TILEC Discussion Paper

DP 2003-011

’Be nice, unless it pays to fight’: a new theory of price determination with implications for competition policy

By
Jan Boone

October 2003

ISSN 1572-4042

'Be nice, unless it pays to fight': a new theory of price determination with implications for competition policy

Jan Boone*
Dept. of Economics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands
Email: j.boone@uvt.nl

13-1-2003

Abstract

This paper introduces a simple extensive form pricing game where firms can react to each others' price changes before the customers arrive. The Bertrand outcome is a Nash equilibrium outcome in this game, but it is not necessarily subgame perfect. The subgame perfect equilibrium outcome features the following comparative static properties. The more similar firms are, the higher the equilibrium price. Further, a new firm that enters the industry or an existing firm that becomes more efficient can raise the equilibrium price.

Keywords: Bertrand paradox, price leadership, mergers, joint dominance, coordinated effects, balance of power

J.E.L. codes: D43, L11, L41

*CentER, TILEC, Tilburg University and CEPR, London. I thank Jan Bouckaert, Eric van Damme, Hans Degryse, Paul Geroski, Simon Grant, Jos Jansen, Paul Klemperer, Johan Lagerlöf, Luca Rigotti, Lars-Hendrik Röller, Karim Sadrieh, Johan Stennek, Frank Verboven, Jörgen Weibull and seminar participants at the European Winter Meeting of the Econometric Society in Berlin, the CEPR/IFS conference 'Innovation and the structure of product market competition'in London, the S.O.M. conference 'Competition and market power' in Groningen, Erasmus University Rotterdam, I.U.I. in Stockholm, K.U. Leuven, Tilburg University, University of Antwerp and W.Z.B. in Berlin for comments. Financial support from NWO, KNAW and VSNU through a Vernieuwingsimpuls grant is gratefully acknowledged. Part of the research for this paper was done while I was in Berlin, and I would like to thank the W.Z.B. for its hospitality.

When a move by one seller evidently forces the other to make a counter move, he is very stupidly refusing to look further than his nose if he proceeds on the assumption that it will not.

Chamberlin (1969: 46).

1 Introduction

This paper aims to contribute to the literature on oligopoly pricing in two ways. First, it introduces the idea that aggressiveness of price competition in an industry is determined by the cost distribution in the industry. The intuition here can be characterized as balance of power. If firms have similar efficiency levels, it is costly for these firms to fight and they will be nice toward each other: they charge a high price. If one firm (or a group of firms) is far more efficient than its opponents, however, there is no balance of power and it pays to fight to keep these inefficient firms out of the market. Second, I observe that the assumption in a standard Bertrand game that firms cannot react to other firms' prices before the consumers arrive, is not compelling in a number of industries. For instance, with retail firms and internet firms it seems more realistic to assume that (with some probability) a firm can react to other firms' price changes before consumers arrive. In this new pricing game which is reminiscent of an English auction, the Bertrand outcome is still a Nash equilibrium but it is not necessarily subgame perfect. The subgame perfect equilibrium (SPE) has the balance of power property above. This is not due to (explicit) collusion. Firms act independently and noncooperatively, but understand Chamberlin's observation above.

Although this be-nice-unless-it-pays-to-fight outcome does not seem unnatural, the SPE has surprising comparative statics results. A rise in the number of firms in the market or an increase in efficiency for some firms in the market can raise the equilibrium price by reducing the incentive to behave aggressively. I argue that this SPE outcome is a natural and simple way to formalize the notions price leadership and joint dominance or coordinated effects.

A simple example illustrates the idea that the cost distribution in an industry is a major determinant of how competitive or aggressive firms' conduct will be in the industry. Consider two industries, denoted I and II, which have the same demand curve for a homogenous good: $X(p) = 1 - p$, where $X(p)$ denotes total demand at price p. In each industry there are three firms. Consumers buy from the cheapest firm(s) only and if more than one firm charges this lowest price p they share the market equally (i.e. consumers randomize to choose a supplier since all firms are identical from their perspective). The industries differ in their cost structure. In industry I the constant marginal costs of firms $0, 1$ and 2 are respectively: $c_0^I = 0, c_1^I = 0.35, c_2^I = 0.4$. In industry II the cost distribution is: $c_0^{II} = 0, c_1^{II} = 0.1, c_2^{II} = 0.4$. In words, in industry II firm 1 is closer to firm 0 in terms of efficiency as compared to industry I. Both Cournot and Bertrand competition predict that the price in industry II is lower than in industry I. This is not always a compelling prediction. As shown below, the SPE of the pricing game introduced in this paper predicts that the outcome in industry I is more competitive than in industry II. The intuition is the following. In industry I, it pays firm 0 to price very aggressively and keep firms 1 and 2 out of the market. That is, firm 0 chooses the limitprice $p^I = c_1^I = 0.35$ and is the only seller in the market. However, in industry II it is very costly for firm 0 to keep both firm 1 and 2 out of the market because firm 1 is so close in efficiency to 0. So, because $c_1^{II} - c_0^{II}$ is small, it is more profitable for 0 (and 1) to keep only firm 2 out of the market and share the market. Thus, $p^{II} = c_2^{II} = 0.4$. Put differently, in industry II there is a balance of power[1] between firms 0 and 1. Because these firms have similar efficiency levels it is very costly for them to fight each other. In industry I, however, firm 0 is far more efficient than firm 1 and hence it pays to fight to keep firm 1 out of the market.

This idea that the cost distribution affects how aggressively firms play, yields five new insights. First, it gives a formalization of the idea of price leadership. The price leader(s) in the framework here is (are) the most efficient firm(s) in the industry. The price leader chooses the price that maximizes its profits (taking into account that other firms leave if the price is below their marginal cost level) and no other firm has an incentive to undercut this price.

Second, the result that prices are higher if firms are more similar (in terms of efficiency) can be seen as a formalization of the joint dominance doctrine (as it is called in Europe) or coordinated effects (as it is called in the US). The idea is the following. Consider an industry with three firms where one is far bigger than the other two. Now the two small firms merge and become big as well. The risk of joint dominance is that the remaining two big firms start to collude because the firms are now more similar in size than before. Compte et. al. (2002), Motta (2002) and Vasconcelos (2002) have formalized this relation between symmetry and collusion using supergames. They show that as firms are more similar in terms of either production capacities or number of productlines sold by each firm, it is easier to sustain collusion using trigger strategies. Although this result is similar to the balance of power result found here, the mechanism is different as I will argue in section 5 below.

Third, entry by a new firm into the market can raise the equilibrium price. This can happen if the new firm establishes a balance of power that was not there before. Thus it becomes less attractive to be aggressive and choose a low price. In other words, this type of entry causes a switch in conduct from playing aggressively to being nice which raises the equilibrium price. This is not the first paper to consider this non-standard effect of entry on price. Other papers are Rosenthal (1980), Amir and Lambson (2000)[2] and Stiglitz (1989). There is also a recent auction literature showing that a rise in the number of bidders can lower the price. I discuss this literature in section 2.4.2. None of the mentioned papers, however, uses the idea of balance of power to formalize a price rise due to entry.

Fourth, the SPE in the pricing game solves the Bertrand paradox. This paradox starts from the observation that Bertrand competition in a duopoly with two firms that produce a homogenous good with the same constant marginal costs yields a price equal to marginal costs. This is called a paradox because it seems counterintuitive that two firms are sufficient to get the perfect com-

[1] Clearly, this is the same intuition for why the USA and USSR have avoided direct military conflict in the 20th century. A real war (instead of a cold one) with a foe of comparable strength would have been too costly. Also, both countries have picked fights with opponents that they considered rather weak.

[2] In the example given by Amir and Lambson (2000) where entry raises the equilibrium price, it reduces the price cost margin due to the assumption of increasing returns to scale. I work with constant marginal cost levels implying that a rise in the equilibrium price raises price cost margins for firms.

petition outcome of price equal to marginal costs. Surely, two firms competing in prices must be able to get to an outcome with a strictly positive price cost margin. This is exactly the prediction of the SPE of the pricing game here. The result that a dynamic extensive form pricing game removes the Bertrand paradox has already been noted by Maskin and Tirole (1988) and Farm and Weibull (1987). The first paper is discussed in section 5. The Farm and Weibull paper has a pricing process that is similar to the one presented here, but they assume that all firms are symmetric. Therefore the monopoly price is always an equilibrium in their paper while this is not the case here.

Finally, the results in this paper can also be seen as contributing to the literature on conjectural variations and, in particular, to the literature on consistent conjectures (see, for instance, Bresnahan (1981) and Klemperer and Meyer (1988)). The subgame perfection requirement in the pricing game below makes players' beliefs (conjectures) about other players' behavior correct (consistent). This leads to an outcome that is less competitive than Bertrand competition for the following reason. Firms understand that when they undercut the current price, the optimal response of all other firms (with marginal costs below the new price) is to follow that price cut. This response is more aggressive than the Bertrand conjecture (firms stick to their higher price) and hence firms behave less aggressively.

This paper is organized as follows. Section 2 introduces the pricing game and its SPE. As this pricing game is rather stylized, section 3 gives examples of more realistic games which yield the same equilibrium outcome. Section 4 analyses the SPE outcome and shows the implications for competition policy. Section 5 discusses some extensions of the model. Section 6 argues that the theory presented here can help explain some empirical findings that are hard to understand with standard Cournot and Bertrand models. Further, I derive an empirical test to see whether the theory here applies in a certain industry. Finally, section 7 concludes the paper. Proofs of all results can be found in the appendix.

2 The model

This section describes an auction style pricing game that determines the industry price. The game is simple, not unplausible from a theoretical point of view and has (under some conditions) a unique SPE. It turns out that the Bertrand outcome is a Nash equilibrium in this game but it is not necessarily subgame perfect. The auction set up, however, does not score high on descriptive realism, making it hard to understand in which industries this pricing game applies. Therefore, the next section introduces two games which are more realistic and I derive conditions under which the SPE in the auction game remains an equilibrium in the more realistic games.

2.1 Industry conditions

Consider an industry where firms produce a homogenous good with constant marginal cost levels. Firm i produces with constant marginal costs $c_i \geq 0$. Let $X(p)$ denote industry demand which is continuously decreasing in p. The number of firms that can be active in the industry is denoted by $N \in \aleph$. Throughout this paper, I make the following assumptions on the industry cost and demand structures.

Assumption 1 $c_0 \leq c_1 \leq ... \leq c_N \leq p^m = \arg\max_p \{X(p)(p - c_0)\}$. *Further,* $X(p)(p - c_0) > X(p')(p' - c_0)$ *for each* p' *and* p *satisfying* $p' < p < p^m$. *Finally, prices are elements of a finite grid, say* $p \in \aleph$, *the monopoly price* p^m *and the cost levels* c_i *belong to this grid as well.*

In words, firms are indexed such that higher indices indicate (weakly) higher marginal cost levels. Because firm 0 is never willing to produce at a price above its monopoly price p^m, firms with marginal cost levels exceeding firm 0's monopoly price are ignored, without loss of generality. Further, I assume that firm 0's profits as monopolist are increasing in $p \in [0, p^m]$. Finally, to ensure that optimal reactions exist (in particular, to make sure that slightly undercutting your opponent is well defined) I make the standard assumption that the price space is discrete and without further loss of generality I assume that prices are integers. This also makes sure that the pricing game considered below is a finite game. To avoid rounding issues, I assume that the monopoly price and marginal cost levels are integers.

2.2 Extensive form pricing game

This section describes the extensive form game that leads to the equilibrium price. I assume that all information in this game is publicly known. That is, the demand function and firms' marginal cost levels are common knowledge. None of the results depend on asymmetric information.

The game starts at a high price, say $p^0 = p^m$, and $N + 1$ firms. A round s in the game is described by the number of players still in the game n^s, their marginal cost levels and the current price p^s. Each round consists of two stages. In the first stage, all remaining bidders simultaneously and independently make bids to undercut the current price p^s (players that do not wish to reduce the price, simply bid p^s again). In the second stage, firms that did not bid the lowest price can either decide to follow this lowest price or leave the game. The end of the game is reached if in a certain round $\bar{s}$ no firm undercuts the current price $p^{\bar{s}}$. The final price is then denoted $p = p^{\bar{s}}$ and the number of firms by $n = n^{\bar{s}}$.

After bidding ends, payoffs in this game are as follows. Any firm that has left the game gets a payoff of 0. A firm i that has not left the game gets a payoff equal to $\frac{X(p)}{n}(p - c_i)$. That is, each of the n firms that are still in the game when the bidding ends, is required to sell at the equilibrium price p and produce $\frac{X(p)}{n}$ units of output. The reason why these firms are required to sell (i.e. they cannot leave anymore) is that assuming otherwise would turn the bidding of prices into cheap talk. The next section replaces this assumption with a probability that the consumers arrive at the end of each stage and then the firm has to sell at the price that it quotes.

The industry story is as follows. Firms can enter and leave the industry without costs, therefore the payoff of a firm that leaves the bidding game is 0. Further, all firms that are willing to sell at a price p share the market equally. This is not unreasonable, because firms produce homogenous goods and consumers faced with n producers offering an identical product at the same price presumably pick a seller at random. I come back to this in section 5.

The idea that this auction style pricing stage captures, is that firms can react before consumers to price changes by other firms. This is the opposite assumption from the standard Bertrand game where firms cannot react at all if opponents' prices differ from what they expected.

Lemma 1 *The Bertrand outcome $p = c_1$ is a Nash equilibrium outcome in this game.*

This result implies that profits are zero if $c_1 = c_0$. The result is not surprising since the pricing game described above is a formalization of the stories that are told to give the intuition for the Bertrand outcome (see, for instance, Scherer and Ross (1990: chapter 6) and Tirole (1988: chapter 5)). But, as the next section shows, the Betrand outcome is not necessarily subgame perfect. Subgame perfection captures Fisher's (1898: 126) critisism of the Cournot and Bertrand equilibrium concepts that "no business man assumes either that his rival's output or price will remain constant any more than a chess player assumes that his opponent will not interfere with his effort to capture a knight. On the contrary, his whole thought is to forecast what move the rival will make in response to one of his own".

2.3 Subgame perfect equilibrium

The following notation is used in characterizing the SPE outcome of the pricing game.

Notation 1 $p_i^* \equiv \arg\max_p \frac{X(p)}{n(p)}(p - c_i)$ *where* $n(p) \equiv \min\{j | c_j \geq p\}$

In words, $n(p)$ is the number of firms with cost levels strictly below p. As shown below, in equilibrium only firms that have strictly positive profits stay in the market. For instance, if $p = c_j$ then firms $0, 1, ..., j-1$ are active in the market: j firms in total. Thus p_i^* denotes the price level that maximizes firm i's payoffs given that only firms with marginal costs below p_i^* will remain active in the production stage. Theorem 1 below shows that these prices form part of firms' subgame perfect strategies and the SPE price equals p_0^*.[3]

Lemma 2 *(Price leadership) The firm with the lowest marginal cost level, denoted by 0, is price leader in the sense that it prefers the lowest price:*

$$p_0^* \leq p_1^* \leq p_2^* \leq ... \leq p_N^*$$

This lemma gives a formalization of price leadership that does not rely on first mover advantages. Firm 0 correctly predicts that firms with marginal cost levels above the equilibrium price p will leave the market. Taking this into account, it chooses the price that is most profitable. Firm 0 is price leader in the sense that no other firm is willing to undercut its profit maximizing price. The intuition is that a low price raises the amount of output a firm produces, and this is most profitable to a low cost firm.

[3] In principle, p_0^* may be a set consisting of more than one element. To simplify the analysis, however, I will ignore this issue here.

This notion of price leadership differs from the one analyzed by Kirman and Schueller (1990), Rotemberg and Saloner (1990), Deneckere, Kovenock and Lee (1992) and Deneckere and Kovenock (1992). In each of these papers, the price leader is a Stackelberg leader and the papers discuss conditions under which this leader-follower outcome is an equilibrium. In these papers it is important that the leader commits to a price. This is not the case in the SPE here. It is, in fact, the reaction by the price leader (and other firms), thus the lack of commitment, that deters firms from undercutting p_0^*.

Theorem 1 *For the pricing game above there exists a SPE in pure strategies. The SPE price equals*

$$p = p_0^*$$

And only firms with $c_j < p_0^*$ produce output.
Strategy profiles leading to this equilibrium outcome prescribe for firm i in any round s of the pricing game to do the following:[4]

$$\begin{array}{lll} \text{if} & p^s > p_i^* & \text{then bid } p_i^* \\ \text{if} & p^s \leq p_i^* & \text{then do not undercut } p^s \\ \text{if} & p^s > c_i & \text{then stay in the game} \\ \text{if} & p^s \leq c_i & \text{then leave the game} \end{array}$$

The SPE price can also be characterized as follows: $p_0^* = c_i$ if

$$\frac{X(c_i)}{i}(c_i - c_0) \geq \frac{X(c_j)}{j}(c_j - c_0) \text{ for each } j \in \{1, 2, ..., N, N+1\} \quad (1)$$

where c_{N+1} denotes the monopoly price p^m.

Equation (1) tells us that a price level strictly inbetween two firm's cost levels, $p \in \langle c_j, c_{j+1} \rangle$ for some j and $j+1$, can never be optimal for firm 0. In such a case, a price equal to c_{j+1} yields higher profits, because it is a higher price (which is more profitable by assumption 1) but does not lead to more firms producing and selling in the industry. Strategy profiles that bring the equilibrium price p_0^* about are the following. Each firm i bids its optimal price p_i^* if the current price is above that price. Further, a firm leaves if the price is equal to or below its marginal cost level and stays in the pricing game as long as the price is above its marginal cost level.

The following corollary shows that the Bertrand paradox cannot happen in this model.[5]

Corollary 1 *If $c_0 = c_1 = ... = c_j < c_{j+1}$ then $p > c_0$.*

One way to understand why the SPE outcome is less aggressive than the Bertrand outcome is to use the idea of conjectural variation. In the Bertrand outcome a firm expects its opponents to keep their price constant. That is,

[4] Strictly speaking this is not complete. In principle, also subgames should be considered where firm i is still active but where firms $j < i$ have already left. This does not affect i's exit strategy but does affect i's bid. Clearly, taking these possibilities explicitly into account makes the analysis notationally heavy, while no additional insights are gained.

[5] Other solutions to the Bertrand paradox are product differentiation, capacity constraints, imperfect information, repeated interactions and staggered price setting. See Tirole (1988) for a discussion.

when the firm underbids the current price it expects its opponents to leave the pricing game and hence a small price reduction gives this firm the whole market. Because it expects its opponents to be soft (they leave the pricing game when it reduces the price) each firm behaves very aggressively and hence the outcome is aggressive. In the SPE, a firm understands that each price reduction will be followed by all firms with marginal costs below the price. Hence a firm expects a rather assertive response from its opponents. This reduces the incentive to be aggressive and hence the outcome is less competitive.

The following proposition gives sufficient conditions for p_0^* to be the unique equilibrium price.

Proposition 1 *Assume that (i) firms do not play weakly dominated strategies, (ii) p_0^* is a singleton and (iii) the expression $\frac{X(p)}{n(p)}(p - c_0)$ is single peaked as a function of p on the set $\{c_0, c_1, ..., c_{i-1}, p_0^*\}$ then p_0^* is the unique SPE price.*

The intuition for the three conditions is as follows. First, since the bidding in each round is simultaneous, two (or more) firms could simultaneously bid a price $p < p_0^*$. Subgame perfection (or one-stage-deviation) has no bite here to remove this price p as an equilibrium price. Given that player i believes that j is going to bid p $(> c_i)$, nothing is lost for player i to bid p himself. However, nothing is gained by bidding p either since this strategy is weakly dominated by bidding p_0^* . In particular, because it is always possible to follow the price p lateron in the game there is no reason to bid p yourself. Assuming that players do not play such weakly dominated strategies, no firm will bid a price below p_0^*.

`Figure 1 approximately here`

If p_0^* is a set with more than one element, it is clear that one can have multiple equilibria. More interesting is the third condition. If firm 0's profits $\frac{X(p)}{n(p)}(p - c_0)$ are not single peaked on $[c_0, p_0^*]$, there is a price p_0' which is a local maximum and a price $\underline{p}$ which is a local minimum as illustrated in figure 1. Then a price $p' > p_0^*$ can also be a subgame perfect outcome. Consider the firm j with $c_j = p_0^*$ (assuming that $p_0^* < p^m$). This firm j will tell firm 0 that it should not make a bid below p' because otherwise firm j will bid $\underline{p}$. At this price $\underline{p}$ it is optimal for firm 0 to reduce the price further to p_0'. Hence firm j will not have to produce at price $\underline{p}$ making firm j's threat credible. If instead the expression $\frac{X(p)}{n(p)}(p - c_0)$ is single peaked on $[c_0, p_0^*]$, any price reduction by firm j to $\underline{p}$ below p_0^* will be followed by the other firms (with $c_i < \underline{p}$) but not strictly undercut by any firm. That implies that $\underline{p}$ will be the equilibrium price and firm j is forced to produce at $\underline{p}$ which leads to a strictly negative payoff for firm j. Hence j's threat to reduce the price below p_0^* is not credible in this case and the equilibrium price p_0^* is unique.

2.4 Two examples

This section presents two examples of the analysis above. First, I discuss the example given in the introduction, then following Klemperer's (2000) suggestion, I use auction theory to illustrate my I.O. results.

2.4.1 Example in introduction

This section analyzes more carefully the example with two industries given in the introduction. Recall that in both industries the demand function is given by $X(p) = 1 - p$, where p is the industry price (the lowest price charged by any firm) and X equals total demand (for the homogenous good) at that price. If n firms are charging this price p then consumers choose their seller randomly in such a way that each firm sells $\frac{X(p)}{n}$ units of output. In industry I the (constant marginal) cost distribution of the three firms is $c_0^I = 0, c_1^I = 0.35, c_2^I = 0.4$ and in industry II it is $c_0^{II} = 0, c_1^{II} = 0.1, c_2^{II} = 0.4$.

Using equation (1), the SPE price is determined as follows. In industry I, firm 0 considers three possible prices: 0.35 and be the only firm in the market, 0.4 and share the market with firm 1 and thirdly a price equal to the monopoly price 0.5 and share the market with firms 1 and 2. The following inequalities show that $p^I = 0.35$ is the equilibrium price. Note that firm 0's profits equal $\frac{X(p)}{n}(p - c_0) = \frac{1-p}{n}p$.

$$\frac{1-0.35}{1}0.35 > \frac{1-0.4}{2}0.4 > \frac{1-0.5}{3}0.5$$

In industry II, however, we have the following inequalities.

$$\frac{1-0.4}{2}0.4 > \frac{1-0.1}{1}0.1 > \frac{1-0.5}{3}0.5$$

Hence the SPE price in industry II equals $p^{II} = 0.4$ which is higher than the equilibrium price in industry I. The intuition is that it is too costly for firm 0 to keep firm 1 out of the market in industry II, while this is not very costly in industry I. Hence firm 0 is more aggressive in industry I and the equilibrium price is lower than in industry II.

2.4.2 PEA auction

Consider the following (slight) modification of the English auction, called a Passive English Auctioneer (PEA) auction. The rules of the game are as follows. The auction starts at a price equal to zero. The buyers indicate whether they are willing to buy at the current price, say by leaving the auction room if they are not willing to buy anymore at that price (or a higher one). In addition to this, any buyer in the auction room can raise the price. If no buyer further raises the price and if there are $n \geq 1$ buyers left, the auctioneer randomly allocates the good to one of the remaining buyers and each buyer has a probability of $\frac{1}{n}$th of winning the good and paying the current price. A buyer who does not get the good pays nothing. In other words, if the number of remaining buyers n equals 2 or more the excess demand is not resolved through increasing the price but through a rationing scheme. This is clearly not an optimal (in the sense of revenue maximizing) auction, as an active auctioneer could raise the revenue of the auction through raising the price until only one buyer were left. That is why the auctioneer here is called passive.

In an I.O. context this passive auctioneer assumption often makes sense. If a number of firms sell the same product at the same price, consumers are indifferent between the sellers and buy randomly from any firm. Moreover, in this context, there is no auctioneer who can actively change the price to resolve excess supply.

There is a Nash equilibrium of the PEA auction where the buyer with the highest valuation gets the good at a price equal to the second highest valuation. But this equilibrium is not necessarily subgame perfect.

I want to stress two properties of the SPE of the PEA auction.[6] First, that the most efficient player does not necessarily bid up the price until all other buyers leave the auction. Second, that adding another buyer (which has in fact a very high valuation) can reduce the equilibrium price. Below these results are related to recent papers in the auction literature.

Example 1 *Consider a private value auction where the valuations of buyers* $0, 1$ *and* 2 *for the auctioned good equal resp.* $q_0 = 2\frac{1}{2}, q_1 = 2$ *and* $q_2 = 1$. *Using the auction equivalent of equation (1), the SPE price equals the price that maximizes bidder* 0*'s payoff. Buyer* 0 *has three relevant choices for the price:* $p = 0$ *and a probability of* $\frac{1}{3}$ *of getting the good,* $p = 1$ *and a probability of* $\frac{1}{2}$ *of winning and* $p = 2$ *in which case buyer* 0 *gets the good for certain. It is straightforward to verify the following inequalities*

$$\frac{2\frac{1}{2} - 0}{3} > \frac{2\frac{1}{2} - 1}{2} > \frac{2\frac{1}{2} - 2}{1}$$

In words, it is most profitable for player 0 *to choose a price equal to* 0 *and 'share' the good with buyers* 1 *and* 2.[7] *Hence the Nash equilibrium where player* 0 *gets the good for certain at a price equal to the second highest valuation is not subgame perfect. Now consider what happens when player* 1 *is removed from the market. In a standard second price auction, removing the buyer with the second highest valuation will reduce the revenue of the auction. Here we get the opposite result. Player* 0 *has now two relevant choices for the price:* $p = 0$ *and a probability of* $\frac{1}{2}$ *of getting the good and* $p = 1$ *in which case buyer* 0 *gets the good for certain. The following inequality implies that player* 0 *chooses* $p = 1$*:*

$$\frac{2\frac{1}{2} - 1}{1} > \frac{2\frac{1}{2} - 0}{2}$$

Hence, as player 1 *leaves the market, the balance of power disappears, player* 0 *becomes more aggressive and the price goes up. Since the gap between* q_0 *and* q_2 *is big, it pays to fight.*

The result that the equilibrium price in an auction can fall as another buyer is added has also been noted in Bulow and Klemperer (2002) and references therein. The mechanism through which this happens is, however, a different one. Bulow and Klemperer consider (almost) common value auctions. In such

[6] Note that to make these points there is no need to introduce asymmetric information into this auction example.

[7] The result that player **0** shares the market at a low price is reminiscent of a paper by Gilbert and Klemperer (2000) where conditions are derived under which rationing is optimal from the auctioneer's point of view. Note that the sharing result is indeed a form of rationing: there is excess demand (three buyers, one product) at the current price ($p = \mathbf{0}$) and yet the price does not go up; the good is allocated randomly. However, the main difference between my result and the one derived by Gilbert and Klemperer is that here rationing is optimal from the point of view of the buyers, not the seller.

an auction the winner may suffer from the winner's curse: the fact that he wins the auction makes it likely that he has overestimated the value of the good. As the number of bidders goes up, it becomes more likely that the winner has overestimated the value of the good: the curse gets worse. Rational players take this effect into account and shade their bids more strongly as the number of players goes up. Hence, a higher number of bidders can lead to lower expected revenues in the auction.

In the example above, there is a private values auction and hence the winner's curse plays no role. Also, with the winner's curse the problem is that players fear that the entrant has a low signal for (bad news about) the value of the auctioned good. In the example above, however, the price reduction is caused by adding a high value player (bidder 1). This bidder with a high valuation makes it less profitable for player 0 to bid aggressively.

Finally, the result in the PEA auction that it can be optimal for bidders to "share" the good at a low price can also be obtained in the context of a simultaneous ascending auction.[8] Consider the case with two bidders and two goods. Bidder 0 values each of these goods at $q_0 = 2\frac{1}{2}$ and bidder 1 values each at $q_1 = 2$. That is, if bidder i gets one (both) good(s) at a price equal to p (prices equal to p and p') his payoff is $q_i - p$ $(2q_i - p - p')$. It is routine to verify that in a simultaneous ascending auction the equilibrium price is 0 in this case and each bidder gets one good. If the valuation of bidder 1 is reduced to $q_1 = 1$, the equilibrium price rises to one and bidder 0 obtains both goods. The balance of power is broken and now it pays to fight for bidder 0.

3 More realistic pricing games

Although the auction style pricing game introduced above is easy to analyse, its disadvantage is a lack of descriptive realism. Therefore it is hard to see in which industries the theory presented here is likely to apply. This section introduces two models that are more realistic and in which the equilibrium derived above remains an equilibrium outcome. This also shows which of the assumptions made above are essential and which are made for ease of exposition.

3.1 Asymmetric information

One assumption made above is that prices can only move in one direction. In the market context, firms can only undercut previous prices. Prices cannot be raised. This gives immediately the result that a deviation by any firm to a price $p < p_0^*$ will be followed by all other firms with marginal costs below p. It is exactly this effect which makes such price reductions less attractive and thereby helps to sustain prices above the Bertrand outcome. Some people may argue that in the real world firms can both increase and decrease their prices. Does this affect the results?

Allowing for both price increases and decreases leads to a technical complication in the model above in the sense that the game is no longer finite. By adjusting prices in every period the game may never come to an end. In order to be able to use the one stage deviation principle to check subgame perfection, we

[8] I thank Paul Klemperer for bringing this simultaneous ascending auction example to my attention.

need to adjust the model to ensure that it is continuous at infinity (see Fudenberg and Tirole (1991:110)). The way in which this is often done is to introduce discounting. However, here I choose another way to do this which also removes the somewhat artificial distinction between the pricing stage and output stage in the game above.

Consider the following change to the model above. The game starts with every firm quoting a price. Then with a probability $q \in \langle 0, 1]$ the customer arrives and she buys the good from the firm(s) with the lowest quoted price.[9] If this happens the game is over. With probability $1 - q$ no customer arrives and every firm is allowed to change its price. After every firm has quoted a new price (or kept the previous price unchanged) there is again a probability q that the customer arrives etc. So the probability that the customer arrives in round T equals $(1-q)^{T-1} q$ which tends to 0 as T goes to infinity and this gives the continuity at infinity. Note that for $q = 1$ the game is a standard Bertrand pricing game.

In the game described so far, a price $p_0^* > c_1$ does no longer survive the one stage deviation test. A firm undercutting p_0^* for one period and then reverting back to p_0^* gains with probability q a strictly positive defection pay off, thereby breaking the equilibrium. Because in the auction pricing game prices could not be raised, such a deviation was not possible.

Using one stage deviation implies that the price cut by the deviating firm is not followed by the other firms and after the price cut (in case the customer did not arrive) all firms charge p_0^* again. This makes the price cut profitable. But this may not be the most natural response for the non-deviating firms. One way to proceed here would be to introduce commitment strategies as in Farm and Weibull (1987). Each firm commits to a strategy saying that all price cuts will be followed as long as the price stays above a firm's marginal costs. Such strategies remove the incentive to undercut and the SPE outcome above remains an equilibrium. In addition to this I give describe two other ways in which deviations from the SPE are not profitable: asymmetric information and menu costs.

First, consider the case where firms do not know what the cost level of the price leader is. A price reduction is then interpreted as a signal that c_0 is rather low and the optimal response to such information is to follow the price reduction because a price leader with such low costs will continue to charge this lower price. This reaction stops a one stage deviation from destroying an equilibrium price above the Bertrand outcome.

Since asymmetric information in general pricing games is rather heavy on notation,[10] I illustrate the point here with the following simple example.

Example 2 *Consider an industry with only one buyer who wants to buy at most one product. The valuation of the buyer is common knowledge, $v = 3$. There are three firms, denoted* $0, 1, 2$. *The cost level of firm 2 is also common knowledge, $c_2 = 2$. Each of the other two firms can have cost level* 0 *or* 1 *with probabilities*

[9] It is not relevant whether one customer arrives or a group of customers arrives, as long as the demand function of the customer(s) is given by $X(p)$.

[10] In particular, in the pricing game described above there would be two types of updating about opponents' cost levels: (i) which firms actively reduce prices and by how much, and (ii) which firms (only) follow price reductions.

π and $(1-\pi)$. The cost levels of these firms are independently distributed. If

$$q < \frac{1}{3}(1-\pi)$$

then the following strategies for prices, $p \in \aleph$,[11] and beliefs, μ, form a perfect Bayesian equilibrium

$$\begin{array}{r} \text{Firm 2 charges a price } p_2 = 3 \\ \text{A firm with cost level } c_1 = 1 \text{ charges a price } p_1 = 3 \text{ and follows a price reduction to } p = 2 \\ \text{A firm with cost level } c_0 = 0 \text{ charges a price } p_1 = 2 \text{ and follows a price reduction to } p = 1 \\ \mu(c_0 = 0 | p = 2) = 1 \\ \mu(c_0 = 1 | p = 3) = 1 \end{array}$$

To see that this is a perfect Bayesian equilibrium, note first that firm 2 has never an incentive to charge a price equal to 2. Hence its optimal strategy is to charge a price equal to 3. A firm with cost level equal to 1 can choose between charging a price equal to 3 or 2. It is routine to verify that the assumption $q < \frac{1}{3}(1-\pi)$ implies that the following inequality holds

$$\pi\left[q0 + (1-q)\frac{2-1}{2}\right] + (1-\pi)\frac{3-1}{3} > \pi\frac{2-1}{2} + (1-\pi)\left[q\frac{2-1}{1} + (1-q)\frac{2-1}{2}\right]$$

making an initial price equal to 3 and only following a price reduction to 2 the optimal strategy.[12] *In words, if this firm charges a price equal to 3 there is a probability π that its opponent has costs equal to 0 and charges a price equal to 2. With probability q the customer arrives and this firm does not sell. With probability $(1-q)$ the customer does not arrive and this firm can adjust its price to 2. Given the strategies above, the price will not be reduced further and when the customer (eventually) arrives the payoff will be $\frac{2-1}{2}$. With probability $(1-\pi)$ the firm's opponent has costlevel 1 as well. Hence all three firms charge a price equal to 3 and when the customer arrives payoffs for this firm equal $\frac{3-1}{3}$. The right hand side of the inequality is derived in a similar manner. The next two inequalities show that it is optimal for a firm with costs equal to 0 to charge a price equal to 2 (instead of 3 and 1 resp.)*

$$\pi\frac{2-0}{2} + (1-\pi)\left[q\frac{2-0}{1} + (1-q)\frac{2-0}{2}\right] > \pi\left[q0 + (1-q)\frac{2-0}{2}\right] + (1-\pi)\frac{3-0}{3}$$

$$\pi\frac{2-0}{2} + (1-\pi)\left[q\frac{2-0}{1} + (1-q)\frac{2-0}{2}\right] > \pi\left[q\frac{1-0}{1} + (1-q)\frac{1-0}{2}\right] + (1-\pi)\frac{1-0}{1}$$

Both inequalities hold because $q > 0$ and $\pi \in \langle 0, 1\rangle$.

The idea of the example is that choosing a low price $(p = 2)$ is only optimal for a price leader with costs equal to 0. Hence a firm with higher costs $(c = 1)$

[11] That is prices are chosen from the same grid as before. Here this gives a very simple grid $\{0, 1, 2, 3\}$. Results go through for a finer grid as well, as long as prices correspond to optimal choices for some realizations of the cost level c_0 for the price leader. That is, players will interpret undercutting as evidence that c_0 is lower than previously thought.

[12] Note that with asymmetric information, the Bertrand equilibrium can also lead to higher prices. However, in this example two firms with costs equal to 1 are able to sustain the monopoly price $(v = 3)$ which is not possible in the Bertrand outcome.

does not try to undercut the monopoly price because that is interpreted as a move by a low cost price leader. Once the firm's opponent has observed a price equal to 2, he interprets this as evidence that the price will stay at 2 (because that is the optimal choice for a low cost price leader). Hence although the rules of the game do not preclude a rise in the price level, asymmetric information makes sure that it will not happen. Hence we get similar results as in the auction type pricing game without the assumption that prices can only move in one direction.

3.2 Costly price adjustments

Another way to keep the SPE outcome derived in the auction style pricing game is to introduce menu costs. Consider the game in the previous subsection, where prices can both be increased and decreased and where there is a probability q that the customer arrives at the end of each round. Now instead of assuming asymmetric information, suppose that quoting and changing a price costs a firm a menu cost $\gamma > 0$. The following result derives a sufficient condition for the SPE in theorem 1 to remain an equilibrium in the game presented here.[13]

Proposition 2 *Let π^{SPE} denote the SPE profit of firm 0,*

$$\pi^{SPE} = \frac{X\left(p_0^*\right)}{n\left(p_0^*\right)}\left(p_0^* - c_0\right)$$

and $n = n\left(p_0^*\right)$ the number of active firms in the SPE in theorem 1. If the probability that the customer arrives is small enough, in particular if

$$q < \frac{\gamma}{(n-1)\,\pi^{SPE}} \tag{2}$$

then p_0^* is a SPE price in the game described above.

The reason why q needs to be small is intuitively clear. If q is close to 1, the probability that the one stage deviation leads to actual sales (because the customer arrives) is high. The additional profit gained from this deviation is of the order $(n-1)\,\pi^{SPE}$ as the firm captures the market share of its $(n-1)$ opponents which are still pricing at p_0^*. Also note that if the number of firms in the market is large, the probability that a deviation leads to actual sales must be very low. The reason is that deviating is very attractive as n is large. Finally, if inequality (2) does not hold, then the fact that p_0^* cannot be sustained, does not imply that the Bertrand outcome ($p = c_1$) is the SPE. There may be a price $p \in \langle c_1, p_0^* \rangle$ that is sustainable as a SPE.

In which markets can one expect inequality (2) to hold? As mentioned in the introduction, I am mainly thinking of retail markets. Consider a High

[13] Above I have ignored the possibility that p_0^* in notation 1 is a set consisting of more than one element. The following argument shows that for γ small enough, the only equilibrium price sustainable in the model here is the smallest element of p_0^*. Consider two elements p^* and p^{**} in p_0^* where $p^{**} > p^*$. Then firms with $c_j < p^*$ will quote a price equal to p^*. Suppose not, that is assume all firms quote a price equal to p^{**}. Then firm **0** has an incentive to undercut its opponents in the next round to p^*. With a probability q firm **0** gains the whole market and with a probability $(1-q)$ it does not lose profits because p^* and p^{**} yield (by assumption) the same profit level. Hence p^{**} is not sustainable as equilibrium price if γ is small.

Street shop selling television sets. The probability that in the next hour, say, a customer walks in who wants to buy a specific TV (of a given brand and type) is small. So changing your price of this specific TV now is likely to be noticed by other TV sellers down High Street before a customer arrives buying that TV. Also, changing the price is costly as catalogues need to be adjusted etc.

Comparing internet firms with traditional retailers, Bailey (1998) finds that prices for books, CDs and software are higher on the internet than for traditional retailers. Using the result in proposition 2 one can give the following explanation for this finding. Assuming that the cost of changing prices is similar for these two types of firm, the main difference is the probability q above. For traditional retailers it is harder to figure out out what prices other retailers are charging and hence it is more likely that a price change goes unnoticed before a customer arrives. For an internet firm, in contrast, it is easy to check opponents' prices (one mouseclick) and prices can be changed very quickly. That is, q is lower for internet firms. Hence proposition 2 predicts that prices are higher for internet firms as it is more likely that inequality (2) holds.

Having introduced the probability q that the customer arrives after a given pricing round, there are also other ways in which the SPE in theorem 1 can be maintained, apart from the menu cost γ. One is to introduce the idea of inertia as used in continuous time games[14] (see, for instance, Bergin and MacLeod (1993)). This can be applied to the situation here by assuming that prices are fixed for $\delta \geq 2$ periods. So starting from the equilibrium price p_0^*, the question is whether a firm i would like to deviate to a price $p < p_0^*$. It is clear to see that for δ big enough, the optimal response of i's opponents is to follow the price reduction because if they did not follow the price reduction, there is a probability $1-(1-q)^{\delta-1}$ that the customer arrives before the price level returns to p_0^*. Hence, although prices can be adjusted both upward and downward, inertia gives the same reaction to a deviation as the auction pricing game: price reductions are followed by all firms with marginal cost levels below the price. The idea that a firm is committed to its price for more than one period is also used in Maskin and Tirole (1988) as discussed below.

4 Implications for competition policy: joint dominance and entry

This section derives a number of properties of the SPE. In particular, I derive conditions under which cost reductions and entry lead to increases in the equilibrium price.

Although the equilibrium price can rise with the number of firms in the industry and fall with firms' cost levels, it is always the case that the price leader is (weakly) better off with less opponents and with less efficient opponents.

[14] There is another relation of the game described above and continuous time games. Consider the case where firms know for sure that the customers arrive at time T. Then one can model the pricing game as one where firms can continuously adjust prices until time T. If changing prices is costless, the Bertrand game can be modelled as one where firms choose prices for every $t \in [0, T]$. That is, the only relevent price is the one quoted at $t = T$. The game in this paper is one where firms choose prices for every $t \in [0, T\rangle$. The open bracket implies that firms can react to every price change before the customers arrive at T. Although this is a neat mathematical characterization of the model above, I prefer the more straightforward interpretation using the probability q.

Proposition 3 *The profits of the price leader (firm* 0*) are non-increasing in the number of other firms* N *and is non-decreasing in the cost levels of the other firms in the industry.*

4.1 Effect of cost reductions on price

Usually we think that a reduction in a firm's cost level never leads to an increase in the equilibrium price in the market. Indeed this can never happen in a Cournot or Bertrand model. However, in the model here, we can have that a fall in production costs for a firm leads to a rise in the equilibrium price.

Consider two prices p and p' with $p' < p \leq p^m$. Let $n = n(p), n' = n(p')$ and $\nu = n - n' \geq 0$. Then the difference in profits for firm 0 between charging p and p' can be written as

$$\Delta(p, p'; c_0) \equiv \frac{1}{n}\left[X(p)(p - c_0) - \frac{n' + \nu}{n'}X(p')(p' - c_0)\right] \tag{3}$$

Proposition 4 *A cost reduction makes* p' *relatively more profitable compared to* $p > p'$ *if*
(i) $c_j \geq p$ *is reduced to* $c_j \in \langle p', p\rangle$ *or*
(ii) $c_j \in [c_0, p'\rangle$ *is reduced to* $c_j < c_0$.
A cost reduction makes p *relatively more profitable compared to* p' *if*
(iii) $c_j \geq p'$ *is reduced to* $c_j \in \langle c_0, p'\rangle$.
Finally, if $c_j > p'$ *is reduced to* $c_j < c_0$ *the effect on the relative profitability of* p *and* p' *is ambiguous.*

Note that p can equal the SPE price before the cost reduction and p' after the cost reduction, but the argument does not depend on this. In fact, the proposition holds for any two prices with $p' < p$. The intuition for these results is as follows. In case (i) a price reduction from p to p' gets rid of an additional firm after the cost reduction. That makes the price reduction relatively more attractive for the priceleader after the cost reduction. In the second case the cost level of the price leader goes down (and the identity of the price leader may or may not change). This also implies that the lower price p' becomes relatively more attractive for the (new) price leader since lower costs make higher sales more attractive.

The third case consists of two subcases. First, consider a firm with $c_j \geq p$ that gets an efficiency gain such that now $c_j \in \langle c_0, p'\rangle$. This implies that the cost reduction has no effect on the number of firms ν pushed out of the market by the price reduction from p to p'. However, the number of firms that the market has to be shared with at p' is now $n' + 1$ instead of n'. This makes the price reduction relatively less profitable. Put more starkly, a given price reduction that removes 10 firms from the market is relatively more profitable if the number of firms before the price cut equals 11 instead of 110. In the second subcase of (iii), $c_j \in [p', p\rangle$ is reduced to $c_j \in \langle c_0, p'\rangle$. That implies that in addition to the effect described above, the price reduction now removes only $\nu - 1$ firms from the market instead of ν. Thus also in this case, the cost reduction makes the higher price p relatively more profitable than the low price p'.

Finally, a cost reduction that replaces the current price leader with a price leader with lower costs but at the same time increases n' and may reduce ν

(if $c_j \in [p', p\rangle$ before the cost reduction) has ambiguous effects on the relative profitability of p and p'. The reason is that this cost reduction can be seen as the 'sum' of cost reductions (ii) and (iii). Reducing the cost level of the price leader makes lower prices relatively more profitable for the price leader. At the same time increasing the number of firms with which the market has to be shared at the low price p' (and reducing the number of firms removed from the market by reducing the price from p to p') makes the high price relatively more attractive for the price leader. Hence the overall effect is ambiguous here.

In other words, a cost reduction only unambigously reduces the price if a relatively high cost level is reduced to a level slightly below p or if a rather low cost level is reduced further.

Proposition 4 provides a new formalization of the joint dominance concept as it is called in EU merger policy, or coordinated effects as it is called in the US. The idea is the following. There is an industry with three firms. One firm is efficient, the other two are rather inefficient. Now the two inefficient firms merge and through economies of scale and scope this merged entity gains in efficiency. Although efficiency gains are generally seen as welfare enhancing, the worry here is that the two low cost firms will be able to charge a high price. Indeed this is exactly the implication of proposition 4. If a high cost level is reduced to a level close to c_0, the price tends to go up.

4.2 Effect of entry on price

As above, compare again the relative profitability of two prices p and p' with $p' < p$ using the expression in equation (3). Now I consider entry by a new firm, not a cost reduction by an existing firm. Let c_e denote the cost level of the entrant.

Proposition 5 *If $c_e \geq p$ then entry has no effect on the relative profitability of p and p';*
if $c_e \in [p', p\rangle$ then entry makes p' relatively more profitable;
if $c_e \in [c_0, p'\rangle$ then entry makes p relatively more profitable;
if $c_e < c_0$ then the effect of entry on the relative profitability of p and p' is ambiguous.

The intuition for these effects is as follows. Entry above an equilibrium price has no effect since price increases become less attractive than before and the attractiveness of price reductions is unchanged. Entry just below price p makes a price reduction more attractive since such a reduction now removes an additional firm from the market. However, entry at a cost level close to the price leader makes price reductions less attractive for the same reason as in case (iii) in proposition 4. Finally, entry by a firm that becomes the new price leader has an ambiguous effect on the price level. On the one hand, a more efficient price leader prefers lower prices. On the other hand, such entry creates more firms with low cost levels making price reductions less attractive.

In section 2.4.2 above, I have already shown that it is possible to construct examples in which entry indeed causes a rise in the equilibrium price level. But one may wonder whether this is just a theoretical possibility or an effect that has practical relevance. I give the following two arguments to show that this is more than a theoretical possibility. In section 6, I discuss empirical evidence suggesting that this effect can be relevant. Here I use simulations to show that it is not unlikely that entry raises the equilibrium price.

Figure 2 summarizes the results of the following simulation exercise. Consider an industry with demand of the form $X(p) = 1 - p$ and where firms' cost levels c_i are drawn from a lognormal distribution with expectation $E(c_i) = e^{-0.75} \approx 0.47$, variance $V(c_i) = e^{-1.5}\left(e^{0.5} - 1\right) \approx 0.14$ and mode equal to $e^{-1.5} \approx 0.22$.[15,16] Now consider the following experiment. First, draw N cost levels from this distribution and calculate the SPE price p_0^*. Next draw an additional firm $N+1$ from the distribution and calculate the new SPE price and see whether the price went up or down due to entry. This experiment is done 100,000 times for each $N \in \{2, 3, ..., 100\}$. Figure 2 focuses on the cases where the cost level of firm $N+1$ is below the price before entry. That is, it considers the cases where entry is relevant. Or, put differently, on the cases where entry has actually an effect on the industry since it follows from proposition 5 that entry by a firm with costs above the equilibrium price has no effect on the equilibrium price. For these cases where entry does have an effect,[17] figure 2 reports for each N the percentage of industries where the price increases and the percentage where the price decreases. Although price decreases are more prevalent, in around 10% of the industries there is a price increase due to entry. This is an indication that this effect may be relevant empirically. Also note that for N above 70 price increases due to entry become less likely. Finally, figure 2 shows that in more than 30% of the cases entry has no effect on the equilibrium price at all. I come back to this below.

`Figure 2 around here`

5 Discussion and extensions

This section discusses the following three issues: price competition with entry costs, rationing rules and the relation between the model introduced here and the literature on dynamic price competition as in Maskin and Tirole (1988) and supergames.

5.1 Entry cost

Introducing sunk entry costs in a Bertrand model with homogenous goods and constant marginal costs gives rise to the following counterintuitive result. No matter how small the entry cost, as long as it is positive, the Bertrand equilibrium outcome is that the most efficient firm is a monopolist and sells at

[15] Put differently, $\ln(c_i)$ is normally distributed with expectation -1 and variance 0.5.

[16] Unfortunately, I am not aware of any empirical evidence on the issue of cost distributions in industries. In public finance it is more or less accepted that skills or wages follow a lognormal distribution (and perhaps a Pareto distribution for the high end tail of the distribution). In I.O., size distributions of firms can be approximated with a lognormal distribution. But there seems to be no corresponding evidence for firms' efficiency distributions. I have tried a number of distributions, but the main effect (that price increases happen in a reasonable number of cases) remains robust. As proposition 5 suggests, this effect is more likely if the entering firm has a cost level in between the cost level of the price leader and the equilibrium price before entry. So with the lognormal distribution used, price increases become less prevalent if one imposes that the price leader has a cost level equal to 0. In that case the distribution has very little mass around c_0 (implying, in fact, that $c_0 = 0$ is very unlikely).

[17] On average, this happens for about $10,000$ out of $100,000$ simulations.

the monopoly price. Once another firm decides to enter, Bertrand competition makes sure that its profits are zero and hence it cannot recover its entry costs. Contestable market theory (Baumol et al. (1982)) can be seen as a reaction to this outcome by arguing that prices should be expected to be closer to marginal costs than predicted by the Bertrand model. The model introduced here gives a similar result.

Consider the model with menu costs γ as introduced in section 3.2. Let f denote the cost of entry into this industry. Assume that in the model without entry costs the price chosen by the price leader equals $p_0^* = c_i < p^m$, that is firm i is (just) priced out of the market. Further assume that at this price p_0^* firm $i-1$ can profitably enter the market and post its price, that is suppose

$$\frac{X\left(p_0^*\right)}{i}\left(p_0^* - c_{i-1}\right) \geq f + \gamma$$

Proposition 6 *Let p^* solve the following equation*

$$p^* = 1 + c_i + \frac{f+\gamma}{qX\left(p^* - 1\right)}$$

Then p^* is a SPE price in this industry.

First note that p^* is not the unique SPE price. In proposition 2 (with $f = 0$) I have shown that p_0^* itself is also a SPE price. The point here is that a higher price p^* can be sustained in this case. The price p^* is defined in such a way that if firm i undercuts p^* (with 1 because $p \in \aleph$ by assumption) this yields not enough profits to earn (strictly) more than $f + \gamma$. With probability q firm i captures the whole market by entering (if the customer happens to arrive at that time) and with probability $(1-q)$ firm 0 reduces its price to p_0^*. This price reduction by firm 0 is credible because p_0^* is -by definition- the SPE price in the game where firm i is present as well (i.e. the game where $f = \gamma = 0$).

So the model above predicts that a rise in entry costs f increases the equilibrium price but not (necessarily) to the monopoly price as in the Bertrand game. Also, the higher q (the more likely it is that the customer arrives before the incumbents can react to entry) the lower the mark up over c_i that can be sustained in equilibrium because hit-and-run behavior by firm i becomes more attractive in this case.

5.2 Rationing rules

Above I have assumed that when n sellers are willing to sell at the lowest price, each seller serves $\frac{1}{n}$th of the market. Although this assumption is reasonable in an I.O. context and more or less standard, one can ask whether the results derived above critically depend on this assumption.

So if there are n firms willing to sell at the lowest price, one can define a more general rationing rule as follows. Firm i gets a market share β_i where the β_i's satisfy

$$\begin{aligned} \beta_i &\geq 0 \text{ for all } i \in \{0, 1, ..., n-1\} \qquad (4) \\ \sum_{i=0}^{n-1} \beta_i &= 1 \end{aligned}$$

For instance, Cournot competition in a homogenous good market where firms differ in efficiency yields an outcome where all firms sell at the same price but with different market shares. The question above can now be phrased as follows: Can we have an equilibrium with $p > c_1$ for all rationing rules satisfying (4)? Clearly, the answer is "no". Consider the following two extreme cases (i) $\beta_0 = 0$ and (ii) $\beta_0 = 1$. In the first case, the most efficient firm will never want to share the market with another firm and the SPE price equals the Bertrand price ($p = c_1$). The second case with efficient rationing is more interesting. Here the other firms will never want to share the market. There is always a firm $i \geq 1$ that undercuts any price $p > c_1$. In other words, although it seems advantegous for firm 0 to have a lot of market power ex post (in the sense that β_0 is high), ex ante firm 0 would like to commit to a lower market share. In particular, if firm 0's profit maximizing price in lemma 2 satisfies $p_0^* = c_i > c_1$ then firm i is better off with a rationing rule $\beta_j = \frac{1}{i}$ for all $j \in \{0, 1, ..., i-1\}$ than with $\beta_0 = 1$.

Although there are rationing rules that destroy any SPE price above the Bertrand price, I now show that the results above do not depend on the equal sharing rationing rule. In particular, I show that for every subgame equilibrium with equal market shares where the price is above the Bertrand price, there exists a subgame equilibrium where firms have different market shares at the same market price.

I focus in the remainder on rationing rules which imply higher market shares for more efficient firms, that is

$$\beta_0 \geq \beta_1 \geq ... \geq \beta_n$$

which seems a reasonable assumption. The following result formalizes the idea that the results derived in this paper do not hinge on the equal sharing assumption.

Lemma 3 *Assume that $c_j > c_{j-1}$ for all $j \geq 1$.*[18] *If it is the case that*

$$\frac{X(c_i)}{i}(c_i - c_0) > \frac{X(c_k)}{k}(c_k - c_0) \text{ for all } k \neq i$$

then there exist functions $\beta_j(i)$ satisfying

$$\begin{aligned} \beta_0(i) &> \beta_1(i) > ... > \beta_{n-1}(i) > 0 \qquad (5) \\ \beta_j(i) &= 0 \text{ for all } j \geq i \\ \sum_{j=0}^{i-1} \beta_j(i) &= 1 \end{aligned}$$

such that $p = c_i$ is the equilibrium price under this rationing rule.

Note that a rationing rule $\beta_j(n)$ has two arguments: the position of the firm (j) and the number of firms in the market (n). The idea of the proof is to construct rationing rules $\beta_j(n)$ close enough to the equal sharing rule that all inequalities continue to hold for the equilibrium $p = c_i$. Since the equilibrium is defined with strict inequalities, this is always possible.

[18] This assumption on costlevels is mainly made for notational convenience as presumably one would like $c_i = c_j$ ($i \neq j$) to imply $\beta_i = \beta_j$.

The introduction of $\beta_j(n)$ gives an exogenous solution to the equal market share result. The next example illustrates how differing market shares can be obtained endogenously by moving away from constant marginal costs. An analysis of general cost functions $c_i(x)$ is beyond the scope of this paper and left for future research.

Example 3 *Consider an industry with $X(p) = 1 - p$ and firms' costs of the form*

$$c_i(x) = c_i x + \frac{1}{2}\phi x^2$$

with $c_0 \leq c_1 \leq ...$ and where x is the output level of firm i. Quoting a price in the pricing game of section 2 now implies a commitment by these firms to collectively serve all the market at this price. Hence a price follower f will set his output level to equate marginal costs with marginal revenue, $c'_f(x_f) = p$ and hence one finds

$$x_f(p) = \begin{cases} \frac{p-c_f}{\phi} & \text{if } p > c_f \\ 0 & \text{if } p \leq c_f \end{cases}$$

A price leader l sets the price p taking these supply decisions into account, i.e. firm l solves

$$\max_p \left(1 - p - \sum_f x_f(p)\right)(p - c_l)$$

It is straightforward to see that firm 0 as price leader sets the lowest price p_0^* of all firms. Hence, as before, no firm wants to undercut p_0^*. Setting a price above p_0^* is here equivalent to leaving the market as the firms committed to p_0^* serve the whole market at this price. Hence one gets similar results as above, but now with endogenously determined market shares.

5.3 Relation with other models of dynamic price competition

The analysis in this paper is related to two strands of dynamic pricing games: the model by Maskin and Tirole (1988) (henceforth MT) and the supergame literature. The main differences between my paper and these approaches is that (1) here the game is one-shot in terms of sales and (2) I allow firms (with some probability) to react to other firms' price changes before the customer arrives. It is worth spelling out what this implies.

The main reason why the game above is one-shot in sales is to stress that repeat sales are not necessary to get a price above the Bertrand outcome. The MT model was designed to analyze price dynamics, not comparative statics with respect to number of firms or firms' efficiency levels. Therefore MT focus on the case of symmetric duopoly. As it is unlikely that meaningful analytical results can be derived on industry structure in the MT model, my model is a lot simpler than MT. Moreover, it is not clear how MT should be extended to more than two firms. In MT firms change prices in an alternating way (firm 1 changing its price in even periods, firm 2 in odd periods). This makes sense with two firms, because the main reason why a firm may want to update its price is that the other firm made a price change. However, when there are five firms it is not convincing to argue that each firm can only adjust its price once every five periods. If one of the five firms undercuts the others in a certain period, it is reasonable to expect that more than one firm will want to react by adjusting its price. But if one allows more than one firm to adjust its price in a period, the equilibrium outcome is again Bertrand.[19]

The supergame literature (see, for instance, Green and Porter (1984), Rotemberg and Saloner (1986) and Abreu (1986)) uses the infinite repetition of the stage (pricing) game to derive the result that prices above the Bertrand price can be sustained. Recently Compte et. al. (2002), Motta (2002) and Vasconcelos (2002) have extended this literature by allowing for asymmetries between firms. Goal of these papers is to formalize the joint dominance argument discussed above. Hence their results are similar to the one obtained here in that more similarity in firms' cost levels leads to higher prices.

Yet, there are a number of differences. First, these papers focus on mergers and thus do not analyze the effects of entry. Second, in the supergame literature if the (expected) duration between sales increases, it becomes harder to discipline firms and the (collusion) price decreases. Here, a longer (expected) duration between sales makes it more likely that firms can react to each other's price changes before the customer arrives (q is lower in section 3) and hence prices tend to be higher. Third, in the supergame literature if the discount factor is close enough to 1 (one) the monopoly price is sustainable (perfect collusion). In section 2 above, the discount factor is 1 and yet p_0^* can be below the monopoly price. The reason is that the supergame literature assumes that firms can decide on the distribution of market shares while here I assume that with n firms selling in the market, each firm's market share is $\frac{1}{n}$. If firms can check each other's production capacity or sales, they can verify which firms stick to the agreement. If this is not feasible, the assumption made in my paper (of an exogenously given rationing rule) becomes more natural. Finally, a disadvantage of the supergame approach is the multiplicity of equilibria, while the game analyzed here has (under some conditions) a unique equilibrium.

6 Empirical implications

The results derived above that cost reductions or entry can lead to higher equilibrium prices are in such contrast to conventional wisdom that one can ask whether the theory presented here has any empirical relevance. This section argues that the implications of the standard models that entry and efficiency gains reduce prices are not as clearly supported by the data than is sometimes assumed. That is not to say that these empirical studies confirm the theory presented here. Indeed there may be other explanations for these results. It does show, however, that the results presented here are not flatly contradicted by the data. Further, I explain how the model here may be helpful in understanding some of these results. The section concludes with a direct test that can be applied to firm level data to see whether the theory presented here is relevant in a certain sector.

[19] MT also consider the case where firm's commitment duration to a price is a random variable (with an exponential distribution). But, as they note, this is less reasonable with commitment to a price than with commitment to a quantity or capacity level. The problem is that it is hard to understand why a firm would stay committed to its price after its opponents have priced it out of the market.

6.1 Empirical findings

First, consider the effect of cost reductions on the equilibrium price. Whereas Cournot competition predicts that cost reductions lead to lower prices this is not always found in empirical research. I discuss here the effects of changes in excise taxes and changes in the exchange rate on the equilibrium price. Ashenfelter and Sullivan (1987) analyze the effects of changes in the excise tax on cigarettes on the cigarette price. They write "Our failure to uniformly find that excise tax increases (decreases) act to increase (decrease) cigarette prices ... calls into question virtually any hypothesis about firm behavior in this industry". Indeed, the standard Cournot and Bertrand models can shed no light on such findings. However, the model introduced here can give intuition for such results. Rewriting equation (3) to introduce excise taxes, one gets the following comparison between two prices p and p' with p the equilibrium price before the rise in excise tax t and p' an arbitrary price below p:

$$\Delta\left(p,p';t\right) \equiv \frac{X\left(p+t\right)}{n}\left(p-c_0\right) - \frac{X\left(p'+t\right)}{n'}\left(p'-c_0\right)$$

The following result gives a sufficient condition for the rise in t to make the lower price p' relatively more profitable compared to p.

Corollary 2 *If $X''(p) \geq 0$ for all $p \geq 0$ then $\frac{d\Delta\left(p,p';t\right)}{dt} < 0$.*

The idea of the proof is that choking off demand (by raising t) is particularly bad for profits if the price cost margin is high. The assumption that the demand function is convex excludes the case where high prices leads to smaller losses in output due to an increase in the price by dt.

Next consider the case where an exchange rate change makes the products of foreign firms more expensive on the domestic market. Goldberg and Knetter (1997) give the following example. Between 1994 and 1995 there was a 34% appreciation of the yen against the dollar, making Japanese products more expensive in the US. In the same period, the suggested retail price of a large screen Sony Trinitron actually fell (by 15%). Although one would expect Japanese firms to try to keep their dollar prices constant, it is seems counterintuitive that such an appreciation of the yen leads to a fall in dollar prices. However, if one assumes that these exporters are price leaders in their export markets, we find the following result. Let e denote the exchange rate in dollars per yen, then a Japanese price leader charging a dollar price p receives a yen price equal to $\frac{p}{e}$. Now the comparison between the optimal price p before appreciation of the yen and a price $p' < p$ becomes

$$\Delta\left(p,p';e\right) \equiv \frac{X\left(p\right)}{n}\left(\frac{p}{e}-c_0\right) - \frac{X\left(p'\right)}{n'}\left(\frac{p'}{e}-c_0\right)$$

The following result shows that depending on the cost distribution and the relation between total revenue $pX\left(p\right)$ and p, an appreciation can make a lower dollar price more attractive

Corollary 3 *If revenue per firm is higher at price p than at p' then $\frac{d\Delta\left(p,p';e\right)}{de} < 0$.*

Another issue in international I.O. is that some exporting industries keep their prices (more or less) constant in the currency of the goal country while others vary their prices with exchange rate changes (see the survey by Goldberg and Knetter (1997) for references). The theory presented here suggests that in the former case, the exporting firms are faced with a domestic (in the goal market) price leader that prices to keep other domestic firms out of the market. Hence p_0^* does not vary with the exchange rate. In the latter case, one of the exporters may be the price leader. As the corollary shows this may imply changing your price in response to exchange rate changes.

Next consider the effect of entry on the equilibrium price. Both Cournot and Bertrand models predict that entry by a firm with a cost level below the industry price will reduce the equilibrium price. However, empirical research has not found such a clear relationship. Consider the following examples. Knetter (1994) tests the following model due to Dixit (1989). The large dollar appreciation of the 1980s made foreign producers' goods a lot cheaper in the US. Hence, one would expect foreign firms to make the sunk investment of setting up an export line to the US in this period. This implies an increase in the number of firms in the US market. Since this entry cost is sunk one expects these firms to stay even when the dollar depreciates. The hypothesis then is that prices in the US market are lower after this appreciation due to entry by foreign firms. Overall Knetter finds only weak evidence in favor of this hypothesis. Moreover, he finds that for some sectors mark-ups actually went up.

Geroski (1989) estimating the effect of entry on price cost margins concludes that "entry shows an enormous amount of variation across industries and over time, while margins are rather predictable over time for any given industry". This is hard to understand within a Cournot or Bertrand framework. However, as the simulations in figure 2 suggest, in the model here entry does not reduce prices in 40% to 50% of the cases and may actually raise the price in some cases. This may help explain explain why the statistical link between entry and price cost margins is rather weak.

Klette (1999) introduces a new econometric framework to estimate simultaneously the variables price cost margin, scale economies and productivity. He finds no significant relationship between markups and industry concentration nor between mark ups and import penetration. Similarly, Lindquist (2001), analyzing the Norwegian aluminium industry, finds that although the international aluminium market has become more competitive over time price cost margins have not fallen. Such results are in line with the older literature based on the structure-conduct-performance framework (see, for instance, Ravencraft (1983) and Schmalensee (1985)). After controling for other effects, there was no separate role left for industry concentration to explain price cost margins. These papers suggest that there is hardly an effect of entry on equilibrium prices. This is not to say that entry never reduces prices. For instance, Bresnahan and Reiss (1990 and 1991) do find a negative effect of entry on price cost margins. However, generally speaking, the effect of entry on price cost margins is a lot smaller than suggested by standard I.O. models. The analysis above suggests that besides the average cost level of firms, the variance in cost levels may be an important determinant of the industry price. I am not aware of any empirical study relating variance in costs to prices.

Finally, there is a recent literature relating market size to price cost margins and firm size. For instance, Campbell and Hopenhayn (2002) find that larger cities have (on average) larger retail establishments and lower prices. They argue that this result is hard to understand with standard I.O. models. To analyze this effect in the framework here, let S denote the size of the market. Then total demand at price p equals $SX(p)$. Substituting this into the equation for the equilibrium price in proposition 6 (the model with the entry cost f) one finds immediately that larger markets lead to lower prices and hence bigger establishments (note that equation (1) implies that S does not affect p_0^*).

The goal of this section has been to illustrate that the surprising comparative statics properties of the SPE outcome may not just be theoretical curiosities. Results that entry or efficiency gains may lead to higher prices can help to understand some empirical findings that are hard to fathom with standard models.

6.2 Empirical test

This section derives an empirical test to see whether the theory presented here applies in a certain industry. More precisely, it gives a necessary condition for the theory to be relevant in a certain industry. The idea behind the test is that in the SPE the gaps between firms j's and $j+1$'s costs cannot be too big. If the gap is big, it would have been better for firm 0 to lower the price to c_{j+1} to keep firm $j+1$ out of the market.

Equation (1) implies the following upperbounds on the average cost level and the variance of cost levels in an industry.

Corollary 4 *The SPE implies that*

$$\begin{aligned}\bar{c}_n &< c_0 + \frac{1}{2}\left(1 - \frac{1}{n}\right)(p - c_0) \\ var(c_i) &< \frac{1}{3}\left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{2n}\right)(p - c_0)^2\end{aligned}$$

where $\bar{c}_n = \frac{1}{n}\sum_{j=0}^{n-1} c_j$ *and* $var(c_i) = \frac{1}{n}\sum_{j=0}^{n-1}(c_j - \bar{c}_n)^2$.

Note that no information is needed here about demand function $X(.)$. The upperbound is derived using that $\frac{X(p)}{X(p')} \leq 1$ for $p' < p$. If information about $X(.)$ is available, the upperbound can be tightened. Although marginal costs are not easily observable in the data, under the assumption of constant marginal costs (made here) average variable costs should give a good approximation. Finally, it is clear that such upperbounds need not hold for Cournot competition.

7 Conclusion

In this paper, I have analyzed two ideas. First, the cost distribution in an industry determines how aggressive firms play. If firms have similar cost levels, there is a balance of power, and firms behave nicely toward each other. If one firm (or group of firms) is far more efficient than the others, then it will fight to keep the inefficient firms out of the market. Second, this be-nice-unless-it-pays-to-fight idea has been formalized by assuming that firms can (with some probability) react to other firms' price changes before the customer arrives.

This leads to a number of implications for competition policy. First, repeat sales are not necessary for firms to sustain prices above the Bertrand outcome.

Hence, in a merger case one should not only check whether there are factors fascilitating collusion (like repeat sales or multimarket contact) but also whether firms in an industry can react to each others' actions before the customer arrives. The second implication is that if firms can react in such a way, then one cannot be sure that entry or efficiency gains by laggards in the industry will discipline incumbents to charge low prices. Third, if a merger makes firms in an industry more symmetric then one should expect higher prices. This is the way I have formalized joint dominance in this paper. Finally, in such a joint dominance case there will be no evidence of explicit or implicit collusion, because each firm acts independently and noncooperatively. Hence, this paper suggests, the merger should be abolished. In other words, in such a case the model does not recommend to let the merger go through and only intervene ex post if evidence of collusion is found.

Although the theory is formulated here in terms of a pricing game, the underlying mechanism of balance of power is more widely applicable. In all situations where aggressive play is not immediately rewarded and hence leaves time for opponents to react before payoffs are realized, it is not obvious that more players or better players leads to a more aggressive outcome. Other examples that one can think of in economics are advertising and R&D races.

References

Abreu, D., (1986), Extremal equilibria of oligopolistic supergames, *Journal of Economic Theory*, 39: 191-225.

Amir, R. and V. Lambson, (2000), On the effects of entry in Cournot markets, *Review of Economic Studies*, Vol. 67 (2): 235-254.

Ashenfelter, O. and D. Sullivan, (1987), Nonparametric tests of market structure: an application to the cigarette industry, *Journal of Industrial Economics*, Vol. XXXV (4), 483-498.

Bailey, J. (1998), Electronic commerce: prices and consumer issues for three products: books, compact discs and software, OECD.

Baumol, W., J. Panzar and R. Willig, (1982), *Contestable markets and the theory of industry structure*, New York: Harcourt Brace Jovanovich.

Bergin, J. and B. MacLeod, (1993), 'Continuous time repeated games', International Economic Review, Vol. 34 (1), pp 21-37.

Bresnahan, T., (1981), Duopoly models with consistent conjectures, *American Economic Review*, Vol. 71 (5), 934-945.

Bresnahan, T. and P. Reiss, (1990), Entry in monopoly markets, *Review of Economic Studies*, Vol. 57, 531-553.

Bresnahan, T. and P. Reiss, (1991), Entry and competition in concentrated markets, *Journal of Political Economy*, Vol. 99 (5), 977-1009.

Bulow, J. and P. Klemperer, (1999), Prices and the winner's curse, *RAND journal of Economics*, Vol. 33 (1), 1-21.

Campbell, J. and H. Hopenhayn, (2002), Market size matters, NBER working paper 9113.

Chamberlin, E., (1969), *The theory of monopolistic competition: A reorientation of the theory of value*, Cambridge, Mass.: Harvard University Press.

Compte, O., F. Jenny and P. Rey, (2002), Capacity constraints, mergers and collusion, *European Economic Review*, Vol. 46, 1-29.

Deneckere, R. and D. Kovenock, (1992), Price leadership, *Review of Economic Studies*, Vol. 59, 143-162.

Deneckere, R., D. Kovenock and R. Lee, (1992), A model of price leadership based on consumer loyalty, *Journal of Industrial Economics*, Vol. XL (2), 147-156.

Dixit, A. , (1989), 'Hysteresis, import penetration and exchange rate pass-through', Quarterly Journal of Economics, Vol. 104, pp. 205-228.

Farm, A. and J. Weibull, (1987), Perfectly flexible pricing in a homogenous market, *Scandinavian Journal of Economics*, 487-495.

Fisher, I., (1898), Cournot and Mathematical Economics, *Quarterly Journal of Economics*, Vol. XII (2): 119-138.

Fudenberg, D. and J. Tirole, (1991), *Game Theory*, MIT Press.

Geroski, P. A., (1989), The effect of entry on profit margins in the short and long run, *Annales d'Économie et de Statistique*, Vol. 15/16, 333-353.

Gilbert, R.J. and P. Klemperer, (2000), An equilibrium theory of rationing, *RAND Journal of Economics*, Vol. 31 (1): 1-21.

Goldberg, P. and M. Knetter, (1997), Goods prices and exchange rates: what have we learned?, *Journal of Economic Literature*, Vol. XXXV (3), 1243-1272.

Green, E. and R. Porter, (1984), Noncooperative collusion under imperfect information, *Econometrica* Vol. 52: 87-100.

Kirman, A. and N. Schueller, (1990), Price leadership and discrimination in the European car market, *Journal of Industrial Economics*, Vol. XXXIX (1), 69-91.

Klemperer, P., (2000), 'Why every economist should learn some auction theory', Invited Symposium paper for the World Congress of the Econometric Society, Seattle: www.nuff.ox.ac.uk/economics/people/klemperer.htm.

Klemperer, P. and M. Meyer, (1988), Consistent conjectures equilibria: a reformulation showing non-uniqueness, *Economic Letters*, Vol. 27: 111-115.

Klette, T.J., (1999), Market power, scale economies and productivity: estimates from a panel of establishment data, *Journal of Industrial Economics*, Vol. XLVII (4), 451-476.

Knetter, M., (1989), Price discrimination by U.S. and German exporters, *American Economic Review*, Vol. 79 (1), 198-210.

Knetter, M., (1994), Did the strong dollar increase competition in U.S. product markets?, *Review of Economics and Statistics*, 192-195.

Lindquist, K.G., (2001), The response by the Norwegian aluminium industry to changing market structure, *International Journal of Industrial Organization* Vol. 19, 79-98.

Maskin, E. and J. Tirole, (1988), 'A theory of dynamic oligopoly, II: price competition, kinked demand curves, and Edgeworth cycles, *Econometrica*, Vol. 56 (3): 571-599.

Motta, M. , (2002), Horizontal mergers, a manuscript chapter for the book *Competition Policy: Theory and Practice*.

Ravenscraft, D., (1983), Structure-profit relationships at the line of business and industry level, *Review of Economics and Statistics*, Vol. 65, 22-31.

Rosenthal, R., (1980), A model in which an increase in the number of sellers leads to a higher price, *Econometrica*, Vol. 48 (6): 1575-1579.

Rotemberg, J. and G. Saloner, (1986), A supergame-theoretic model of business cycles and price wars during booms, *American Economic Review* Vol 76: 390-407.

Rotemberg, J. and G. Saloner, (1990), Collusive price leadership, *Journal of Industrial Economics*, Vol. XXXIX (1), 93-111.

Scherer, F. and D. Ross, (1990), *Industrial market structure and economic performance*, Houghton Mifflin Company.

Schmalensee, R., (1985), Do markets differ much?, *American Economic Review*, Vol. 75 (3), 341-351.

Stiglitz, J., (1989), Imperfect information in the product market, In. R. Schmalensee and R. Willig, ed, *Handbook of Industrial Organization*, Volume I, Elsevier Science Publishers.

Tirole, J., (1988), *The theory of industrial organization*, MIT Press.

Vasconcelos, H., (2002), Tacit collusion, cost asymmetries and mergers, mimeo European University Institute.

8 Appendix: proof of results

This appendix contains the proofs of the results in the main text.

Proof of lemma 1

It is routine to verify that the following strategy profiles form a Nash equilibrium and that they yield the Bertrand outcome. Any player i with cost level $c_i > c_0$ stays in the pricing game at any price p strictly above c_i and leaves the pricing game if $p \leq c_i$. Further, such a player i undercuts any price p strictly above $c_i + 1$ to $c_i + 1$. Any player j with $c_j = c_0$ stays in the pricing game at any price $p \geq c_j$ and leaves the pricing game at $p < c_j$. Further, such a player j bids the price $p = c_1$. Q.E.D.

Proof of lemma 2

This is proved by contradiction. Suppose that there exists $i > j$ (i.e. $c_i \geq c_j$) such that $p_i^* < p_j^*$ (and thus $n_i = n(p_i^*) < n(p_j^*) = n_j$). Then

$$\begin{aligned}
0 &> \frac{X(p_i^*)}{n_i}(p_i^* - c_j) - \frac{X(p_j^*)}{n_j}(p_j^* - c_j) \\
&= \frac{X(p_i^*)}{n_i}p_i^* - \frac{X(p_j^*)}{n_j}p_j^* - c_j\left(\frac{X(p_i^*)}{n_i} - \frac{X(p_j^*)}{n_j}\right) \\
&\overset{(*)}{\geq} \frac{X(p_i^*)}{n_i}p_i^* - \frac{X(p_j^*)}{n_j}p_j^* - c_i\left(\frac{X(p_i^*)}{n_i} - \frac{X(p_j^*)}{n_j}\right) \\
&= \frac{X(p_i^*)}{n_i}(p_i^* - c_i) - \frac{X(p_j^*)}{n_j}(p_j^* - c_i)
\end{aligned}$$

The first inequality follows from the fact that p_j^* is the price preferred by firm j. The second inequality (labelled $(*)$) follows from the observations that $c_i \geq c_j$ and that $p_i^* < p_j^*$ implies that $\frac{X(p_i^*)}{n_i} - \frac{X(p_j^*)}{n_j} > 0$. It follows that

$$\frac{X(p_j^*)}{n_j}(p_j^* - c_i) > \frac{X(p_i^*)}{n_i}(p_i^* - c_i)$$

contradicting the initial assumption that $p_i^* < p_j^*$. Q.E.D.

Proof of theorem 1

Consider a firm $i \in \{0, 1, ..., N\}$. Given the other firms' strategies, can this firm gain by a one stage deviation? First, by definition of p_i^* (and given the other firms' exit strategy) firm i cannot gain by quoting a price different from

p_i^*. It is straightforward to see thatr firm i will strictly lose profits of it quotes a price $p < p_0^*$. Further, unilaterally changing its exit strategy (either leaving at a price $p > c_i$ or staying in the market at a price $p < c_i$) does not raise i's profits and may reduce its profits. Since no firm can gain by a one stage deviation, the strategies form a SPE.

Finally, consider equation (1). The following reasoning shows that it cannot be optimal for firm 0 to choose a price $p \in \langle c_j, c_{j+1} \rangle$. By raising the the price to c_{j+1} the number of firms in the market is unchanged $(n(p) = n(c_{j+1}) = j + 1)$ while $X(c_{j+1})(c_{j+1} - c_0) > X(p)(p - c_0)$ by assumption 1. Q.E.D.

Proof of corollary 1

Since $\frac{X(c_{j+1})}{j+1}(c_{j+1} - c_0) > 0$, equation (1) implies that $p = c_0$ cannot be a SPE price. Q.E.D.

Proof of proposition 1

Suppose not, that is suppose there is a price $p' \neq p_0^*$ which is also a SPE price. This implies that at this price no firm can gain by a one stage deviation. So in particular, it must be the case that all firms with $c_i > p'$ have left the game (if not, they could avoid a negative profit by leaving). Next, consider a firm with $c_i < p'$. This firm must be active in the market. If not, it could gain (and would never lose) by staying in the market at the current price. In other words, for such a firm i to have left at a price $p' > c_i$ is a weakly dominated strategy (which is ruled out by assumption). Hence, it follows that the number of active players at this price p' equals

$$n(p') = \min\{j | c_j \geq p'\}$$

as defined in notation 1.

Given that at a price p' there are $n(p')$ firms active in the market, is it possible to rule out $p' < p_0^*$. First, consider the case where one firm, say firm j, reduces the price to p'. Then this firm j could do strictly better by not reducing the price. This can be seen as follows. Even firm 0 will not undercut the price $p' < p_0^*$ (because by assumption $\frac{X(c_i)}{i}(c_i - c_0)$ is increasing in i to the point where $c_i = p_0^*$). Hence firm j has to produce at the price p' and would have been strictly better off to produce at a price above p'. Hence one stage deviation rules out that one firm reduces the price to $p' < p_0^*$. Now consider the case where 2 or more firms simultaneously bid $p' < p_0^*$. This cannot be ruled out by one stage deviation. However, the same reasoning implies that these firms play weakly dominated strategies. They can never lose by not bidding p' (and may gain if the other firms refrain from bidding p' as well). Hence the assumption that players do not play weakly dominated strategies rules out $p' < p_0^*$.

Finally, consider $p' > p_0^*$. In this case it is always optimal for firm 0 to reduce the price to p_0^* (by definition of p_0^*). Hence a price $p' > p_0^*$ does not survive a one stage deviation by firm 0. What may stop firm 0 from reducing p' to p_0^* is the threat by another firm that such a price reduction will trigger a further price reduction to $p'' < p_0^*$. But as noted above, such a threat is not credible. Q.E.D.

Proof of proposition 2

Consider a one stage deviation by firm 0:

$$\pi^{dev} = qX(p_0^* - 1)(p_0^* - 1 - c_0) + (1 - q)\pi^{SPE} - \gamma$$

Hence firm 0 does not want to deviate (and therefore no other firm i wants to deviate either) if

$$\begin{aligned} \pi^{SPE} &> \pi^{dev} \\ &< \; => q\pi^{SPE} > qX(p_0^* - 1)(p_0^* - 1 - c_0) - \gamma \end{aligned}$$

Since $X(p_0^* - 1)(p_0^* - 1 - c_0) < n\pi^{SPE}$ a sufficient condition for this inequality to hold is

$$q < \frac{\gamma}{(n-1)\,\pi^{SPE}}$$

Q.E.D.

Proof proposition 3

Effect on firm 0's profits follows from a revealed preference argument. Any price firm 0 chooses after entry (or cost reduction by opponent), it could have chosen before but it did not. Moreover, any p is weakly less profitable after entry (or cost reduction) than before.Q.E.D.

Proof proposition 4

Note that $p' < p \leq p^m$ toghether with assumption 1 imply that

$$X(p)(p - c_0) > X(p')(p' - c_0)$$

(i) If $c_j \geq p$ is reduced to $c_j \in \langle p', p\rangle$, then ν goes up and hence $\Delta(p, p'; c_0)$ falls, making p' relatively more attractive compared to p.

(ii) Reducing c_0 while keeping n' and ν constant, one gets

$$\frac{d\Delta(p, p'; c_0)}{dc_0} = \frac{1}{n}\left[-X(p) + \frac{n' + \nu}{n'}X(p')\right] > 0$$

because $X(p') > X(p)$ and $\frac{n'+\nu}{n'} > 1$.

(iii) In this case, ν falls while n' rises. Hence $\Delta(p, p'; c_0)$ rises.

Finally, if $c_j > p'$ is reduced to $c_j < c_0$, then ν falls and n' rises tending to make p' less profitable compared to p. On the other hand, the cost level of the price leader falls, making p' relatively more profitable. Hence the overall effect is ambiguous. Q.E.D.

Proof of proposition 5

Entry with cost level above the higher price p has no effect on n' or ν and hence no effect on the relative profitability of p and p'.

If $c_e \in [p', p\rangle$ then ν goes up while n' is unchanged. This makes p' relatively more profitable than p.

If $c_e \in [c_0, p'\rangle$ then ν is unchanged while n' increases. This makes p relatively more profitable than p'.

Finally, if $c_e < c_0$ then there are two effects. On the one hand (as in proposition 4) the reduction in the cost level of the price leader makes p' relatively more profitable. On the other hand, entry with $c_e < c_0$ raises n' while ν is unchanged. This makes p relatively more profitable. Hence the overall effect is ambiguous. Q.E.D.

Proof of proposition 6

The price p^* is chosen such that firm i cannot make strictly positive profits by entering. Entry by firm i is only profitable at a price $p > c_i$. Once the firm has entered, it is optimal for firm 0 to reduce the price in the next period to $p_0^* = c_i$ by definition of p_0^* and because this price change is in fact profitable (because

$\frac{X(p_0^*)}{i}(p_0^* - c_{i-1}) \geq f + \gamma$ it follows that $\frac{X(p_0^*)}{i}(p_0^* - c_0) > \gamma$). Hence the only profit that firm i can make by entering is in the event where the customer arrives immediately after it enters. To maximize this profit, firm i quotes a price equal to $p^* - 1$. Hence i's expected profits from entering equal

$$\pi^e = q(p^* - 1 - c_i)X(p^* - 1) + (1 - q)0 - f - \gamma$$

Setting this equal to 0 yields the expression for p^* in the proposition. Q.E.D.

Proof of lemma 3

It is routine to verify that for ε close enough to 0 the following rationing rule

$$\beta_j^\varepsilon(n) = \begin{cases} \frac{1}{n} + \varepsilon\left(\frac{n-1}{2} - j\right) & \text{if} \quad j \leq n-1 \\ 0 & \text{if} \quad j \geq n \end{cases}$$

satisfies $\beta_j^\varepsilon(n) \geq 0$ for all $j \in \{0, 1, ..., n-1\}$, $\sum_{j=0}^{n-1} \beta_j^\varepsilon(n) = 1$ and does not violate the equilibrium inequalities at $p = c_i$. Q.E.D.

Proof of Corollary 2

Differentiating $\Delta(p, p'; t)$ with respect to t yields

$$\frac{d\Delta(p, p'; t)}{dt} = \frac{X'(p+t)}{n}(p - c_0) - \frac{X'(p'+t)}{n'}(p' - c_0)$$

Hence $\frac{d\Delta(p,p';t)}{dt} < 0$ if and only if

$$\left|\frac{X'(p+t)}{X'(p'+t)}\right| < \frac{\frac{p-c_0}{n}}{\frac{p'-c_0}{n'}} \tag{6}$$

The following proof by contradiction shows that the right hand side of this inequality is bigger than 1. Suppose not, that is suppose that $\frac{p-c_0}{n} < \frac{p'-c_0}{n'}$. Then $p' < p$ implies that

$$\frac{p - c_0}{n}X(p+t) < \frac{p' - c_0}{n'}X(p+t) < \frac{p' - c_0}{n'}X(p'+t)$$

which contradicts the fact that p is the optimal price (before t is increased).

Hence a sufficient condition for inequality (6) to hold is

$$\left|\frac{X'(p+t)}{X'(p'+t)}\right| \leq 1$$

which is satisfied if $X''(p) \geq 0$ for all p. Q.E.D.

Proof of Corollary 4

Suppose $p_0^* = c_n$ then equation (1) implies that

$$c_j - c_0 < \frac{j}{n}\frac{X(c_n)}{X(c_j)}(c_n - c_0)$$

for all $j < n$. Since $c_j < c_n$ implies that $X(c_j) > X(c_n)$, a necessary condition for this inequality to hold is

$$c_j - c_0 < \frac{j}{n}(c_n - c_0) \tag{7}$$

Summing over j (from $j = 0$ to $j = n - 1$), writing $p = c_n$ and dividing by the number of firms n yields

$$\bar{c}_n < c_0 + \frac{1}{2}\left(1 - \frac{1}{n}\right)(p - c_0)$$

To get the expression for the variance, substract $\bar{c}_n$ on both sides of (7). Since $\bar{c}_n \geq c_0$ a necessary condition for the resulting inequality to hold is

$$c_j - \bar{c}_n < \frac{j}{n}(p - c_0)$$

To get to the variance, note that this implies that

$$\frac{1}{n}\sum_{j=0}^{n-1}(c_j - \bar{c}_n)^2 < (p - c_0)^2 \frac{1}{n}\sum_{j=0}^{n-1}\left(\frac{j}{n}\right)^2$$

which can be written as the expression in the corollary. Q.E.D.

Figure 1: profits price leader

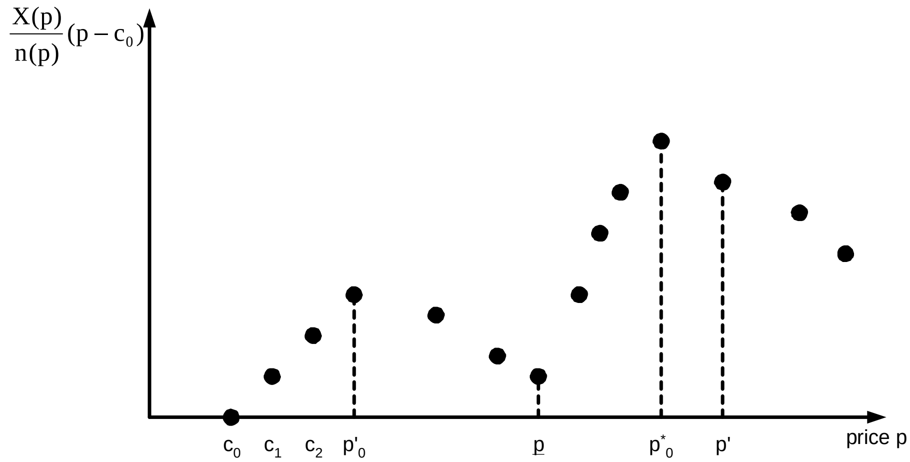

Figure 2: Effect of entry on price